HW3-SOLUTION KEY

Question 1: maximize $x+5y$

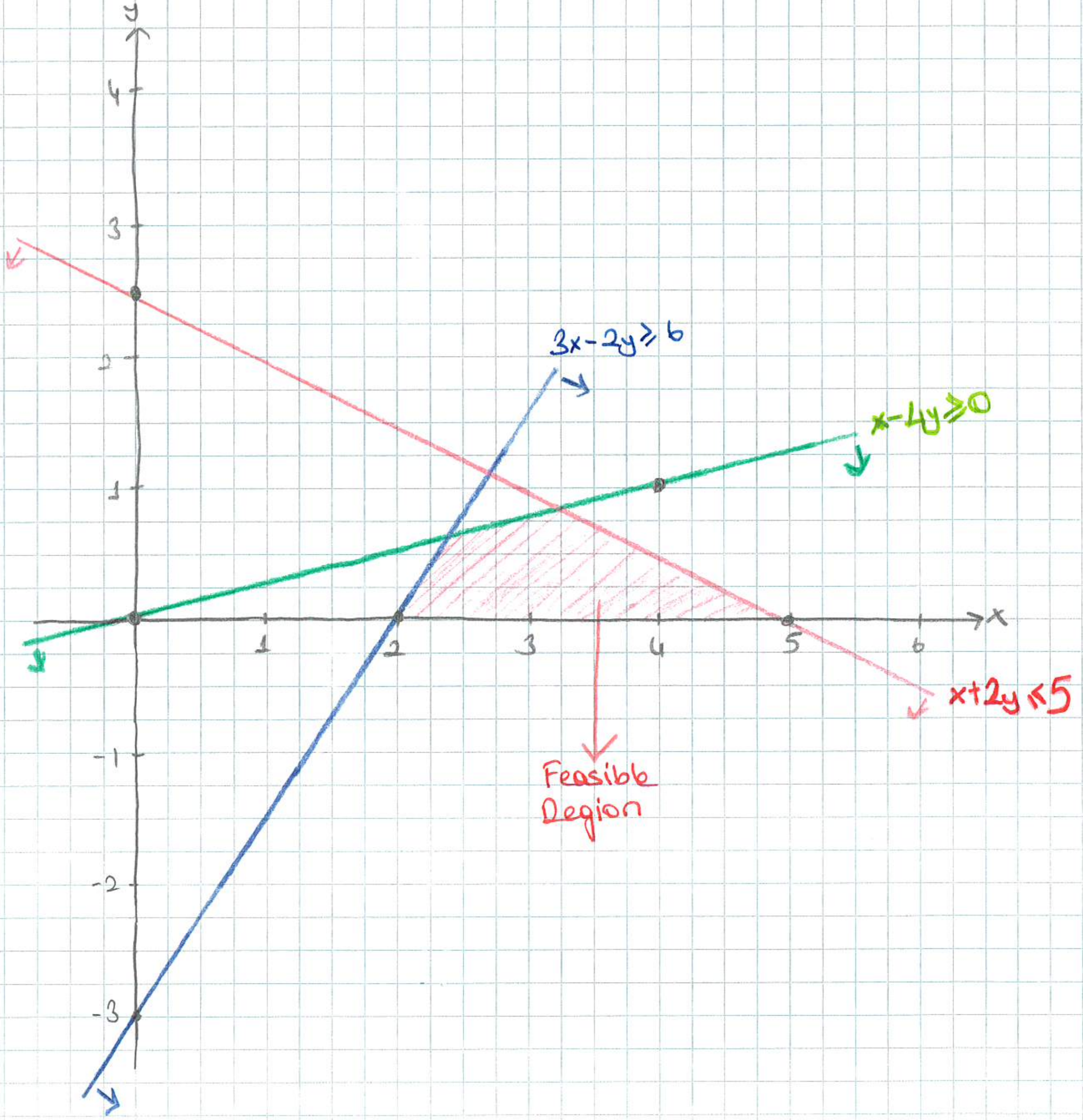
subject to $x+2y \leq 5$

$3x-2y \geq 6$

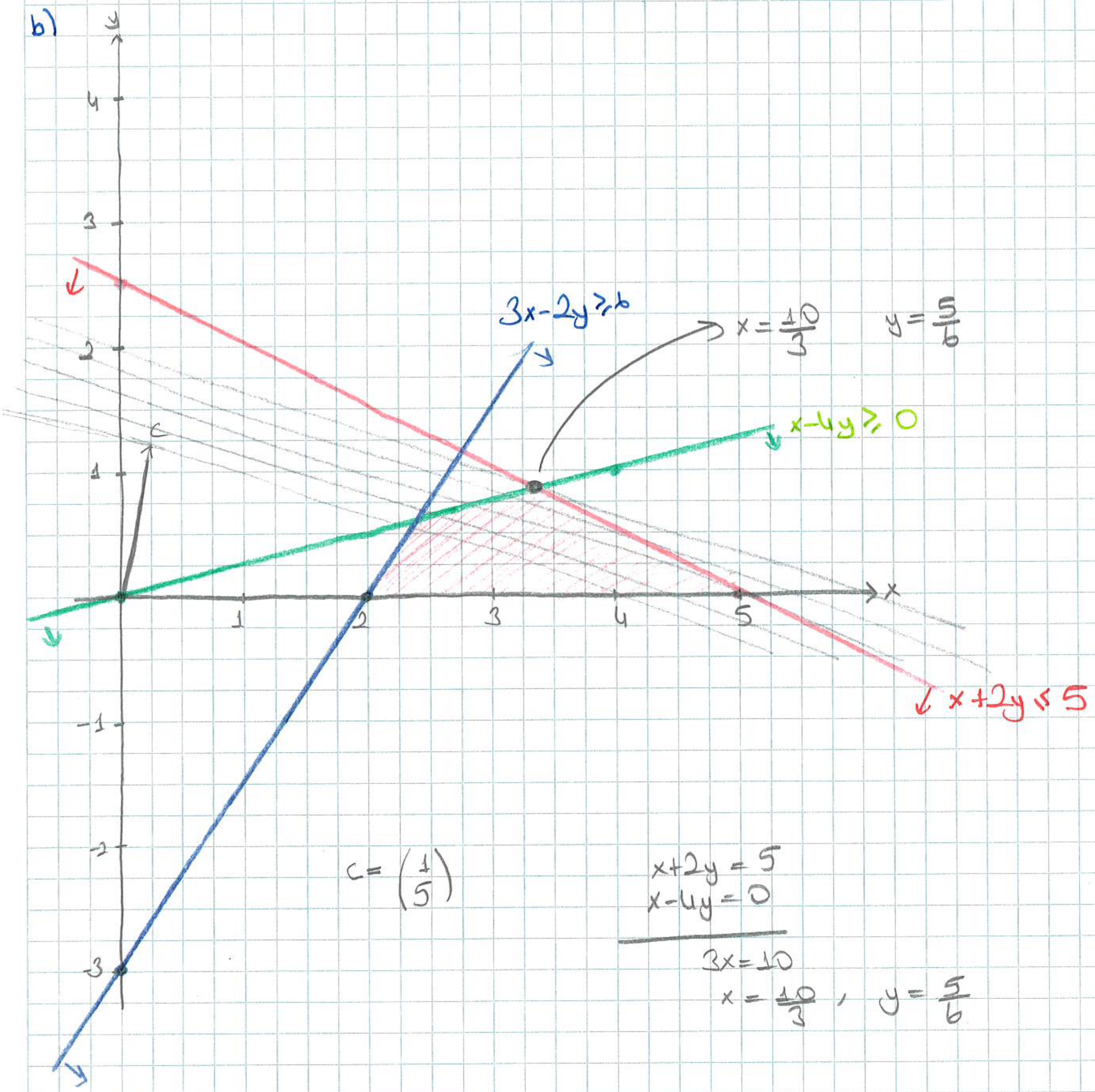
$x-4y \geq 0$

$x, y \geq 0$

a) Feasible Region:



b)



$c = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

$$\begin{aligned} x+2y &= 5 \\ x-4y &= 0 \\ \hline 3x &= 10 \end{aligned}$$

$x = \frac{10}{3}$, $y = \frac{5}{6}$

Optimal Solution: $(x, y) = \left(\frac{10}{3}, \frac{5}{6}\right)$

Objective Function Value: $z = x + 5y = \frac{15}{2}$

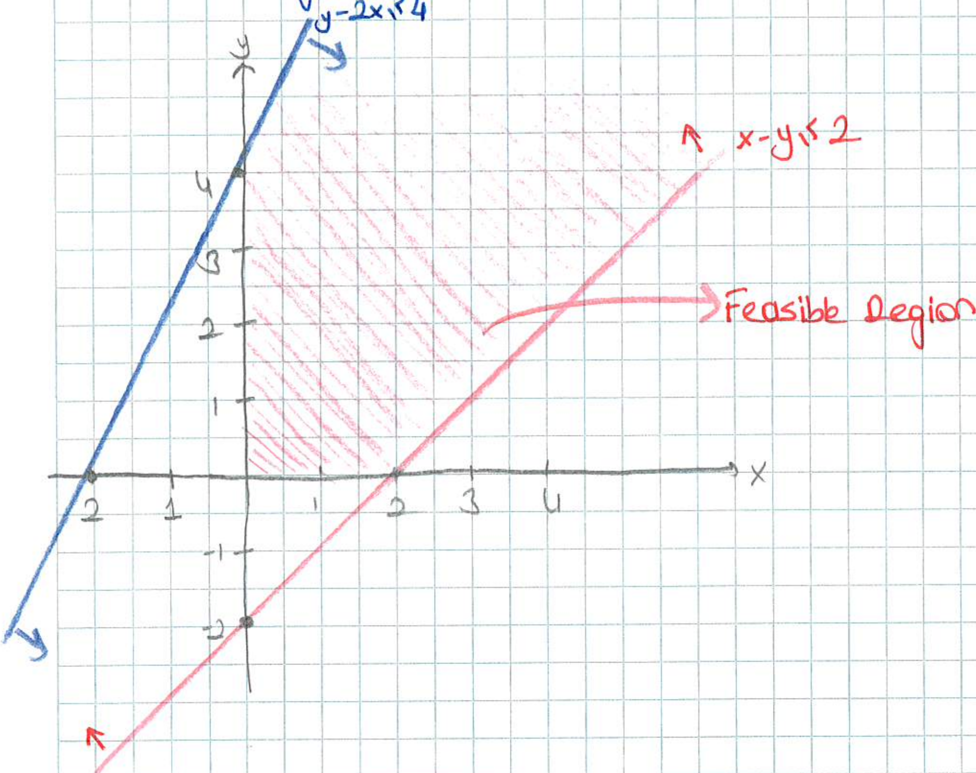
Question 2:

Constraints: $x - y \leq 4$

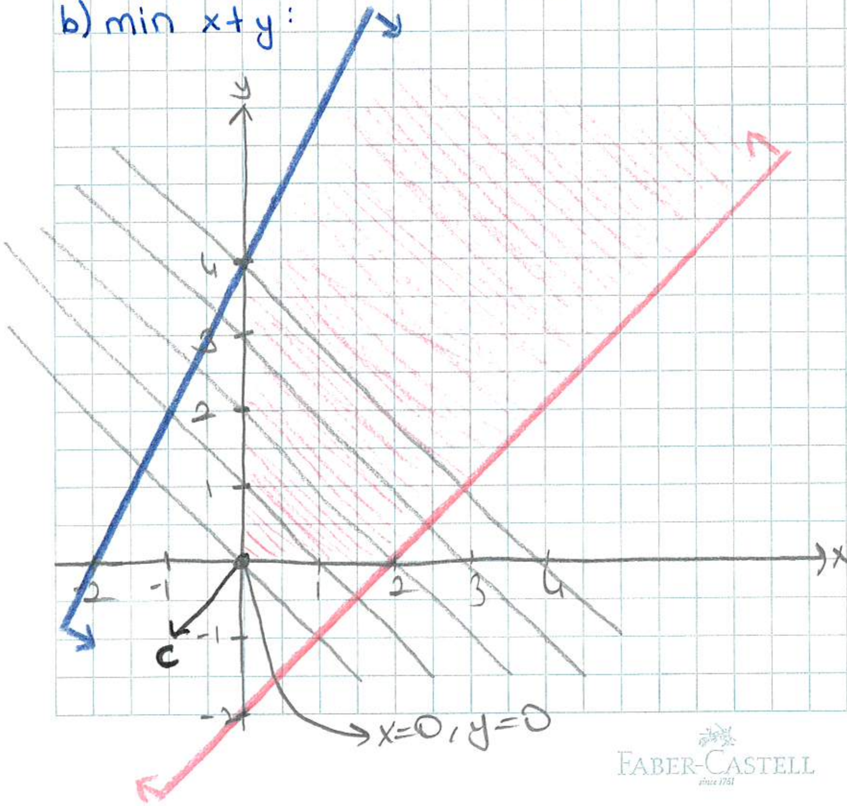
$y - 2x \leq 4$

$x, y \geq 0$

a) Feasible Region:



b) min $x + y$:



$-c = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

Optimal Solution: $(x, y) = (0, 0)$

Objective Function Value: $z = x + y = 0$

- Other possible solutions:

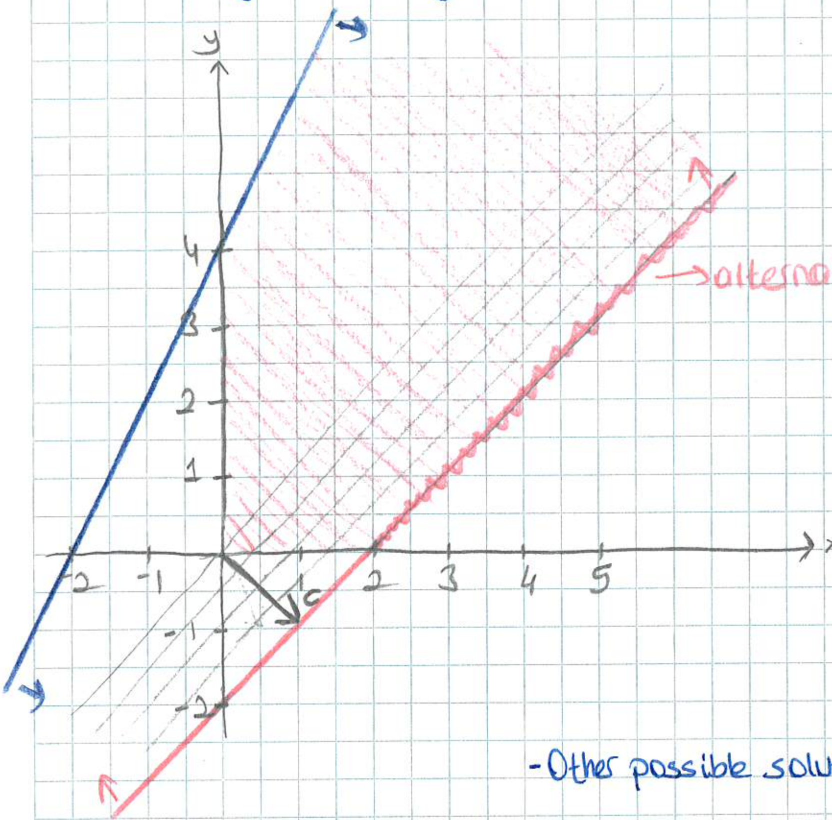
→ max $-x - 3y$,

→ min $5x + y$,

→ max $-x - y$,

etc.

c) max $x-y$



alternative optima

$c = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

- 2 optimal solutions:
  1 - $(x,y) = (2,0)$
  2 - $(x,y) = (3,1)$
- Objective Function Value:
  $z = x - y = \underline{\underline{2}}$

- Other possible solutions:

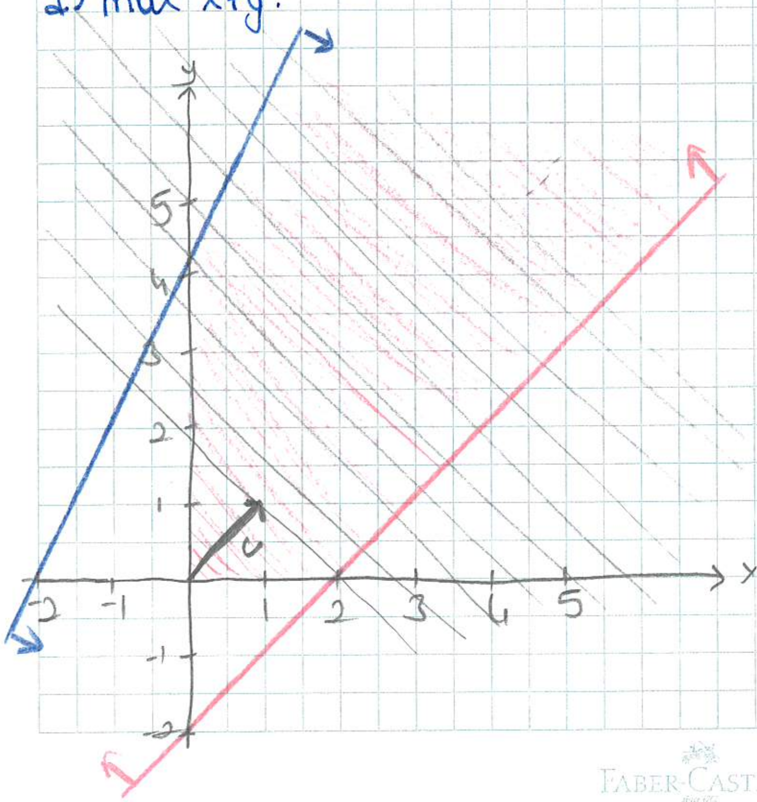
→ max $y-2x$ → min $2x-y$

→ min $y-x$

→ max $-x$ → min x

→ max $-y$ → min y

d) max $x+y$:



$c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

The problem is unbounded

- Other possible solutions:

→ min $-x-2y$,

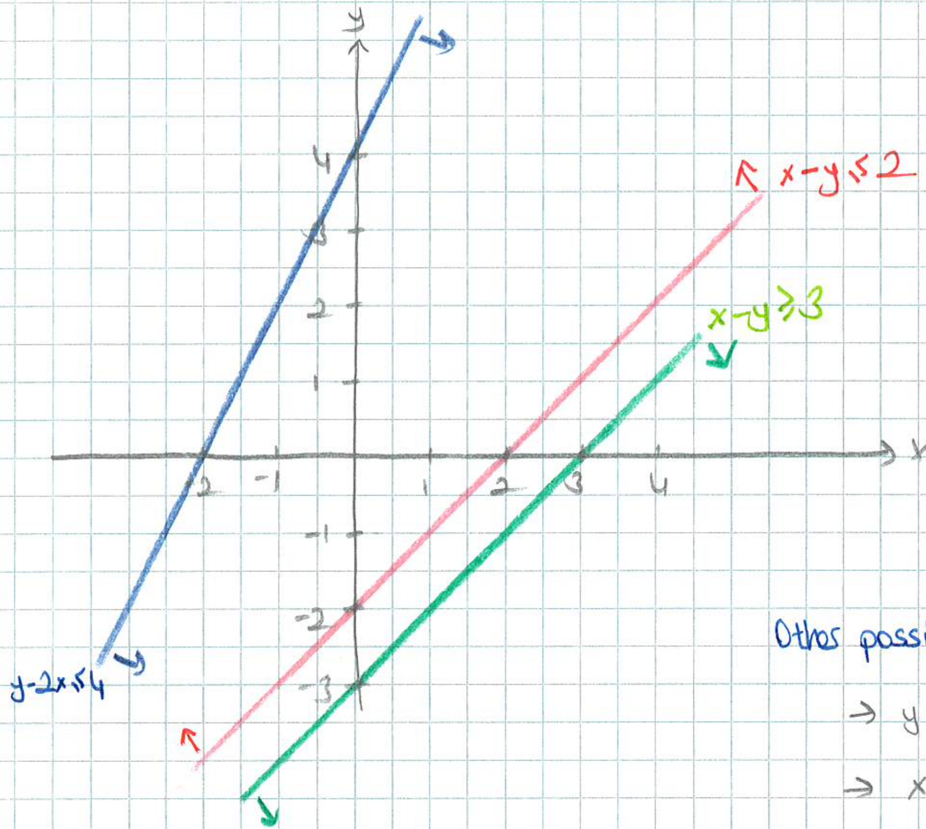
→ max $3x-y$,

→ min $-x-y$,

etc.

e) This problem cannot be infeasible.

To make it infeasible, we need to add another constraint such as $x-y \geq 3$



→ The feasible region is empty. The problem is infeasible.

Other possible constraints:

→ $y-2x \geq 5$,

→ $x+y \leq -3$,

→ $x-y \geq 4$,

etc.