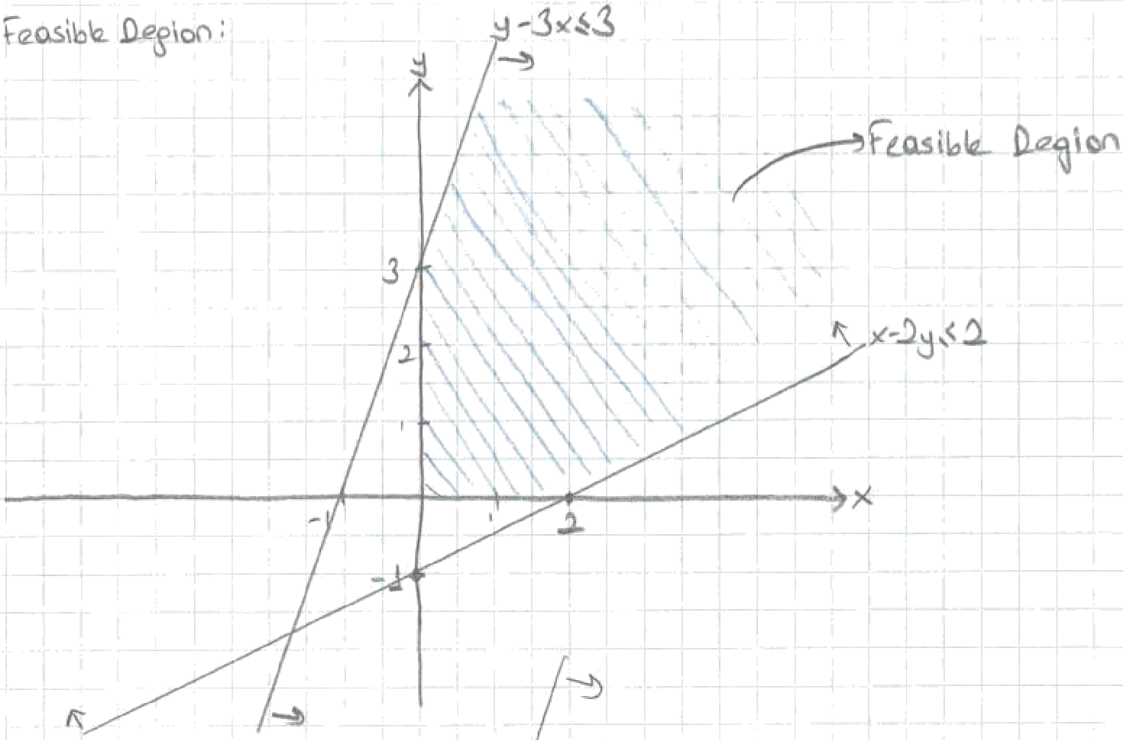
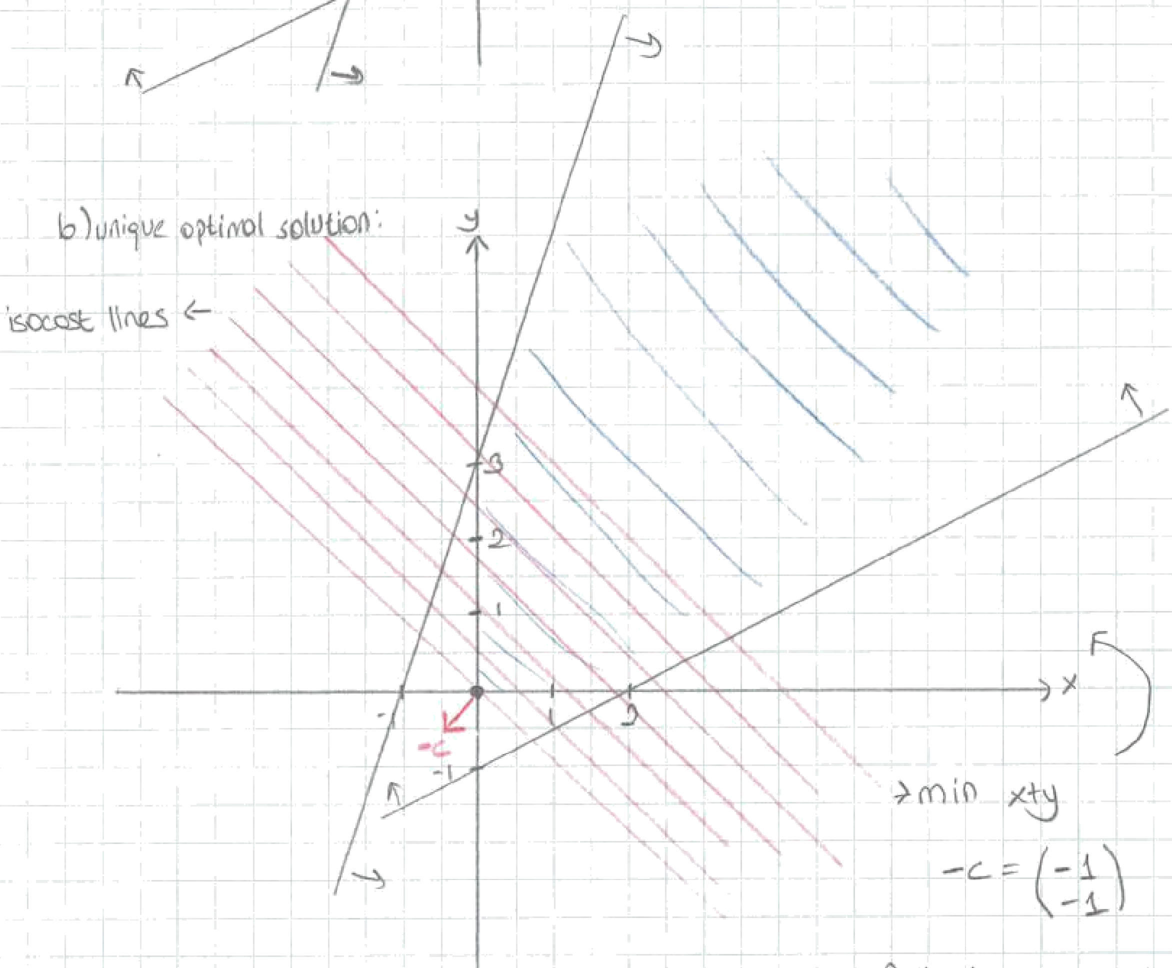
1E102-Final Study Set Solutions

Question 1:

a) Feasible Region:



b) unique optimal solution:



→ min $x+y$

$-c = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

Optimal solution: $(x,y) = (0,0)$

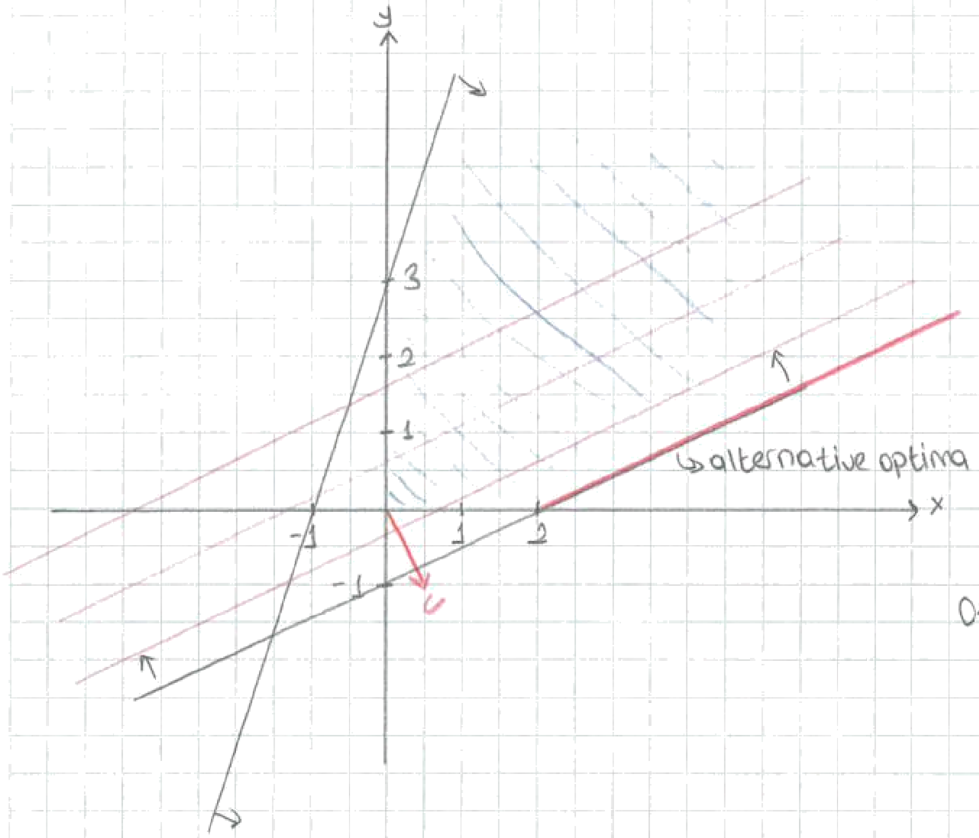
Optimal soln. value: $z = x+y = 0$

Other possible solutions:

- max $-x-y$
- min $2x+y$
- max $-x-3y$,

etc.

c) Alternative Optimal solution:



→ max $x - 2y$

$c = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Two optimal solutions:

- $(x,y) = (2,0)$
- $(x,y) = (4,1)$

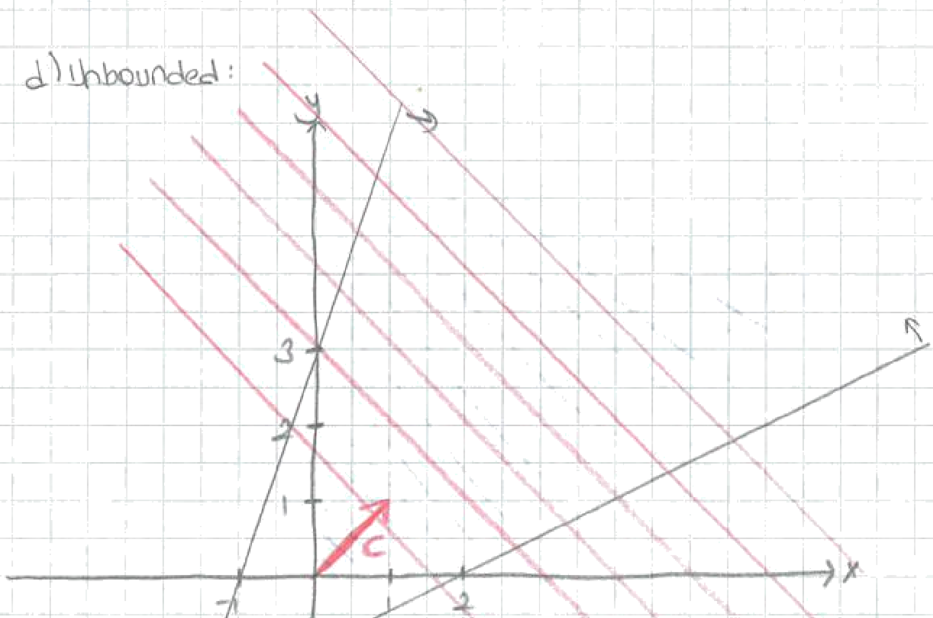
Other possible solutions:

→ min $-x + 2y$

→ max $y - 3x$

→ min $3x - y$

d) Unbounded:



→ max $x + y$

$c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Other possible solutions:

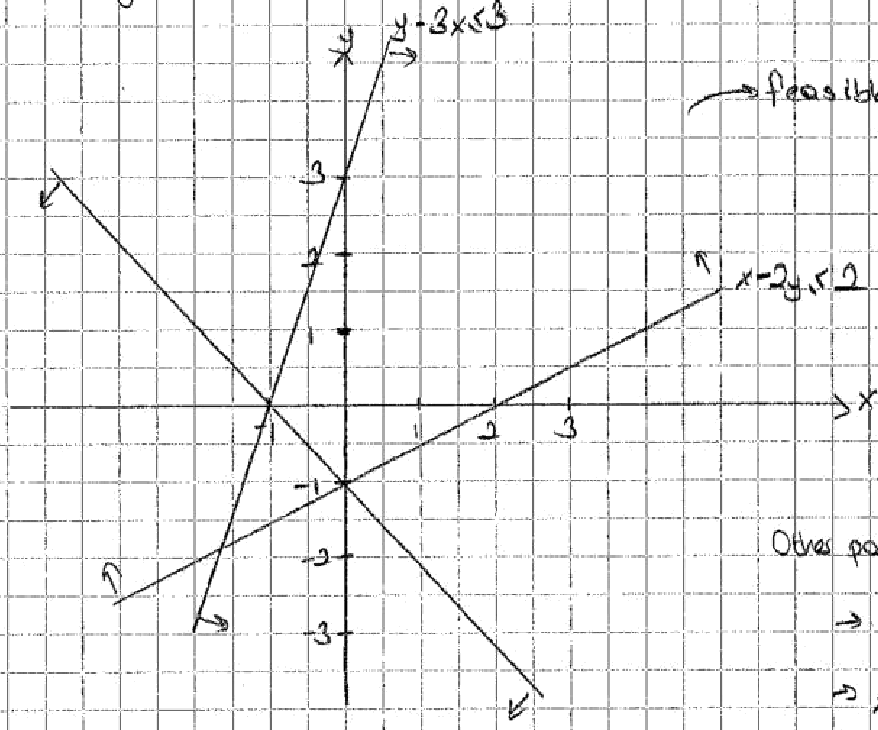
→ min $-x - y$

→ max $2x - y$

→ min $-x - 3y$

e) Infeasibility:

- Add $x+y \leq -1$ to the constraints:



→ feasible region is empty

Other possible solutions:

→ $-x+2y \geq -4$

→ $x \leq -1$

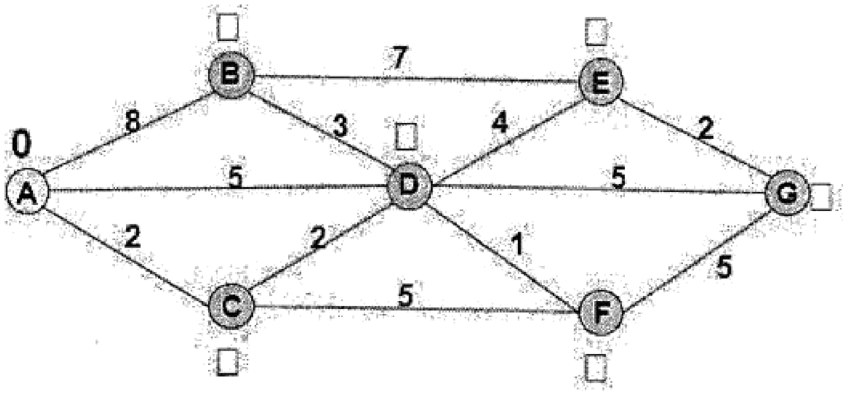
→ $-y \geq 2$

Question 2: NN tour: From node S, nearest unvisited node is G. From node G, nearest unvisited node is F. From node F, nearest unvisited node is T. From node T, nearest unvisited node is E. All nodes are visited, we have to return to node S. Then the TSP tour with NN is SGFTES with cost:

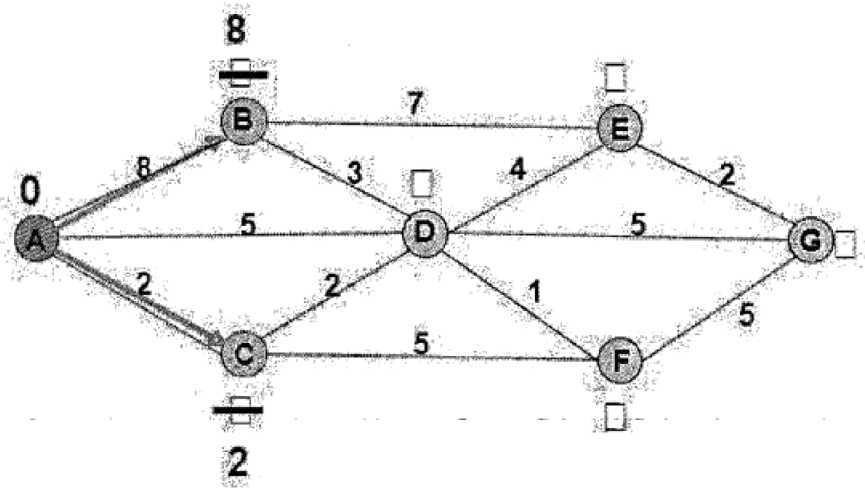
(S to G) + (G to F) + (F to T) + (T to E) + (E to S through G without visiting G) = (58) + (132) + (201) + (113) + (217 + 58) = 779.

When we apply 2-opt, we switch GF and ES with GE and FS – as we do not want to have edges crossing each other. So the new TSP tour is SGETFS with cost 58 + 217 + 113 + 201 + 79 = 668.

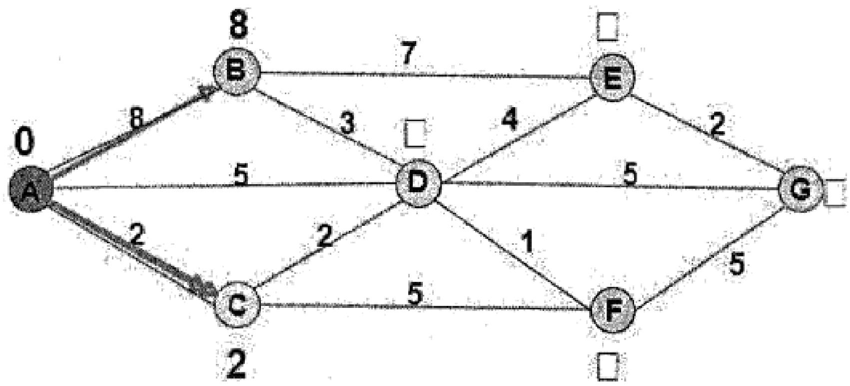
Question 3: First we initialize the network:



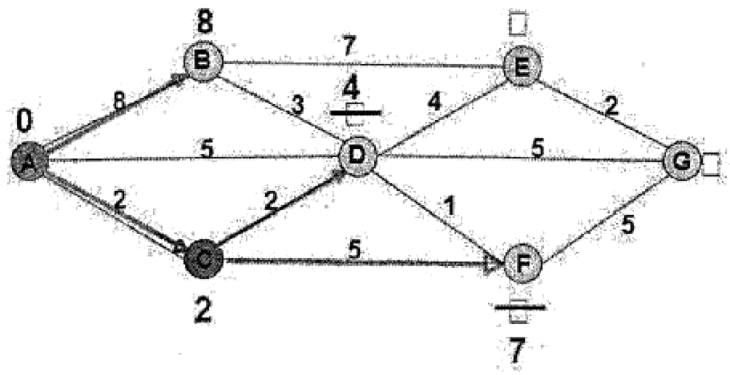
Then choose the node with the minimum temporary distance label in each step.



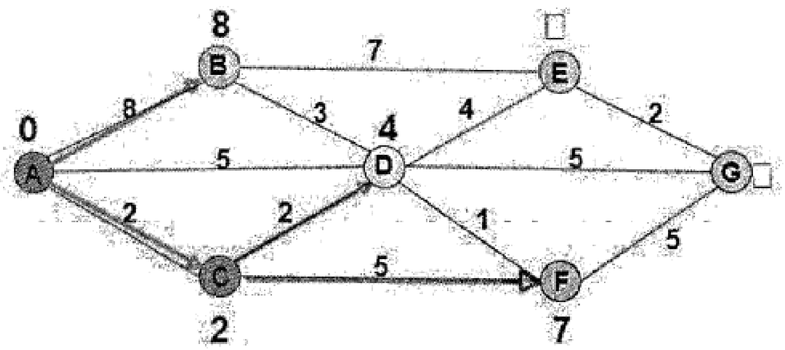
C is the nearest.



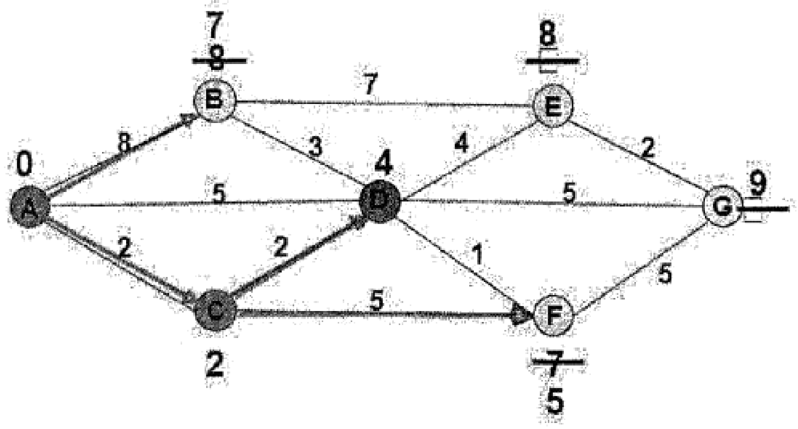
Then update the network. The predecessor of node C is now node A.



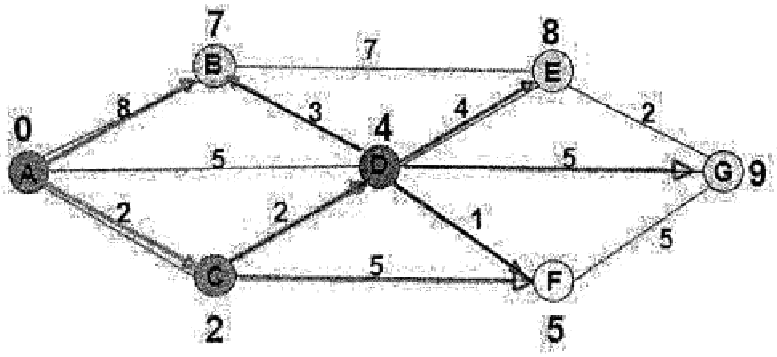
Again choose the node with the minimum temporary distance label.



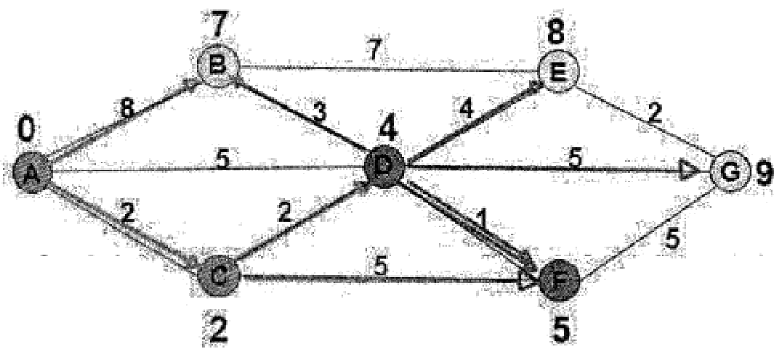
Then update the network. The predecessor of node D is now node C.



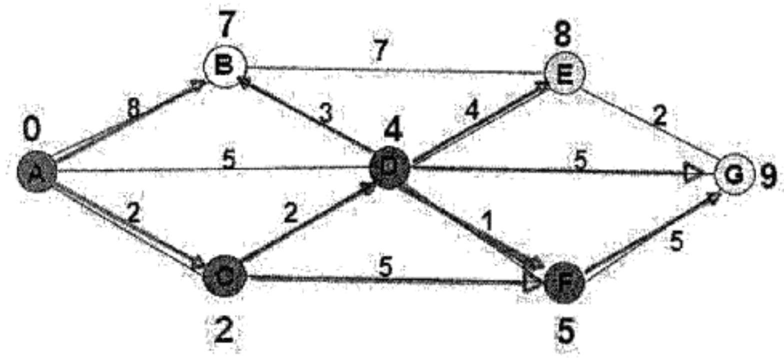
Choose the node with the minimum temporary distance label.



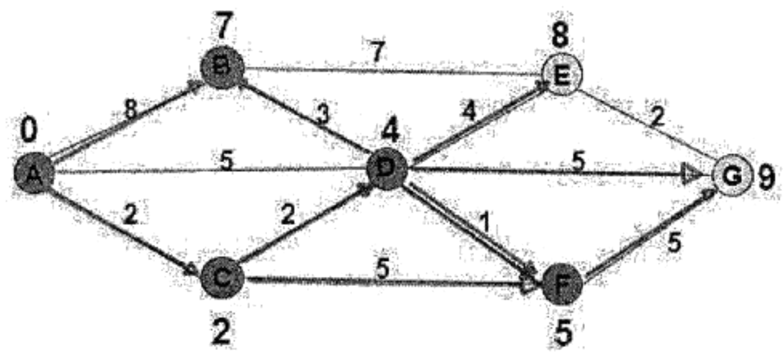
Then update the network. The predecessor of node F is now node D.



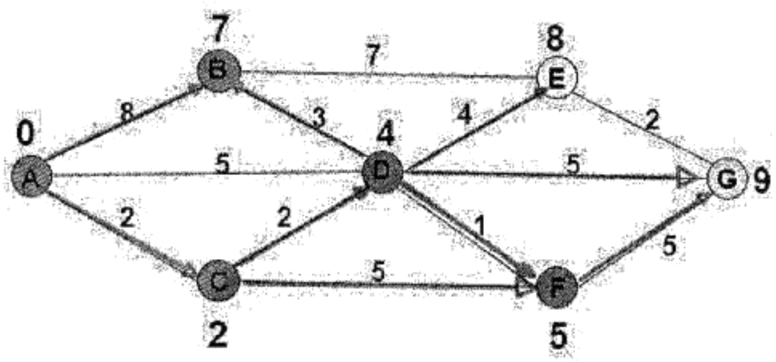
Choose the node with the minimum temporary distance label. (Do not update node G since 5 + 5 > 9)



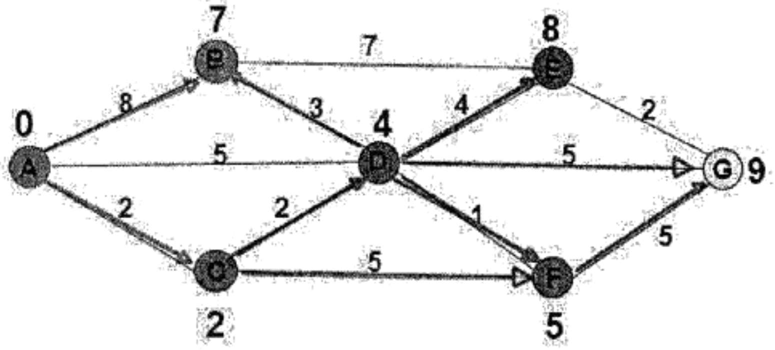
Then update the network. The predecessor of node B is now node D.



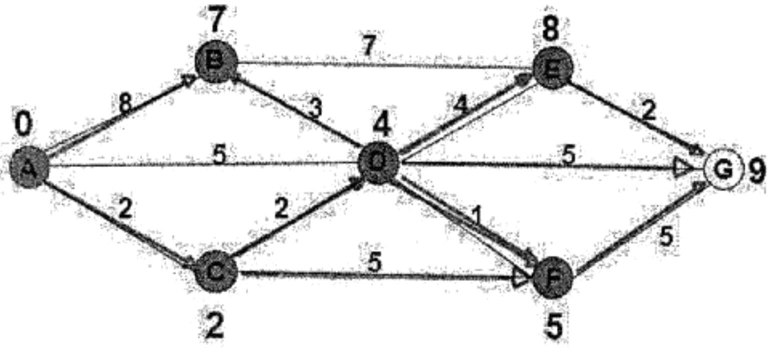
Choose the node with the minimum temporary distance label.



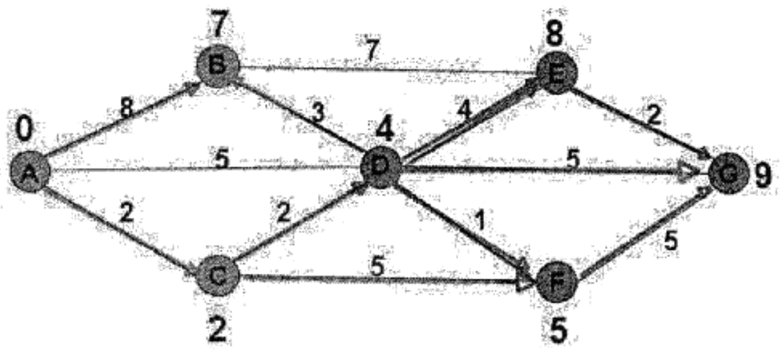
Then update the network. The predecessor of node E is now node D.



Choose the node with the minimum temporary distance label. (Do not update node G since 8 + 2 > 9)



Then update the network. The predecessor of node G is now node D.



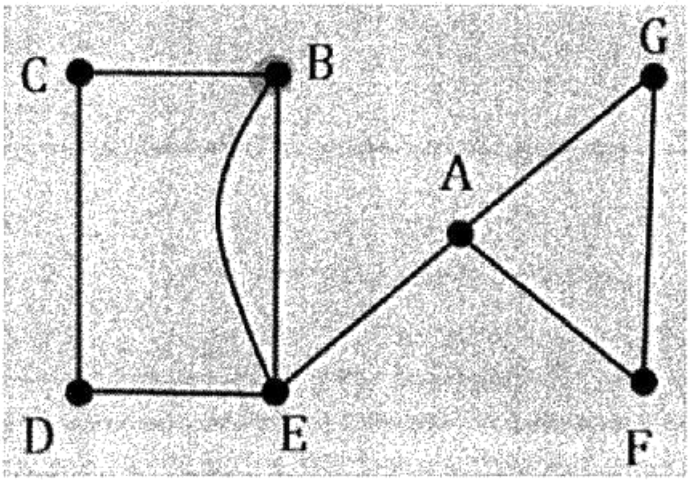
All nodes are now permanent. The predecessors form a tree. The shortest path from node A to node G can be found by tracing back predecessors; in this case, the shortest path from node A to node G is ACDG with a cost of 9. Dijkstra's algorithm is finished.

Question 4:

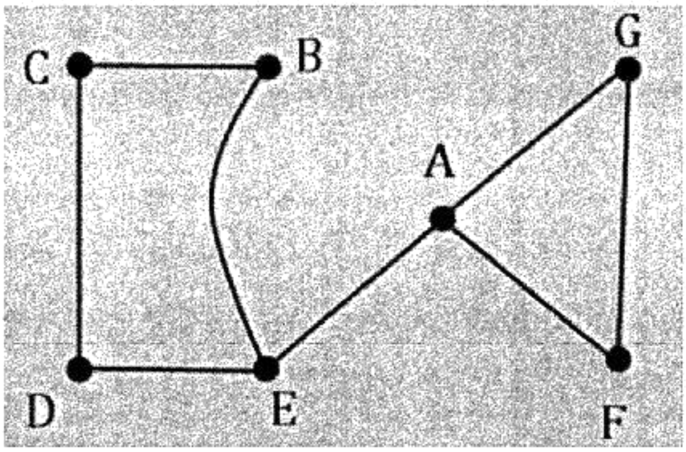
a) You cannot form an Euler circuit/cycle using that network – Euler circuit/cycle requires you to form a tour starting and finishing at the same node visiting every edge – since there are four nodes with odd degree. The number of edges with odd degree should be exactly 0 to allow forming of an Euler circuit/cycle.

b) Let's first look at the degrees of all vertices: A has a degree of 4, B has a degree of 4, C has a degree of 2, D has a degree of 2, E has a degree of 4, F has a degree of 2 and finally G has a degree of 2. So this graph allows us to form an Euler circuit/cycle as it is connected.

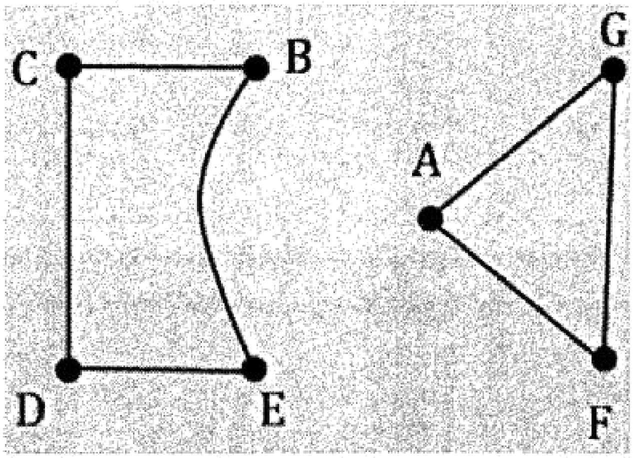
Let's start at vertex A. We have the freedom to choose edges AB, AC, AF and AG. Let's choose edge AB, circuit so far (csf) is AB.



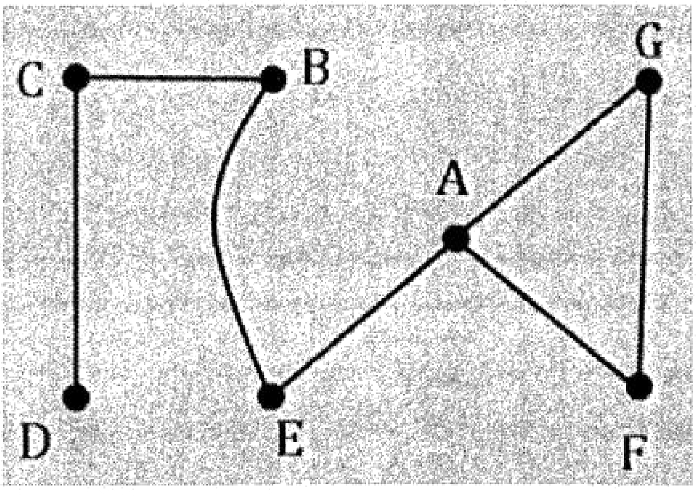
Edge AB is deleted. Now, let's choose edge BE and csf is ABE.



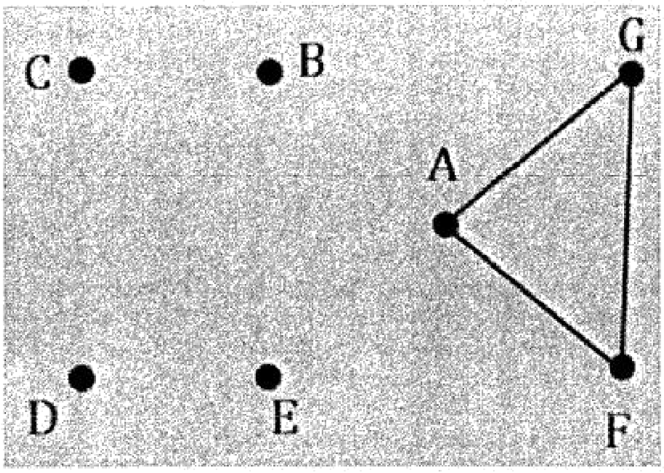
Edge BE is deleted. Now we need to be careful, if we choose edge EA, the graph will not be connected – there will be two disconnected graphs.



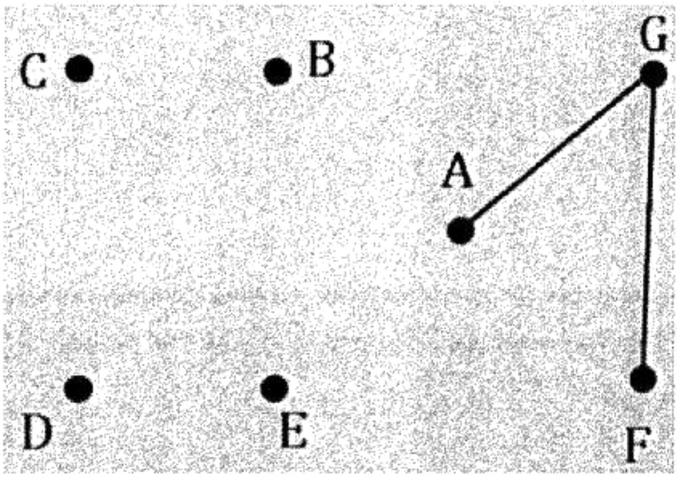
So, let's choose edge ED and csf is ABED.



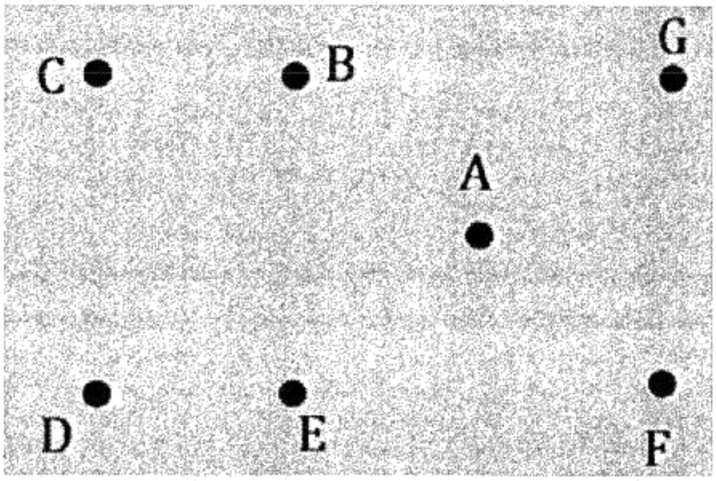
Remaining graph has one option for the next several choices, so let's skip those steps and have csf as ABEDCBEA.



Now, we can choose AF or AG. Let's choose AF and **csf** is ABEDCBEAF.



Again we have one option in each iteration from now on. So, **csf** is ABEDCBEAFGA.



Notice that all of the edges has been deleted, hence the Euler circuit is complete and it is ABEDCBEAFGA

Question 5:

a) Joey Crusher dominates Lenny Leonard.

Joey has more work experience and a higher CGPA.

All remaining attributes are the same.

That's why Joey will always be preferred over Lenny.

b) There are many ways to scale the criteria. The most basic one is to use the "best" and "worst" values of each criteria as its scale. However, for performance measures like CGPA, we know that there is already a preferred scale; then we could also make use of it in this context. A sample scale is shown below:

	Best	Worst	
Work Experience	8	0	→ max
CGPA	4.0	2.0	→ max
WTW Night Shifts	Willing	Not Willing	→ max
Salary Expectation	2800	3500	→ min
Eagerness to Work	Enthusiastic	Not motivated enough	→ max

You might come up with a different scale (as long as you explain your reasoning)

c) Normalize the values

$$\text{Rating} = \frac{\text{Score} - \text{Worst}}{\text{Best} - \text{Worst}} \rightarrow \text{if max}$$

$$\text{Rating} = \frac{\text{Worst} - \text{Score}}{\text{Worst} - \text{Best}} \rightarrow \text{if min}$$

	Work Exp.	CPGA	WTW Night Shifts	Salary Exp.	Eagerness to work
Herbert Powell	1	0.1	0	0	0
Lisa Simpson	0	0.7	0	1	1
Wayne Slater	0.75	0.6	1	0.14	0.5
Joey Crusher	0.5	0.5	0.5	0.71	0.5
Eugene Fisk	0.25	0.8	1	0.71	1

d) $0.2+0.25+0.15=0.6$

Let w denote the weight for the "Eagerness to work" criterion.

Then $(0.4-w)$ is the weight for "WTW Night Shifts" criterion.

Using this substitution, we could calculate the overall scores of each candidate (in terms of w):

- Herbert Powell: $0.2+(0.25)0.01=0.2025$
- Lisa Simpson: $(0.7)0.25+(0.15)1+w.1=0.325+w$
- Wayne Slater: $(0.75)0.2+(0.6)0.25+1(0.4-w)+(0.14)0.15+0.5w=0.721-0.5w$
- Joey Crusher: $(0.5)0.2+(0.5)0.25+(0.5)(0.4-w)+(0.71)0.15+0.5w=0.5315$
- Eugene Fisk: $(0.25)0.2+(0.8)0.25+1(0.4-w)+(0.71)0.15+1w=0.7565$

We know that $w \geq 0$; therefore $0 \leq w \leq 0.4$

- 3 of the candidates have overall scores independent from w: Herbert, Joey, Eugene.

No candidate could reach 0.7565 even the w is 0.4.

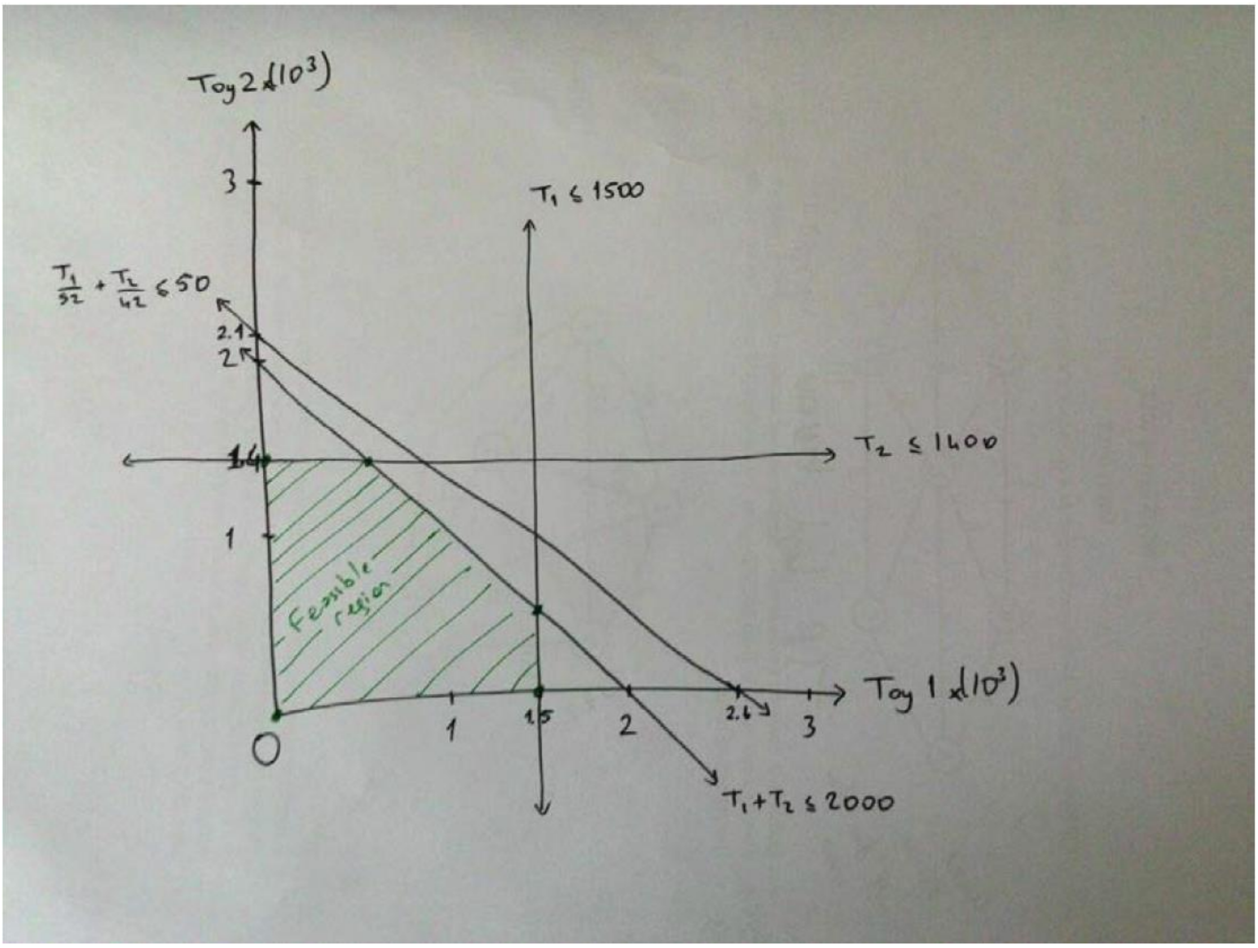
Therefore Eugene Fisk is the candidate to hire no matter what the w value is.

Question 6:

a) First constraint is for relating production rates with total available hours for production. We cannot use more than the given time.

Second constraint is for market research which says that at most 2000 toys can be sold combined.
Third constraint is for maximum demand of Toy1, which is 1500.
Fourth constraint is for maximum demand of Toy2, which is 1400.
Fifth and the constraint is non-negativity (integrality) constraint. It denotes that the produced amounts for each type of toy cannot be nonnegative.

b) A correction here, let's use the right hand side of first constraint as 50 rather than 420, for scaling reasons. And let's denote the axes as to the power of 10 to the 3, just to have a better visualization.



The objective function is 12T1 + 16T2, so if we put the isoprofit line onto the graph, we obtain (600, 1400) as our optimal solution. And the total profit is $29,600.

Question 7:

a)

	a	b	c	d	e	f
a	–	1	4	4	7	3
b	1	–	3	3	6	2
c	4	3	–	2	4	3
d	4	3	2	–	3	1
e	7	6	4	3	–	4
f	3	2	3	1	4	–

b) There are $\binom{6}{2} = 15$ combinations: List all combinations and respective travel times to reach each district.

	ab	ac	ad	ae	af	bc	bd	be	bf	cd	ce	cf	de	df	ef
a	0	0	0	0	0	1	1	1	1	4	4	3	4	3	3
b	0	1	1	1	1	0	0	0	0	3	3	2	3	2	2
c	3	0	2	4	3	0	2	3	3	0	0	0	2	2	3
d	3	2	0	3	1	2	0	3	1	0	2	1	0	0	1
e	6	4	3	0	4	4	3	0	4	3	0	4	0	3	0
f	2	3	1	3	0	2	1	2	0	1	3	0	1	0	0
TOTAL TIME	14	10	7	11	9	9	7	9	9	11	12	10	10	10	9

If total sum of time reaching every district is minimized, ad OR bd should be selected as fire station location combinations.

c) To cover every district in shortest possible time, the worst time accessing a district should be minimum. Therefore, we should focus on column (or row) maximums to find the worst time for accessing a district from an open facility.

From a : 7 (to e)
From b : 6 (to e)
From c : 4 (to a or e)
From d : 4 (to a)
From e : 7 (to a)
From f : 4 (to f)

The fire station should be located at C OR D OR F in order to obtain minimum coverage time (which is 4).

d) Construct 0/1 coverage matrix for $\gamma = 3$.

If time $> 3 \Rightarrow 0$.
time $\leq 3 \Rightarrow 1$.

	a	b	c	d	e	f
a	1	1	0	0	0	1
b	1	1	1	1	0	1
c	0	1	1	1	0	1
d	0	1	1	1	1	1
e	0	0	0	1	1	0
f	1	1	1	1	0	1

i) The locations that satisfy full coverage of the city are:

ad, bd, be, df, ef. (when you sum up the rows or columns in 0/1 matrix for those combinations, all entries are at least 1.)

ii)

	ad	bd	be	df	ef
Sum of time reaching each district	7	7	9	10	9
Worst time for reaching a district	3	2	3	3	3
Number of twice covered districts	2	4	1	4	1

iii)

	Best	Worst	
Scale of criterion 1:	7	10	→ To be minimized
Scale of criterion 2:	2	3	→ To be minimized
Scale of criterion 3:	4	1	→ To be maximized

To normalize the ratings apply the following formulas: $\text{Rating} = \frac{\text{Score} - \text{Worst}}{\text{Best} - \text{Worst}}$ (if max)

$\text{Rating} = \frac{\text{Worst} - \text{Score}}{\text{Worst} - \text{Best}}$ (if min)

	ad	bd	be	df	ef
Crit 1	1	1	0.67	0	0.67
Crit 2	0	1	0	0	0
Crit 3	0.34	1	0	1	0

The results are rounded to the nearest 2 decimals.

IV) bd has 1's all over its column, that's why we could infer that it is the best combination without making any computations. To rank the others, let's make the necessary computations.

$$\text{Score}(ad) = 1 \cdot 0{,}2 + 0 + 0{,}34 \cdot 0{,}5 = 0{,}37$$

$$\text{Score}(bd) = 1 \cdot 0{,}2 + 1 \cdot 0{,}3 + 1 \cdot 0{,}5 = 1$$

$$\text{Score}(be) = 0{,}67 \cdot 0{,}2 + 0 + 0 = 0{,}134$$

$$\text{Score}(df) = 0 + 0 + 1 \cdot 0{,}5 = 0{,}5$$

$$\text{Score}(ef) = 0{,}67 \cdot 0{,}2 + 0 + 0 = 0{,}134$$

Then the preference order is:

$$bd \succ df \succ ad \succ be \sim ef$$

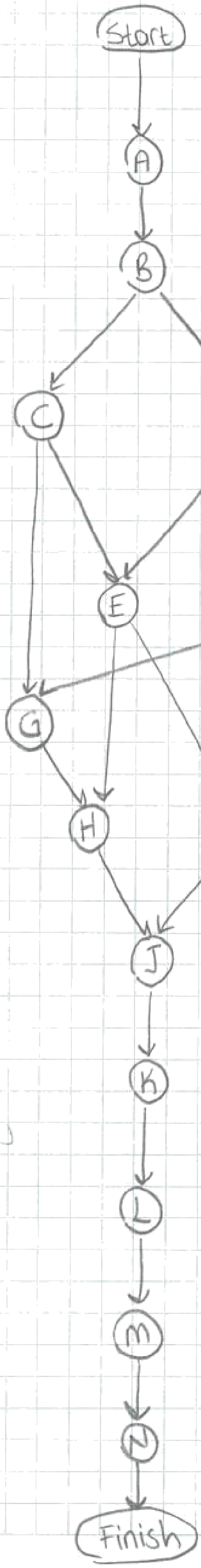
Remark: "$\succ$" is the preference operator.

$x \succ y$ denotes that x is strongly preferred over y.

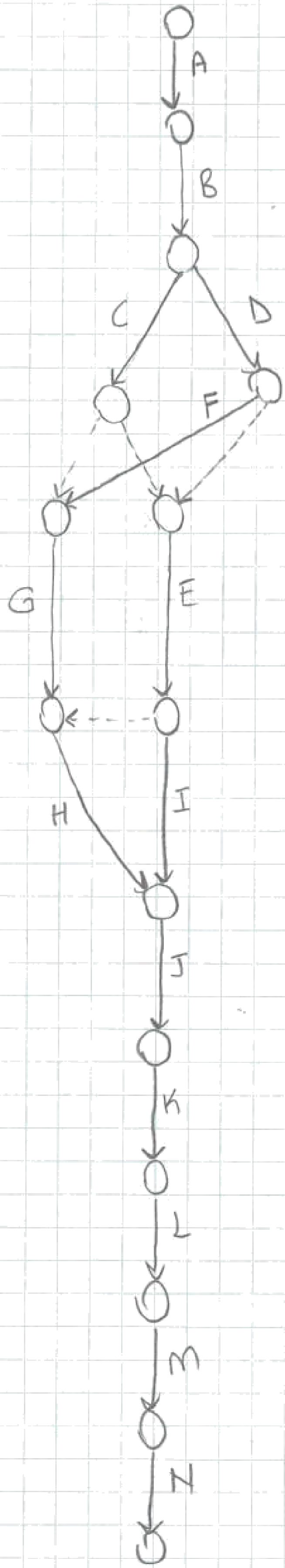
Similarly, "$\sim$" shows the indifference between two options.

Question 8:

a) AON network:



b) AOA network:



c) & d)

Activity	ES	EF	LS	LF	Slack	On Critical Path?
A	0	5	0	5	0	YES
B	5	8	5	8	0	YES
C	8	10	20	22	12	NO
D	8	12	8	12	0	YES
E	12	20	22	30	10	NO
F	12	24	12	24	0	YES
G	24	30	24	30	0	YES
H	30	34	30	34	0	YES
I	20	22	32	34	12	NO
J	34	39	34	39	0	YES
K	39	42	39	42	0	YES
L	42	43	42	43	0	YES
M	43	46	43	46	0	YES
N	46	48	46	48	0	YES

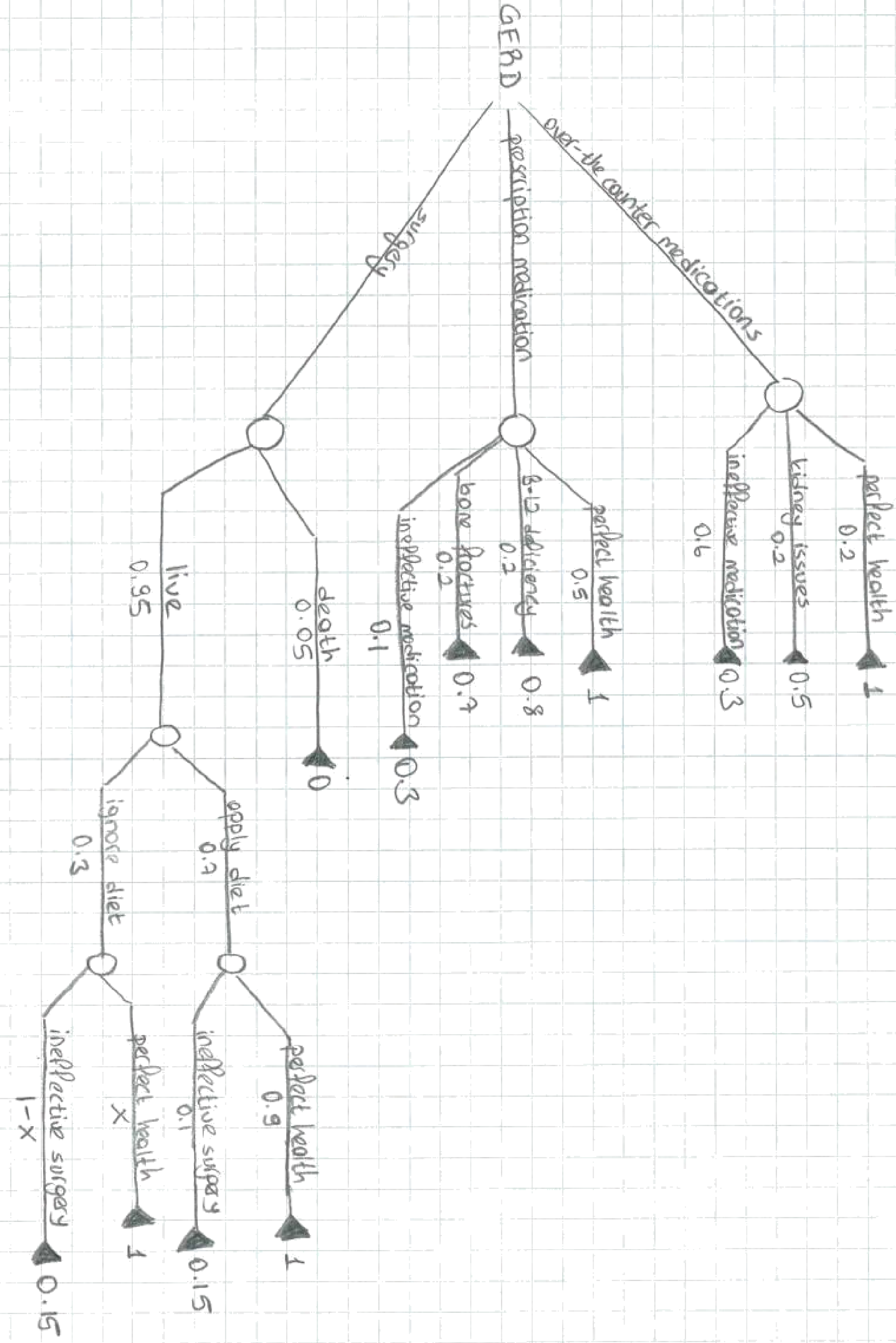
e) Critical Path:

A, B, D, F, G, H, J, K, L, M, N

Completion Time=48 days

Question 9:

Decision Tree:



$u(\text{over-the counter medications}) = 0.2\times1 + 0.2\times0.5 + 0.6\times0.3$

$= 0.48$

$u(\text{prescription medication}) = 0.5\times1 + 0.2\times0.8 + 0.2\times0.7 + 0.1\times0.3$

$= 0.83$

In order to select surgery option, utility of surgery should be greater than 0.83.

$u(\text{surgery}) = 0.05\times0 + 0.95\times[(0.7)\times(0.9\times1 + 0.1\times0.15) + (0.3)\times(x + (1-x)0.15)]$

$= 0.95(0.6855 + 0.3\times0.85x)$

$= 0.651 + 0.242x$

$0.651 + 0.242x > 0.83$

$0.242x > 0.179$

$x > 0.74$

Question 10:

i)

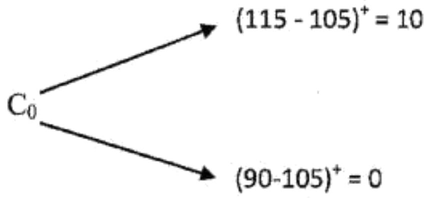
a) $u = 115/100 = 1.15$

$d = 90/100 = 0.9$

$1+r = 105/100 = 1.05$, $r = 0.05$

b) $d - 1 = -0.1 < r = 0.05 < u - 1 = 0.15$. By the fundamental theorem of asset pricing, we conclude that the model satisfies the principle of no-arbitrage.

c) The payoff price tree for the call is as follows:

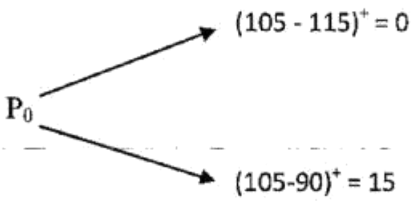


Let (x,y) be a replicating strategy. It satisfies the system.

$$\left.\begin{aligned} 115x + 105y &= 10 \\ 90x + 105y &= 0 \end{aligned}\right\} \; 25x = 10, x = \frac{2}{5} \;\&\; y = -\frac{12}{35}$$

Hence the call price is $C_0 = 100x + 100y = 40/7$.

d) The payoff price tree is as follows:



Let (x,y) be a replicating strategy. It satisfies the system.

$$\left.\begin{aligned} 115x + 105y &= 0 \\ 90x + 105y &= 15 \end{aligned}\right\} \; 25x = -15, x = -\frac{3}{5} \;\&\; y = \frac{23}{35}$$

Hence the price is $P_0 = 100x + 100y = 40/7$.

e)

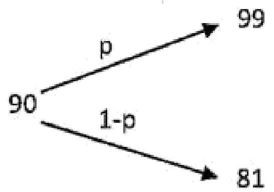
$115x + 105y = 115 - F$ (1)

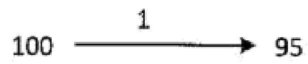
$90x + 105y = 90 - F$ (2)

$100x + 100y = 0$, $y = -x$

$$\left.\begin{aligned}(1)\,115x - 105x = 10x = 115 - F\\(2)\,90x - 105x = -15x = 90 - F\end{aligned}\right\} -\frac{2}{3} = \frac{115 - F}{90 - F}, F = 105 = 100(1 + 0.05)$$

ii)





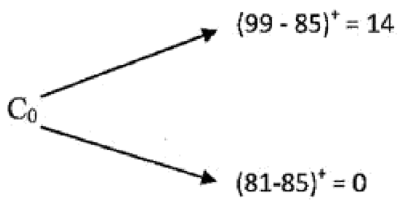
a) $u = 99/90 = 1.1$

$d = 81/90 = 0.9$

$1+r = 95/100 = 0.95$, $r = -0.05$

b) $d - 1 = -0.1 < r = -0.05 < u - 1 = 0.1$. By the fundamental theorem of asset pricing, we conclude that the model satisfies the principle of no-arbitrage.

c) The payoff price tree for the call is as follows:

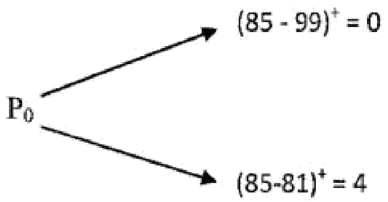


Let (x,y) be a replicating strategy. It satisfies the system.

$$\left.\begin{aligned}99x + 95y &= 14\\81x + 95y &= 0\end{aligned}\right\} 18x = 14, x = \tfrac{7}{9} \,\&\, y = -\tfrac{63}{95}$$

Hence the call price is $C_0 = 90x + 100y = 70/19$.

d) The payoff price tree is as follows:



Let (x,y) be a replicating strategy. It satisfies the system.

$$\left.\begin{aligned} 99x + 95y &= 0 \\ 81x + 95y &= 4 \end{aligned}\right\} \; 18x = -4, x = -\frac{2}{9} \,\&\, y = \frac{22}{95}$$

Hence the price is $P_0 = 100x + 100y = 60/19$.

e)

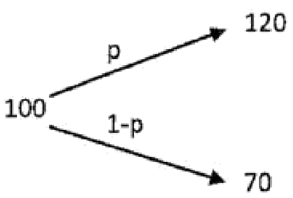
$99x + 95y = 99 - F$ (1)

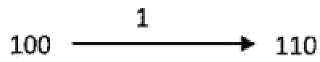
$81x + 95y = 81 - F$ (2)

$90x + 100y = 0$, $x = -10y/9$

$$\left.\begin{aligned} (1) - 110y + 95y &= -15y = 99 - F \\ (2) - 90y + 95y &= 5y = 81 - F \end{aligned}\right\} -3 = \frac{99-F}{81-F}, F = 85.5 = 90(1-0.05)$$

iii)





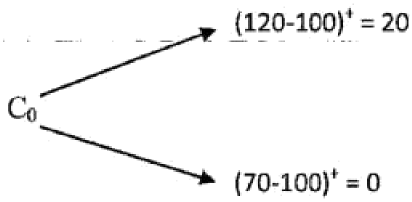
a) $u = 120/100 = 1.2$

$d = 70/100 = 0.7$

$1+r = 110/100 = 1.10$, $r = 0.1$

b) $d - 1 = -0.3 < r = 0.1 < u - 1 = 0.2$. By the fundamental theorem of asset pricing, we conclude that the model satisfies the principle of no-arbitrage.

c) The payoff price tree for the call is as follows:

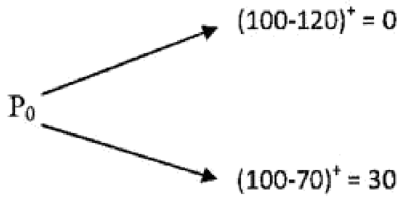


Let (x,y) be a replicating strategy. It satisfies the system.

$$\left.\begin{aligned} 120x + 110y &= 20 \\ 70x + 110y &= 0 \end{aligned}\right\} \; 50x = 20, x = \frac{2}{5} \,\&\, y = -\frac{14}{55}$$

Hence the call price is $C_0 = 100x + 100y = 160/11$.

d) The payoff price tree is as follows:



Let (x,y) be a replicating strategy. It satisfies the system.

$$\left.\begin{aligned} 120x + 110y &= 0 \\ 70x + 110y &= 30 \end{aligned}\right\} \; 50x = -30, x = -\frac{3}{5} \,\&\, y = \frac{36}{55}$$

Hence the price is $P_0 = 100x + 100y = 60/11$.

e)

$120x + 110y = 120 - F$ (1)

$70x + 110y = 70 - F$ (2)

$100x + 100y = 0$, $y = -x$

$$\left.\begin{aligned} (1)\,120x - 110x &= 10x = 120 - F \\ (2)\,70x - 110x &= -40x = 70 - F \end{aligned}\right\} \; -\frac{1}{4} = \frac{120 - F}{70 - F}, F = 110 = 100(1 + 0.1)$$