

IE102 Spring 2017 Homework IV - Due: April 21, Friday 17:00

IMPORTANT: READ BEFORE YOU START WORKING ON THE HOMEWORK

Information on Homework IV

- Homework IV is 3%. Make sure that you review course material before you start. Cite sources used if any.
- Homework assignment will be conducted in groups of 2. We randomly form the groups. You can find your group member and his/her contact information at the course website. **If you cannot reach your group member by all means, indicate this on the cover page of your submission.**
- Grading Policy: The homework grades will not be only based on the correctness of the answers provided, but on the perceived degree of effort on the returned assignment. There exists no motive for the students to copy the answers from someone else, since even without obtaining the correct answers one can obtain full credit. The grading scale for each question is 2 points (full effort), 1 point (some effort), and 0 (no effort or no submission).
- Academic Dishonesty (taken from IE102 Course Information): **Academic misconduct is not tolerated, and violations will be treated in accordance with the university policy.** For assignments: Discussion of the course material and working in groups are effective ways of learning. However, this does not mean that you can copy the assignment from one of your friends. You can discuss with your friends, but you should be alone, without any written solution in front of you, when you actually prepare your assignment. It is extremely important to understand the value of this exercise in the process of learning. Any violation of the above understanding will result measures outlined in the student rules and regulations.
- Submission Information: An assignment submission box will be placed in front of office EA327 on the morning of the submission date and will be removed at the time of the deadline. You need to put your assignment in the designated box before the submission deadline. **Submission via e-mail is NOT accepted.**

Contact TA Eren Özbay if you have questions related to this HW: eren.ozbay@bilkent.edu.tr

There will be an office hour prior to the submission deadline.

Question 1:

Sunco Oil manufactures three types of gasoline (gas 1, 2, and 3). Each type is produced by blending three types of crude oil (crude 1, crude 2, and crude 3). Sunco can purchase up to 5000 barrels of each type of crude daily.

The selling price per barrel of gasoline and the purchase price per barrel of crude oil are given below:

Selling Price per Barrel		Purchase Price per Barrel	
Gas 1	\$70	Crude 1	\$45
Gas 2	\$60	Crude 2	\$35
Gas 3	\$50	Crude 3	\$25

The three types of gasoline differ in their octane rating and their sulfur content. The crude oil blended to form **Gas 1** must have an average octane rating of at least 10 and contain at most 1% sulfur. The crude oil blended to form **Gas 2** must have an average octane rating of at least 8 and contain at most 2% sulfur. The crude oil blended to form **Gas 3** must have an average octane rating of at least 6 and contain at most 1% sulfur. The octane rating and sulfur content of the three types of crude oil are given below:

	Octane Rating	Sulfur Content
Crude 1	12	0.5%
Crude 2	6	2.0%
Crude 3	8	3.0%

It costs \$4 to transform 1 barrel of oil into 1 barrel of gasoline, and Sunco's refinery can produce up to 14000 barrels of gasoline daily. Sunco's customers require the following amounts of each gasoline: **Gas 1** - 3000 barrels per day; **Gas 2** - 2000 barrels per day; and **Gas 3** - 1000 barrels per day. The company considers it an obligation to meet these demands.

The firm came up with the following mathematical program (where constraints are grouped in sets) to maximize its daily profits.

$$\text{Max} \quad 70G_1 + 60G_2 + 50G_3 - 45C_1 - 35C_2 - 25C_3 - 4(C_1 + C_2 + C_3)$$

Subject to

$$\begin{array}{ll} \text{Set I} & 12X_{11} + 6X_{21} + 8X_{31} \geq 10G_1 \\ & 12X_{12} + 6X_{22} + 8X_{32} \geq 8G_2 \\ & 12X_{13} + 6X_{23} + 8X_{33} \geq 6G_3 \\ \text{Set II} & 0.005X_{11} + 0.02X_{21} + 0.03X_{31} \leq 0.01G_1 \\ & 0.005X_{12} + 0.02X_{22} + 0.03X_{32} \leq 0.02G_2 \\ & 0.005X_{11} + 0.02X_{21} + 0.03X_{31} \leq 0.01G_3 \\ \text{Set III} & G_1 \geq 3000 \\ & G_2 \geq 2000 \\ & G_3 \geq 1000 \\ \text{Set IV} & G_1 + G_2 + G_3 \leq 14000 \end{array}$$

Set V	$C_1 = X_{11} + X_{12} + X_{13}$
	$C_2 = X_{21} + X_{22} + X_{23}$
	$C_3 = X_{31} + X_{32} + X_{33}$
Set VI	$G_1 = X_{11} + X_{21} + X_{31}$
	$G_2 = X_{12} + X_{22} + X_{32}$
	$G_3 = X_{13} + X_{23} + X_{33}$
Set VII	$C_1 \leq 5000$
	$C_2 \leq 5000$
	$C_3 \leq 5000$

- a) What are the decision variables of the program? Be precise in your descriptions.
- b) Explain the meaning of the constraints in each Set. Be precise in your descriptions.
- c) The non-negativity constraints are not provided in the constraint set above. Write them with respect to your answer to part (a) (Hint: If you conclude an element of the set above is a decision variable, you should write its non-negativity constraint as “variable ≥ 0 ”)

Question 2:

A symptomatic patient who has a condition called hydrocephalus (an abnormal accumulation of cerebrospinal fluid (CSF) within the brain) is seeking your medical attention between deciding the treatment options. Several neurosurgeons agree on the urgency of treating the condition but disagree on the preferred treatment. They can either choose to treat by placing a drainage tube (shunt) or drug therapy.

The neurosurgeons who recommend treatment by placing a shunt in the brain provide these information: If the condition subsides by the time of surgery, they do not need to do any operations on the brain, but since the surgery requires general anaesthesia, there is a risk of operative death, with a probability of 3%. If the condition is continuing by the time of surgery, with a probability of 85%, there is still a risk of operative death with a probability of as high as 10%. If the patient survives the operation, he may recover fully or not recover at all. He recovers fully with a probability of 60%.

The neurosurgeons who recommend drug therapy provide these information: With a probability of 15% the condition may subside after the start of drug therapy and the patient may recover fully or with side effects. He recovers fully with a probability of 80%. If the condition is still present after a 6-month long of drug therapy, the condition may subside with a probability of 80% with side effects or it may have no effect on the condition of the patient.

Given these information and the utility of each outcome (in the table below) answer the following questions.

Possible Outcome	Utility
Complete recovery	1
Recovery with side effects	0.7
No recovery	0.4
Operative death	0

a) Draw the decision tree.

b) Calculate the expected utility of each decision. Which treatment should the patient choose? Why?

c) Perform one-way sensitivity analysis on the utility of “recovery with side effects”. What is the threshold value of the utility of “recovery with side effects”? What is the expected utility of the corresponding threshold value of the utility of “recovery with side effects”? When the patient should prefer surgery over drug therapy? What is the upper limit on the utility of “recovery with side effects”? Provide a graphical solution as in the lecture notes.