IE 102 Spring 2017

Routing Through Networks - 2

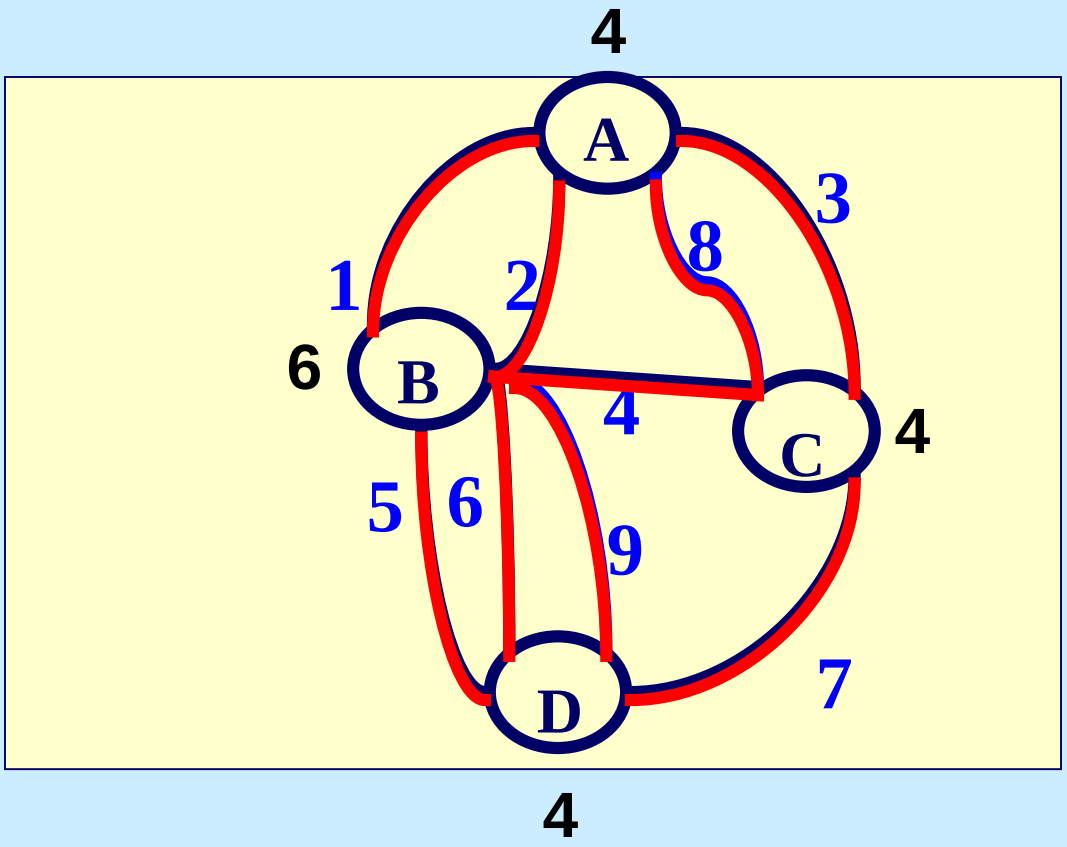
Hamiltonian Path Problem

- Assume a network with all possible arcs – called complete graph
- Hamiltonian Circuit (path, graph) – a path that visits each node exactly once
- Find the path that minimizes total “distance” written on the arcs.
- An approach: Brute-force approach (total enumeration) Try every unique circuit

Eulerian Cycle Problem

- Eulerian Circuit (path, graph) – a path that visits each edge exactly once

On Eulerian Cycles



The degree of a node in an undirected graph is the number of incident arcs

Theorem. An undirected graph has an eulerian cycle if and only if

(1) every node degree is even and

(2) the graph is connected (that is, there is a path from each node to each other node).

Eulerian Cycle Problem

- Fleury’s algorithm

Shortest Path Problem

Performance measure that can be used:

- distance traveled
- cost
- time
- ....

Performance measure is whatever the number on the arc represents.

Minimize one of the above

Shortest Path Problem

- We have a systematic method to solve the problem, an algorithm (a detailed procedure – Dijkstra'a Algorithm)
- What is the meaning of optimal solution?

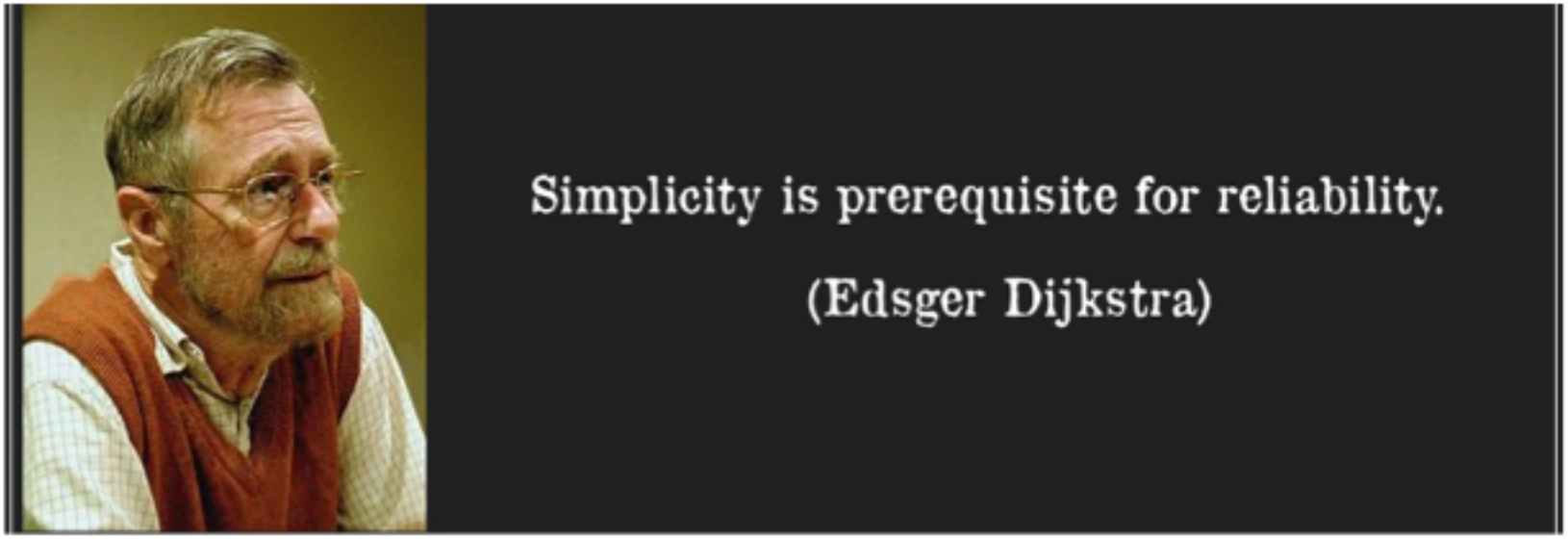
  A solution that yields the minimum value of the performance measure considered

VERY LUCKY TO KNOW/PROVE IF THE ALGORITHM YIELDS AN OPTIMAL SOLUTION

Dijkstra’s Algorithm

Edsger Wybe Dijkstra

- May 11, 1930 – August 6, 2002
- Dutch computer scientist from Netherlands
- Received the 1972 A. M. Turing Award, widely considered the most prestigious award in computer science
- Known for his many essays on programming



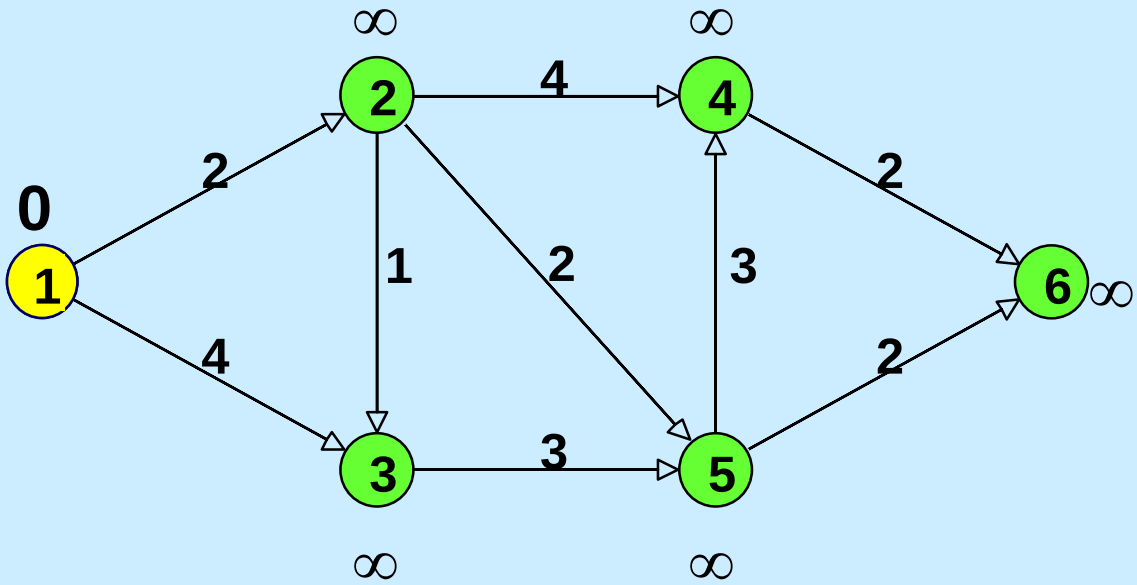
Dijsktra's Algorithm

General Description
Suppose we want to find a shortest path from a given node s to other nodes in a network (one-to-all shortest path problem)

- Node s is called a starting node or an initial node

- Dijkstra's algorithm starts by assigning some initial values for the distances from node s and to every other node in the network
- It operates in steps, where at each step the algorithm improves the distance values.
- At each step, the shortest distance from node s to another node is determined

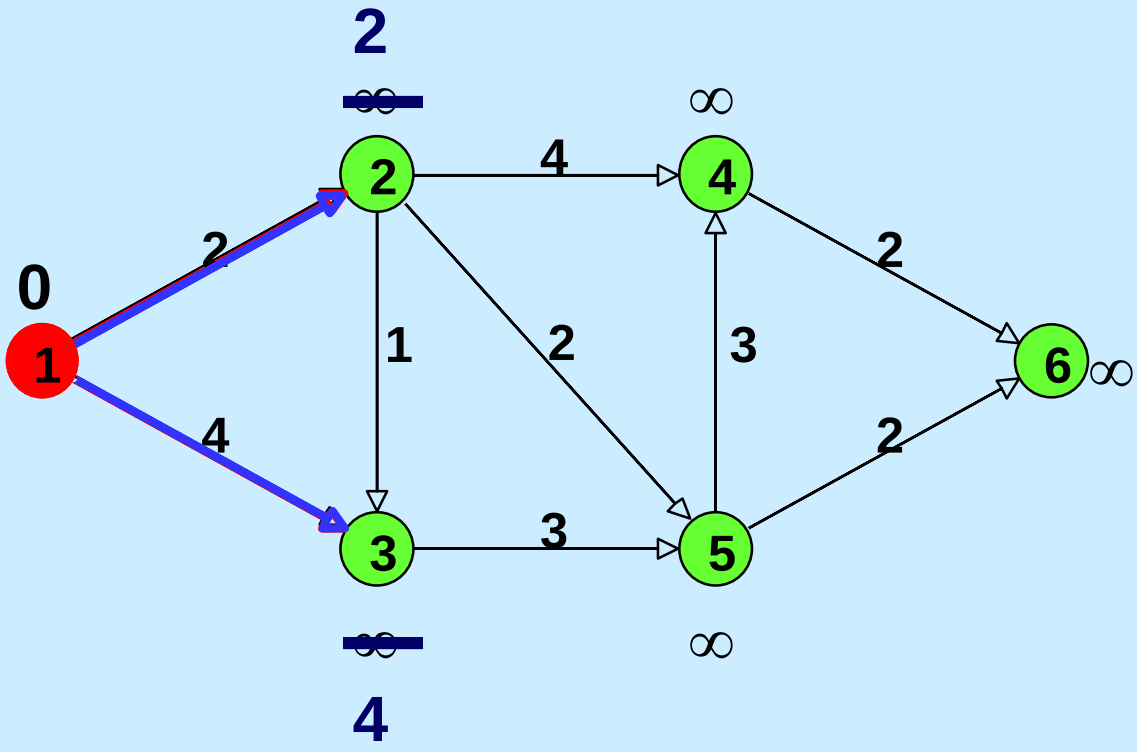
An Example



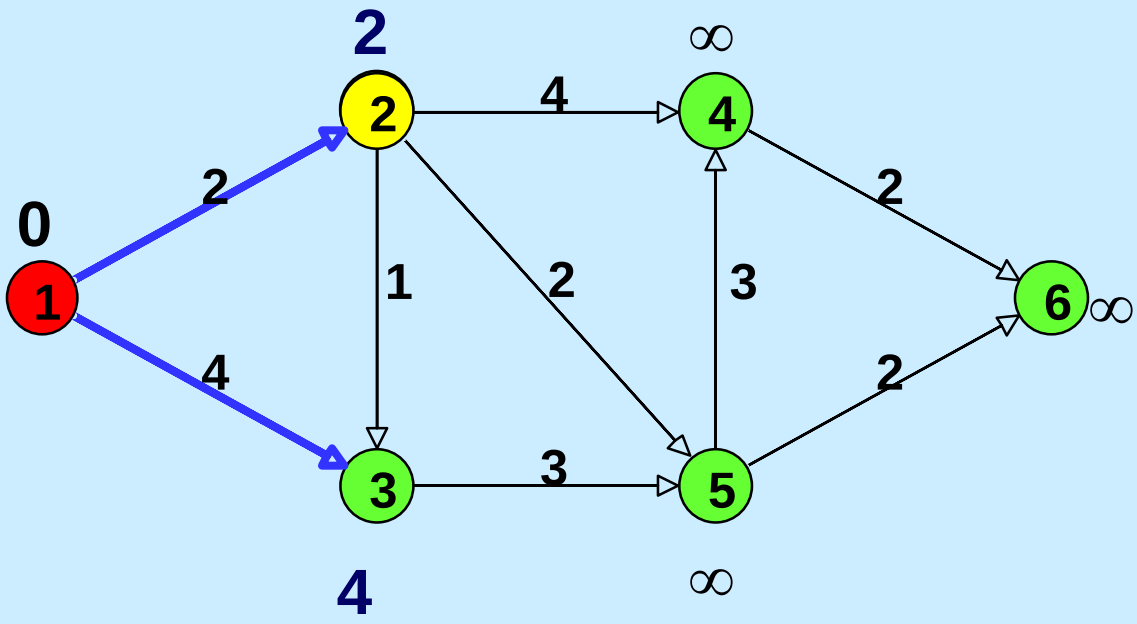
Initialize

Select the node with the minimum temporary distance label.

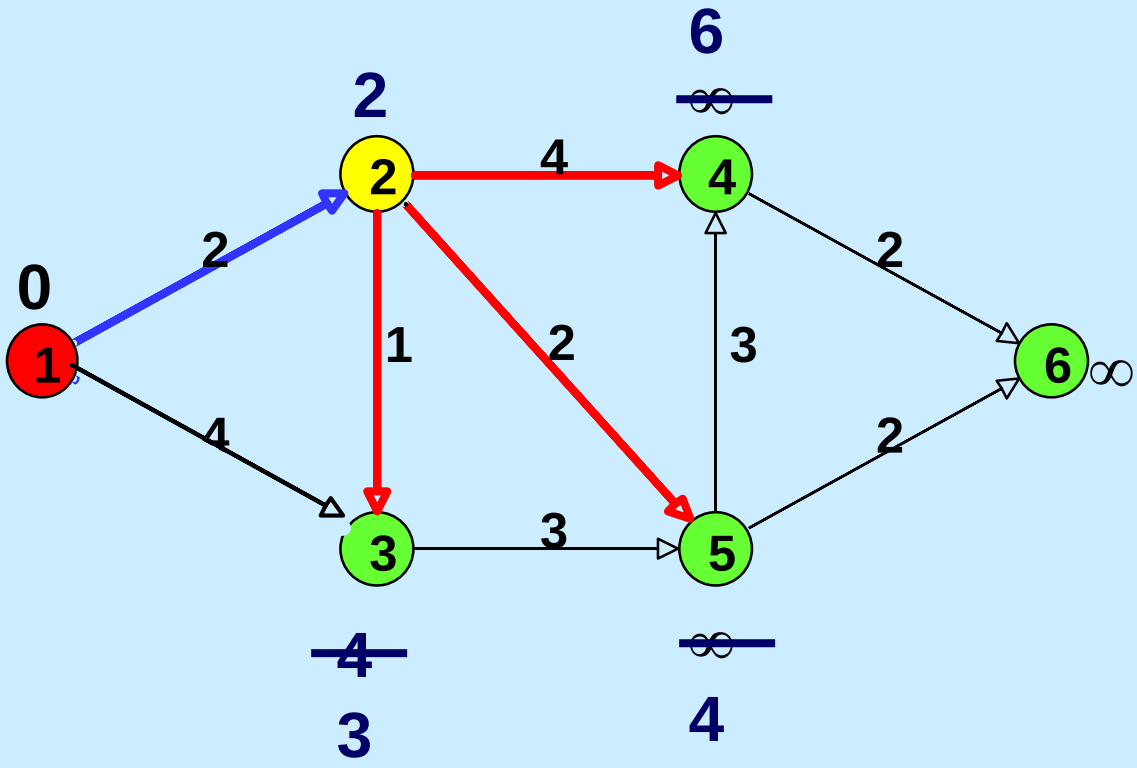
Update Step



Choose Minimum Temporary Label

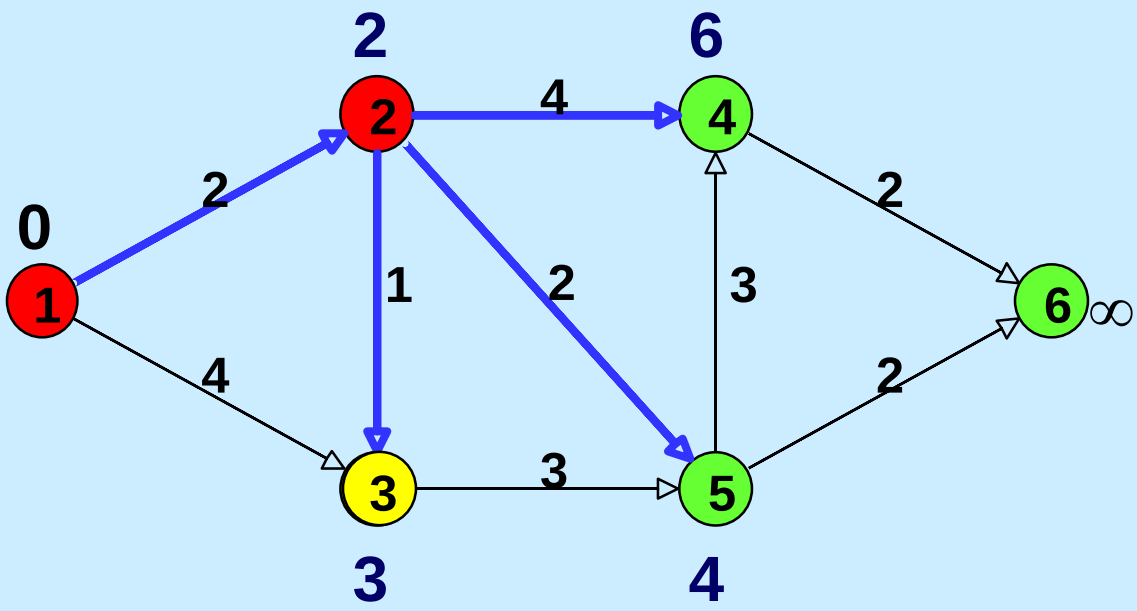


Update Step

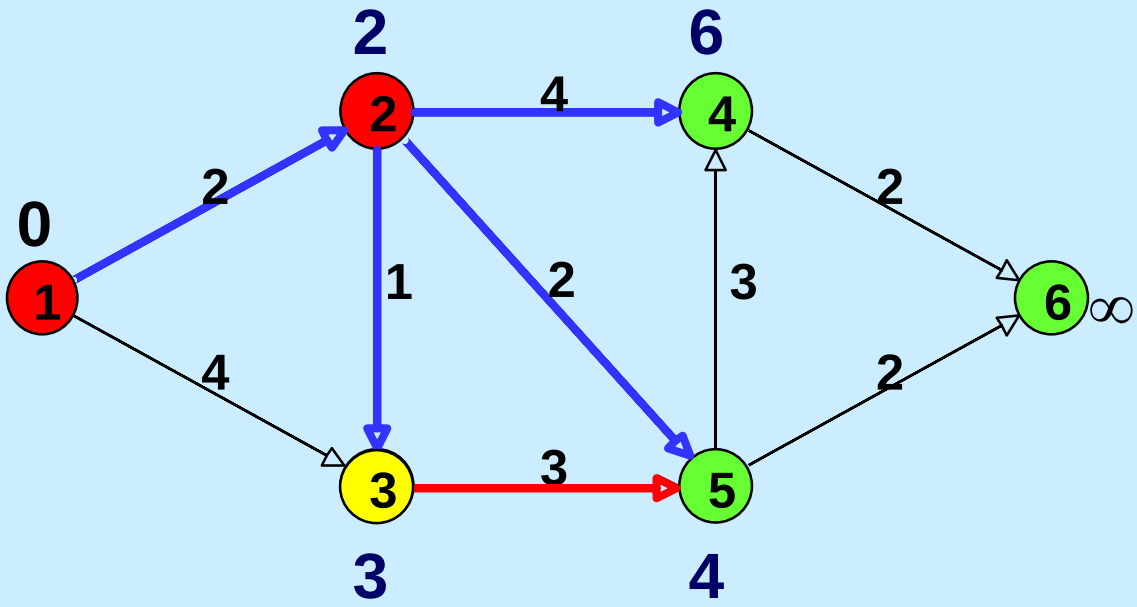


The predecessor of node 3 is now node 2

Choose Minimum Temporary Label

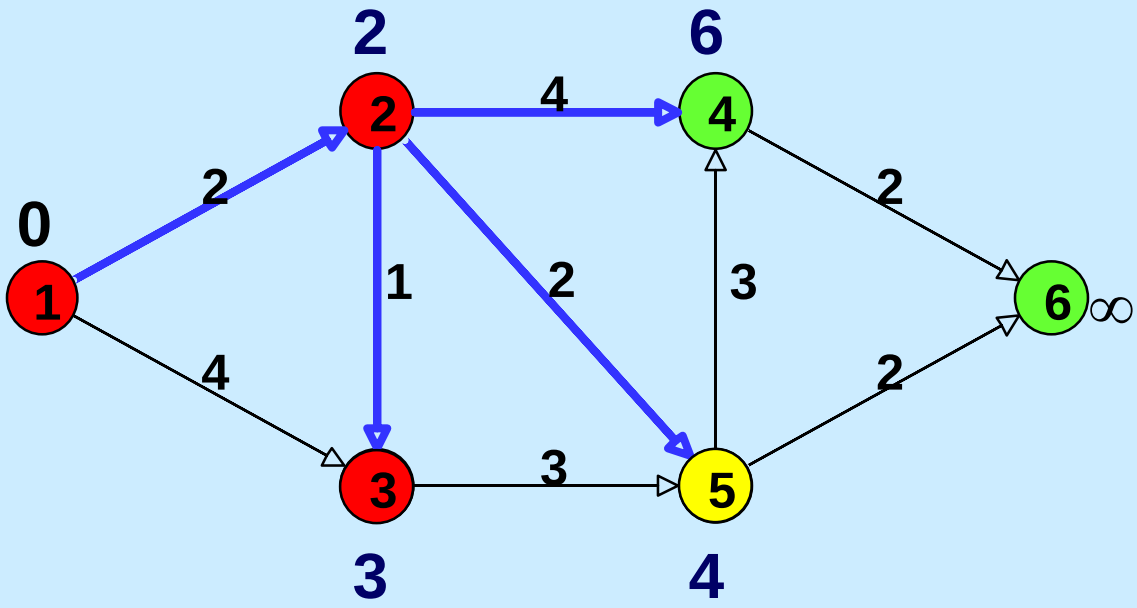


Update

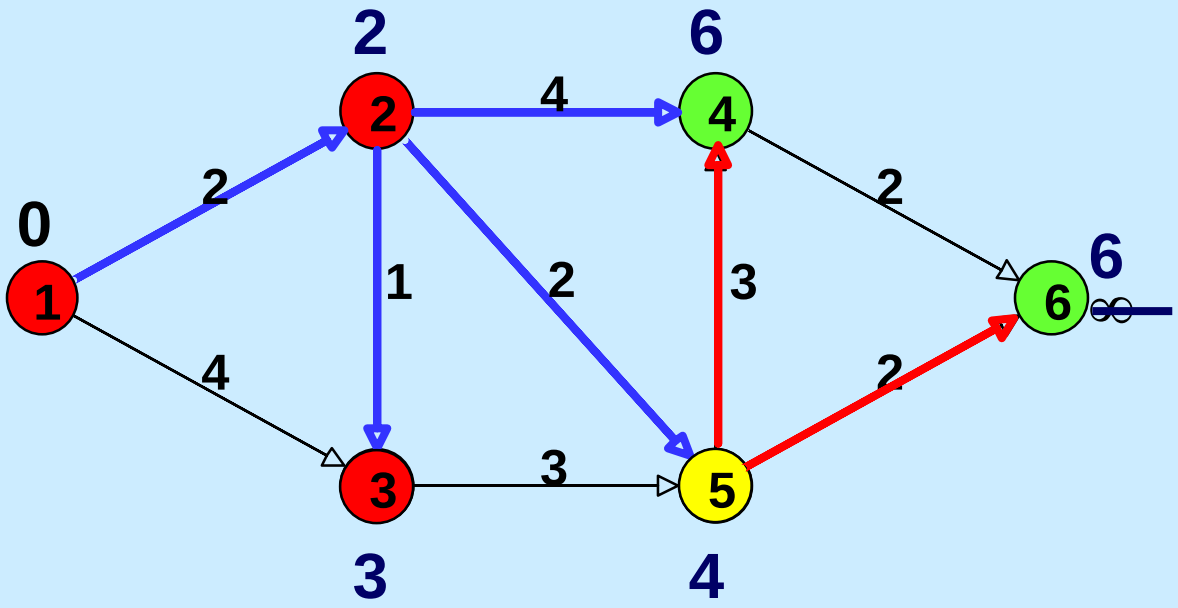


d(5) is not changed.

Choose Minimum Temporary Label

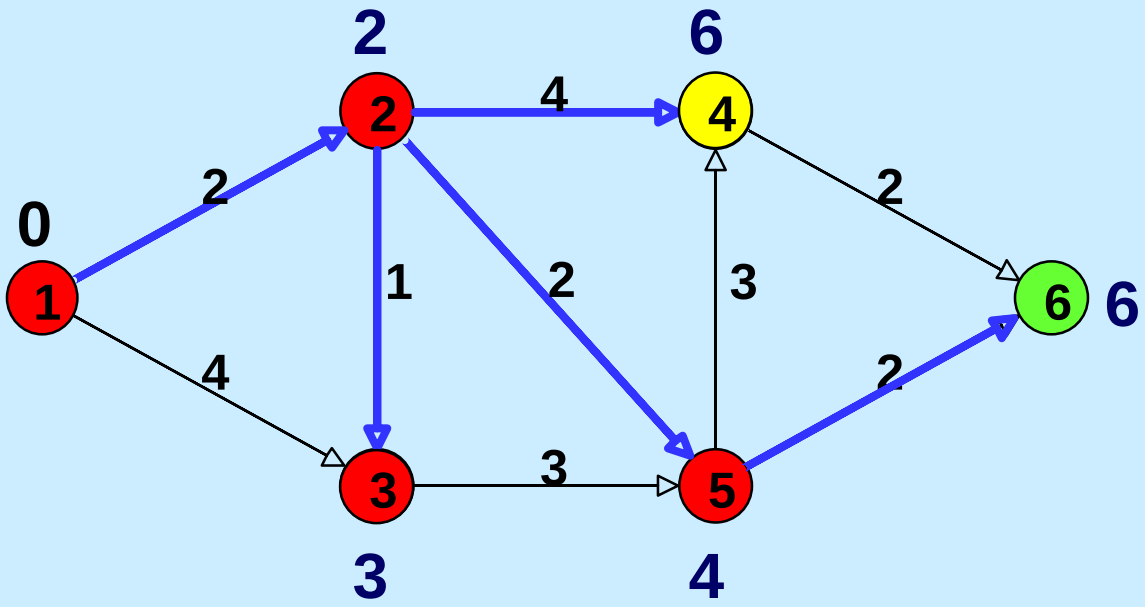


Update

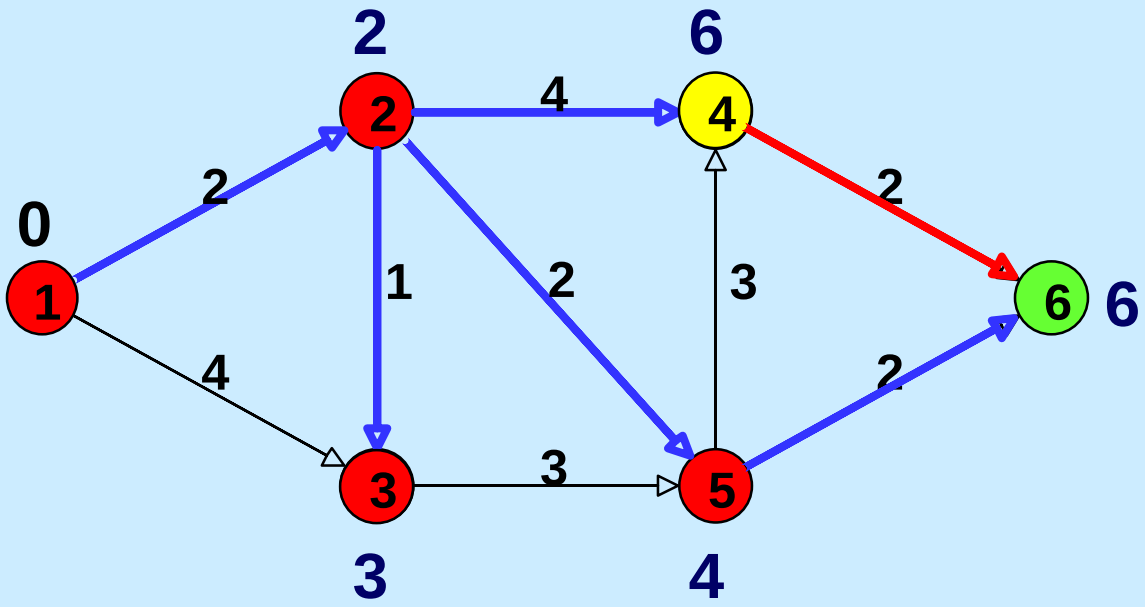


d(4) is not changed

Choose Minimum Temporary Label

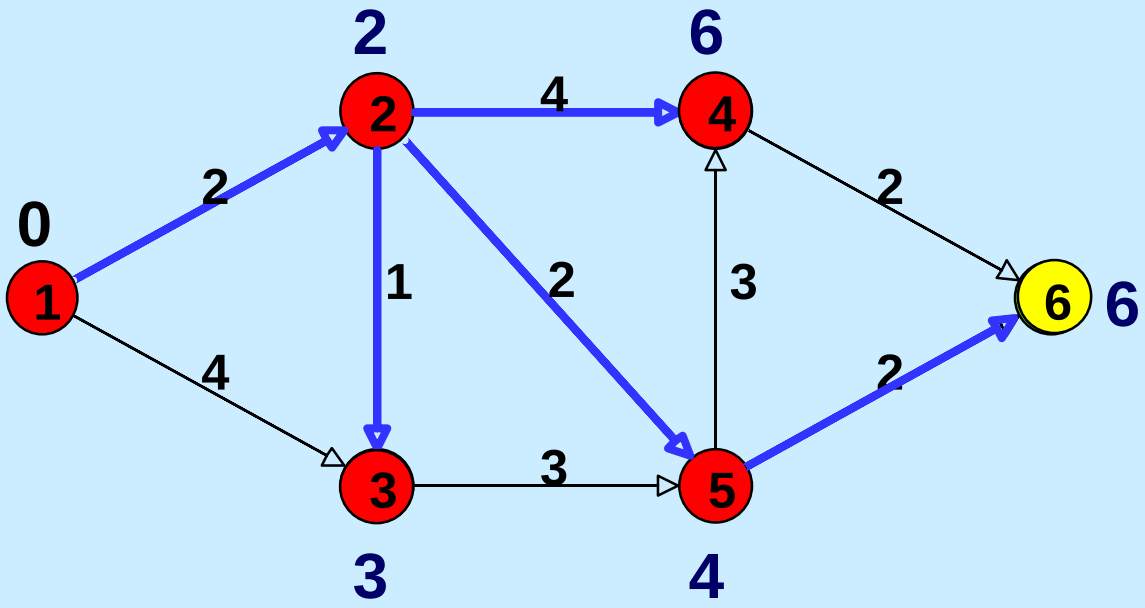


Update



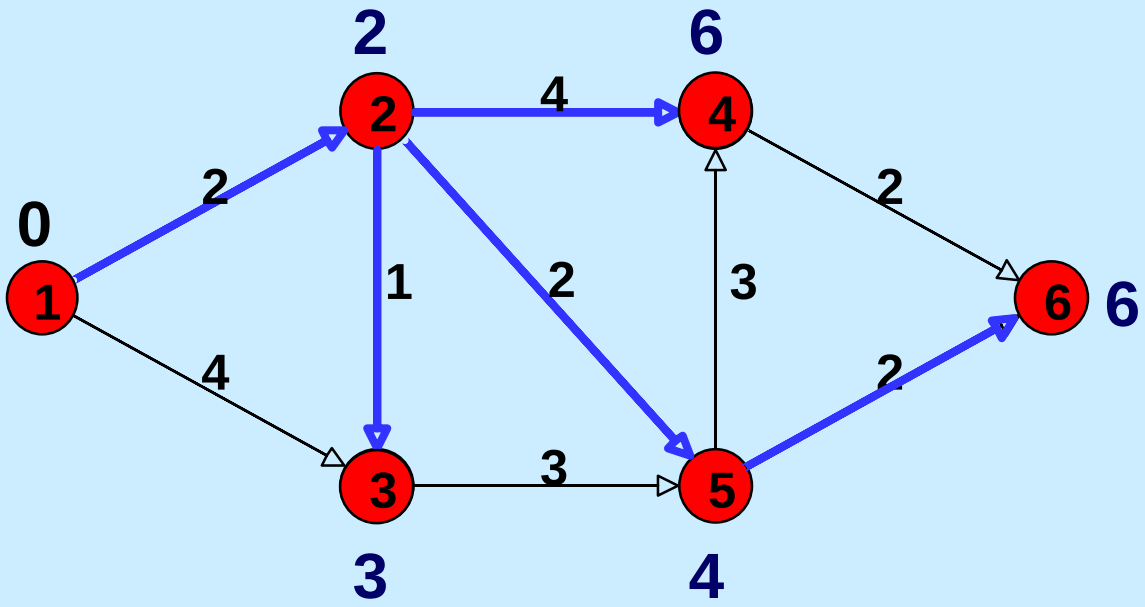
d(6) is not updated

Choose Minimum Temporary Label



There is nothing to update

End of Algorithm

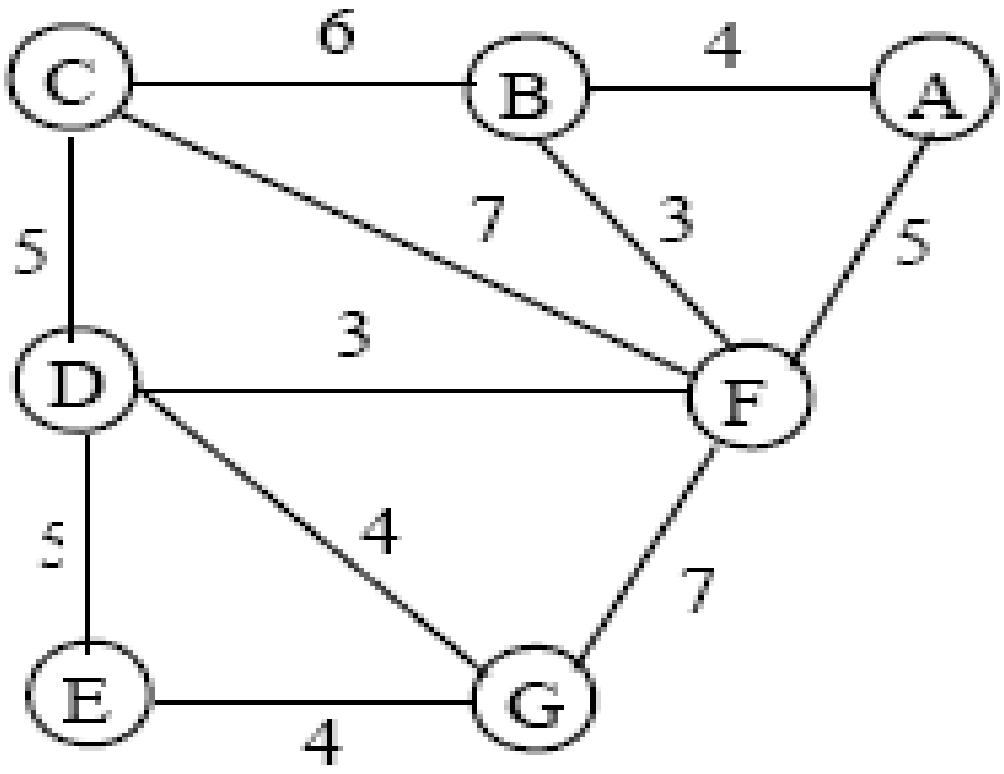


All nodes are now permanent

The predecessors form a tree

The shortest path from node 1 to node 6 can be found by tracing back predecessors

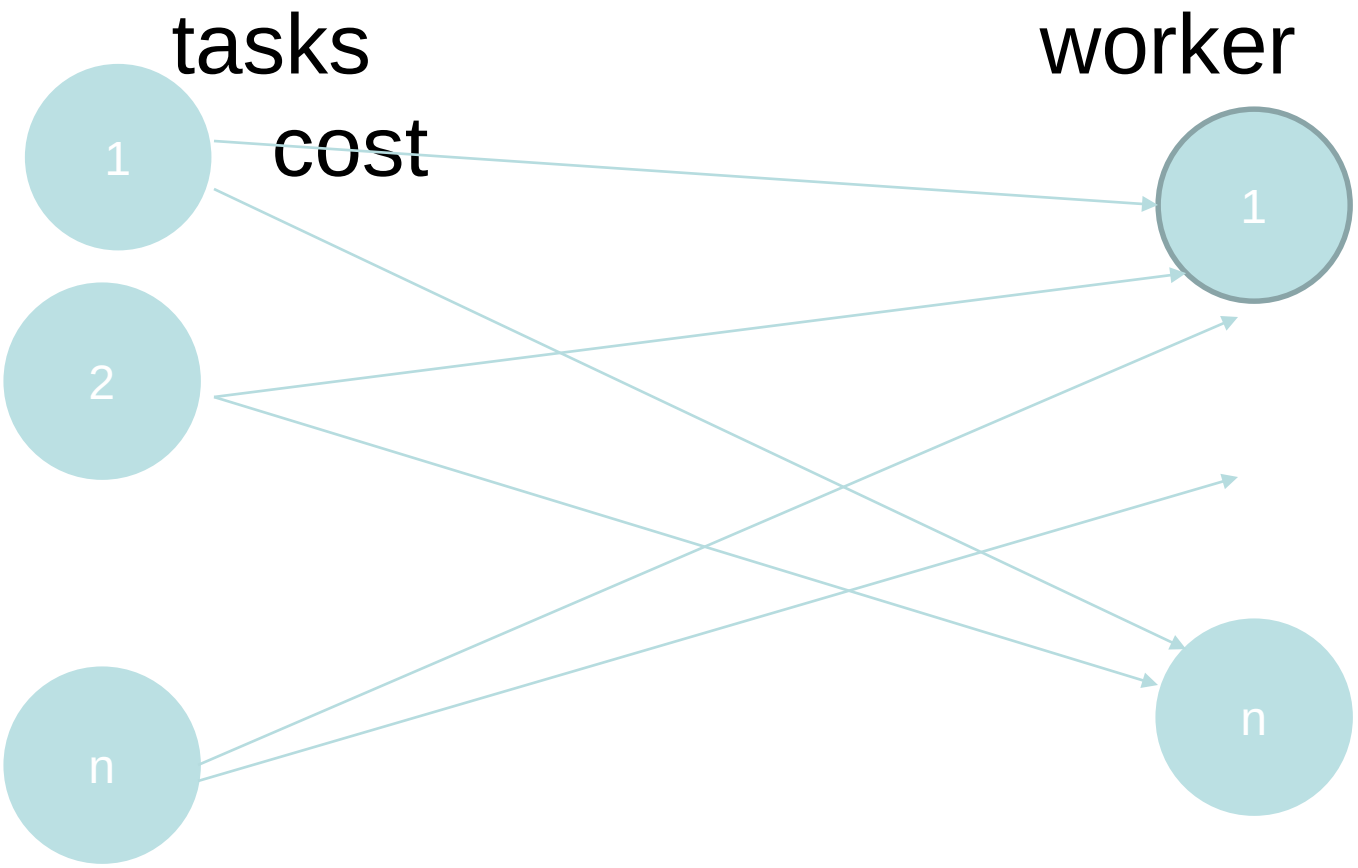
Exercise



Assignment Problem

- n tasks – n workers
- A worker is assigned only one task
- Each task is assigned to a single worker
- “Cost” of performing a task is a function of the worker and task.
- Examples:
  - ➢tasks / workers at Mini
  - ➢jobs/employees at airport
  - ➢lawyers/cases
  - ➢students/topics
  - ➢swimmers/strokes for medley

Assignment Problem –
as a network



Solution

- Matrix representation of costs

Workers / Tasks	1	2	...	n
1	c_{11}	c_{12}		c_{1n}
...				
n	c_{n1}	c_{n2}		c_{nn}

- One of the lucky cases
- Hungarian Algorithm
- We'll see tomorrow and Friday in class

A few ideas on the problem

- Minimize total cost
- Only one worker for a task, one task for a worker

  (1) Find the minimum for each row and add – Is it possible to have total cost less than that?

  (2) Find the minimum in each column and add - Is it possible to have total cost less than that?
- Both are lower bounds on the optimal cost