IE102 FALL 2019

STUDY SET

Question 1: Suppose that a problem with decision variables x and y consists of the following objective function and constraints:

maximize $x + 5y$

subject to $x + 2y \leq 5$

$3x - 2y \geq 6$

$x - 4y \geq 0$

$x, y \geq 0$

a) Draw the feasible region.
b) Solve the problem graphically. What is the optimal solution and objective function value?

Question 2: Suppose that a problem with decision variables x and y consists of the following constraints:

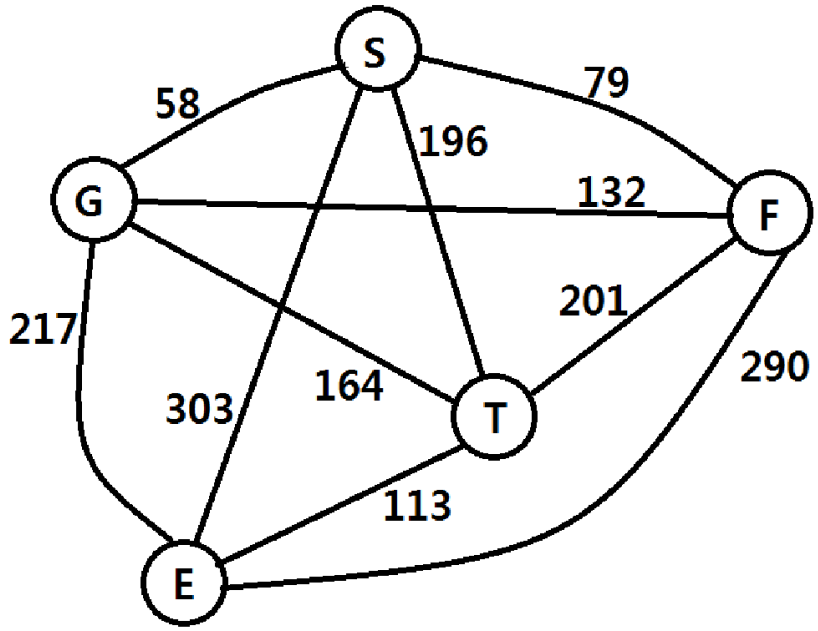
$x - y \leq 2$

$y - 2x \leq 4$

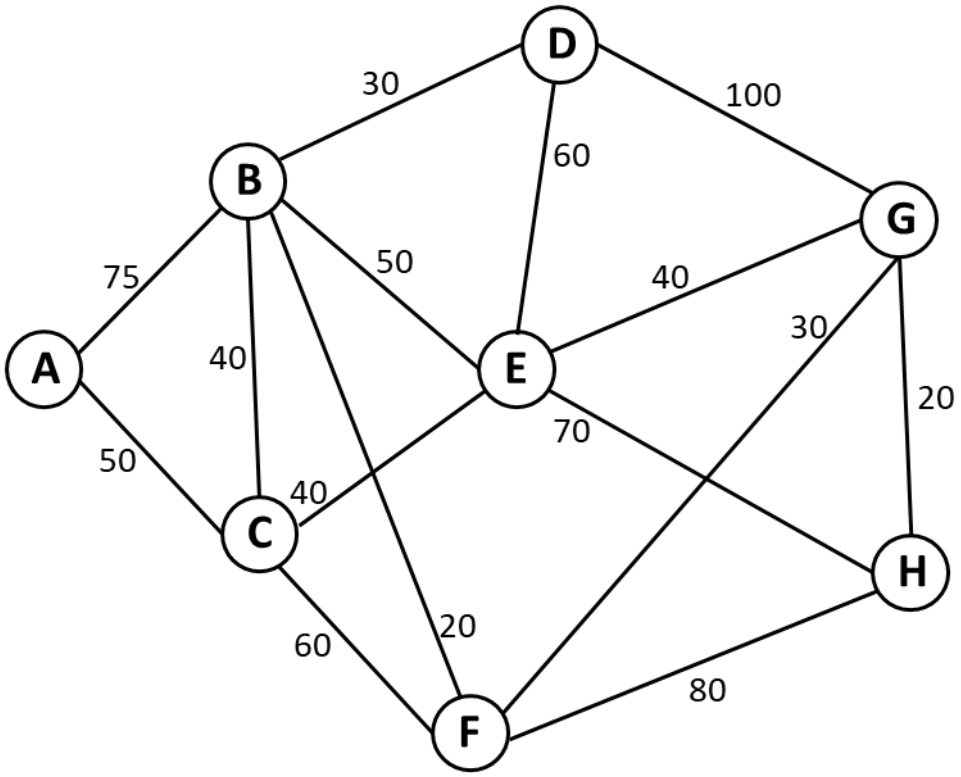
$x, y \geq 0$

a) Draw the feasible region.
b) Find a linear objective function for which the problem has a unique optimal solution and justify your answer by solving the problem graphically.
c) Find a linear objective function for which the problem has alternative optimal solutions and justify your answer by solving the problem graphically. Provide two optimal solutions and objective function value.
d) Find a linear objective function for which the problem is unbounded and justify your answer by solving the problem graphically.
e) Can this problem be infeasible? If not, what do you need to do to make it infeasible?

Question 3: Solve the below network applying Nearest Neighbour algorithm to obtain a TSP tour starting from node S. Then apply 2-opt to the obtained TSP tour. Provide the tours and the corresponding costs.

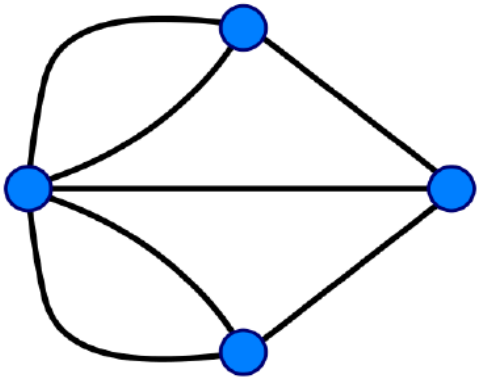


Question 4: In the network represented below, the numbers on the arcs correspond to the distance between the nodes it is connecting. Find the shortest path from Node A to Node H by Dijkstra's algorithm.

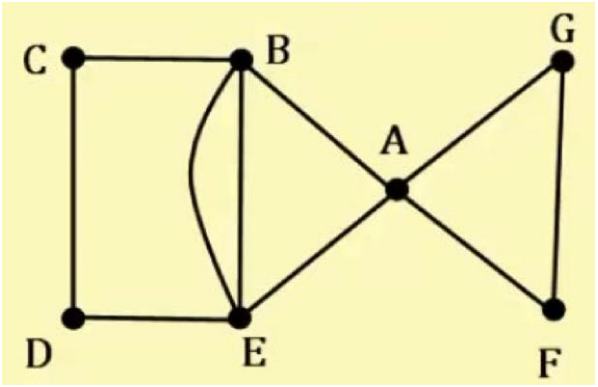


Question 5: Try and answer following questions:

a) Can you form an Eulerian circuit/cycle for the following network? Why/Why not? Note that finding a Hamiltonian path and circuit/cycle is trivial for this network.



b) One can find an Eulerian circuit/cycle by trial and error but you will find one for the below network using Fleury's Algorithm, which is useful when the network is rather complicated. First check whether an Eulerian circuit/cycle can exist and create one starting from vertex A. Justify each step.



Question 6: The head physician of Springfield General Hospital, Mr. Burns, would like to hire a new nurse for the cardiology unit. He interviewed 6 job applicants and recorded the noteworthy (good or bad) attributes of each candidate. His secretary Jacqueline has put together all the information that Mr. Burns has conveyed to her about the applicants. The collected data is shown below:

Applicant Name	Work Experience	CGPA	Willingness to Work Night Shifts	Salary Expectation (per month)	Eagerness to Work
Herbert Powell	8 years	2.2	Not Willing	$3500	Not motivated enough
Lisa Simpson	Not available	3.4	Not Willing	$2800	Enthusiastic
Lenny Leonard	2 years	2.5	Neutral	$3000	Motivated
Wayne Slater	6 years	3.2	Willing	$3400	Motivated
Joey Crusher	4 years	3	Neutral	$3000	Motivated
Eugene Fisk	2 years	3.6	Willing	$3000	Enthusiastic

"**Enthusiastic**" is a **stronger** emotion compared to "**Motivated**".

a) Help Jacqueline to eliminate the applicant(s) which is (are) dominated without doing any computations. Clearly indicate who dominates who and why. Eliminate that applicant(s) from the data table.
b) Jacqueline was unsure on how to scale each criterion. Develop a scaling methodology for each and explain your logic briefly.
c) Rescale the given data to common units with the scale you developed.
d) Mr. Burns has a difficulty coming up with the weights of the criteria. He is unsure about 2 of them but determined the weights for other 3 as follows:

Work Experience: 0.2 CGPA: 0.25 Salary Expectation: 0.15

Make an analysis on the weight of the "Eagerness to Work" criterion. Indicate the range of values that makes each applicant be the best choice for the job.

Question 7: ToysRS is planning to produce 2 new toys. Total available hours for production are 420 hours. Production rates are 52 toys/hour and 42 toys/hour for Toy1 and Toy2, respectively. Profit per unit is $12 and $16 for Toy1 and Toy2, respectively.
Demands for these toys are expected to be pretty high. However, market research reveals that total number of toys that can be sold is 2000. Moreover, maximum demands are estimated to be 1500 and 1400 for Toy1 and Toy2, respectively.
The decision variables are:

T_1: Amount produced of Toy1
T_2: Amount produced of Toy2

The mathematical model for this problem is given as:

Max $12T_1 + 16T_2$
Subject to

$$\frac{T_1}{52} + \frac{T_2}{48} \leq 420$$

$$T_1 + T_2 \leq 2000$$

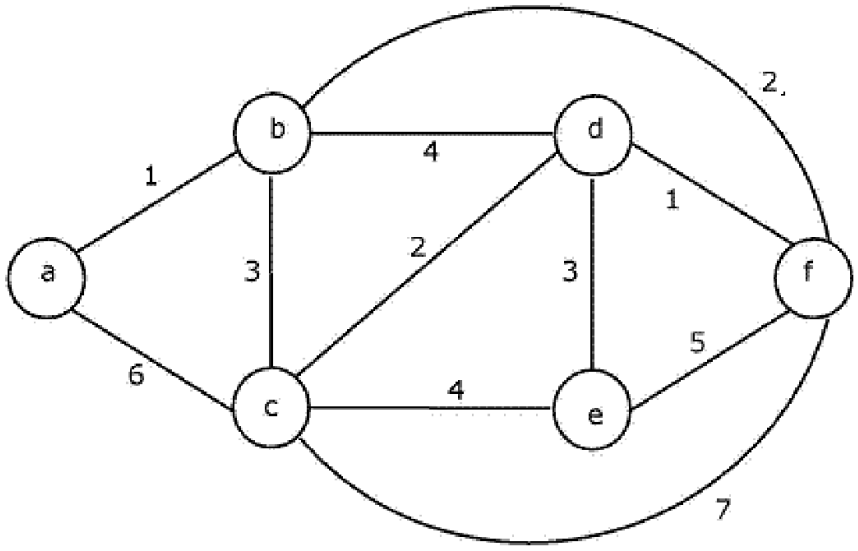
$$T_1 \leq 1500$$

$$T_2 \leq 1400$$

$$T_1, T_2 \geq 0$$

a) Explain the meaning of each constraint.
b) Solve the model graphically and provide the optimal solution along with the corresponding amounts for each type of toy.

Question 8: Central City is going to establish new fire stations. It is known that there are 6 major districts in the city (a…f). The exact locations of these districts are shown on the graph below, along with the travel time values between pairs (in minutes). The stations can only be located on the nodes, not on the edges.



a) Construct the travel time matrix.

b) What are the optimal fire station locations if we want to minimize the total time of reaching every district while locating two stations.

c) What is the minimum allowable time to cover every district in the shortest possible time using a single station? (i.e. indicate the minimum time that every city is reachable by a station) On which location(s) should the fire station be located at in order to obtain this minimum time?

d) Suppose that the allowable time is 3 minutes for full-coverage.

i. Find the feasible solutions with two stations. Please indicate all the combinations satisfying the full-coverage of the city.

ii. In order to evaluate each feasible location combination, find the values for the following criteria.

- Total of time reaching every district
- The worst time of reaching a district
- Number of twice covered districts

iii. Scale the values you found on the previous part to common units. The range of each criterion should be between the minimum and maximum values found in part ii.

iv. Suppose that the weight of each criterion is as shown below. List the location combinations in preference order according to their total scores.

- Total sum of time reaching each district: 0.2
- The worst time of reaching a district: 0.3
- Number of twice covered districts: 0.5

Question 9: Consider the activities, their predecessors and completion durations given below:

Activity	Immediate Predecessors	Estimated Duration
A	-	5 days
B	A	3 days
C	B	2 days
D	B	4 day
E	C, D	8 days
F	D	12 days
G	C, F	6 days
H	E, G	4 days
I	E	2 days
J	H, I	5 days
K	J	3 days
L	K	1 days
M	L	3 days
N	M	2 days

a) Draw the AON (Activity on Node) network.
b) Draw the AOA (Activity on Arc) network.
c) Perform the critical path activities (i,e., calculate Earliest start (ES), Earliest finish(EF), Latest start(LS) and Latest finish(LF) for each activity).
d) After computing the ES, EF, LS, and LF times for all activities, compute the slack or free time for each activity.
e) Decide the critical path and state the completion time of entire project.

Question 10: A patient having gastroesophageal reflux disease (GERD) goes to hospital for remedy. Several gastroenterologists agree that the GERD should be treated but disagree on the preferred treatment. The choice is between three options:

1. lifestyle modifications and over-the-counter medications
2. prescription medication
3. surgery

One gastroenterologist recommends over-the-counter medications by taking antacids that neutralize stomach acid. There is no operative mortality in this treatment, however; overuse of some acids can cause side effects. The gastroenterologist estimates that there is 20% chance of recovering perfect health, 20% chance of recovering with kidney issues, and a %60 of over-the-counter medication being ineffective.

Second gastroenterologist recommends prescription medications. Again, there is no operative mortality in this treatment, but side effects can be encountered. This gastroenterologist estimates that there is %50 chance of recovering with perfect health, 20% chance of recovering with vitamin B-12 deficiency, 20% chance of recovering with bone fractures, and a 10% chance of prescription medication being ineffective.

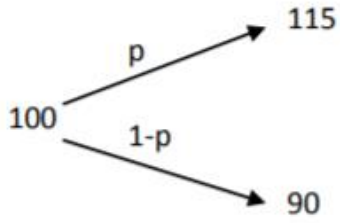
Third gastroenterologist recommends surgery called fundoplication. This surgery requires general anaesthesia. The operative mortality for this patient is as high as 5%. The doctor states that, if the patient lives, the patient needs to change his diet for surgery to be effective. There is 30% chance of patient to obey to the recommended diet and 70% chance of to ignore the recommendations. If the patient applies appropriate diet, there is 90% chance of recovering fully, 10% chance of operation being ineffective. If the patient ignores recommendations, there is **X%** chance of recovering fully, or otherwise the operation is ineffective.

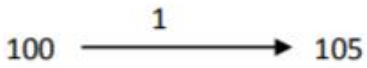
Using the utilities in the table below, draw the decision tree and decide on the X value for surgery option to be selected. What is the expected utility of the surgery with the corresponding X value?

Outcome	*Utility*
Complete recovery	1
Recovery with B-12 deficiency	0.8
Recovery with bone fractures	0.7
Recovery with kidney issues	0.5
Ineffective over the-counter medication or prescription medication	0.3
Ineffective surgery	0.15
Operative death	0

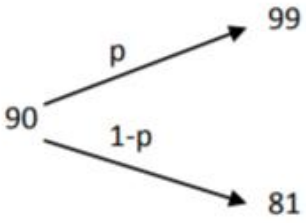
Question 11: Consider the following one-period binomial models for a stock and a bond with the price trees.

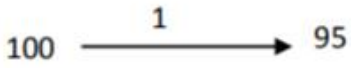
i)



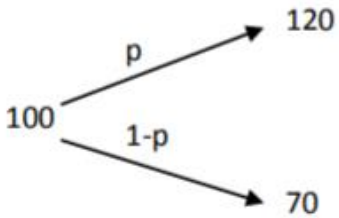


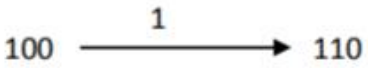
ii)





iii)





a) For each model, what are the up factor, down factor and interest rate?
b) Does these models satisfy the principle of no-arbitrage?
c) Consider a call option written on the stock with strike price K = 105 for (i), K = 85 for (ii) and K = 100 for (iii). Find a replicating strategy and the price of this option for each model.
d) Consider a put option written on the stock with strike price K = 105 for (i), K = 85 for (ii) and K = 100 for (iii). Find a replicating strategy and the price of this option for each model.
e) Consider a forward contract with a predetermined price F. The contract has zero price today as a rule. What should F be in each of these markets?

Question 12:

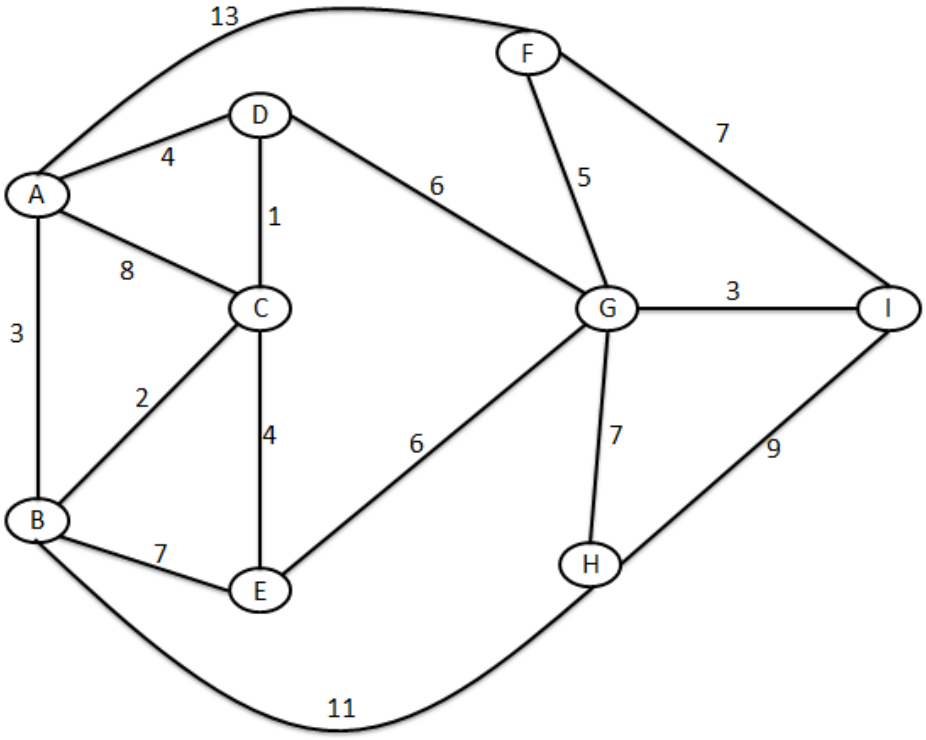
You are the maintenance and construction superintendent responsible for the project given below. The following data has been provided for your benefit.

Symbol	Time (Hrs.)	Precedents
A	8	-
B	200	A
C	12	-
D	4	I,M
E	8	-
F	40	B
G	32	F,L
I	8	G,K
J	225	A
K	8	J,L
L	35	C,E
M	24	G,K
N	6	I
O	4	D,N

a) Draw the AON (Activity on Node) network.
b) Draw the AOA (Activity on Arc) network.
c) Perform the critical path activities (i,e., calculate Earliest start (ES), Earliest finish(EF), Latest start(LS) and Latest finish(LF) for each activity).
d) After computing the ES, EF, LS, and LF times for all activities, compute the slack or free time for each activity.
e) Decide the critical path and state the completion time of entire project.

Question 13:

The aim is to establish the ambulance stations. It is known that there are 6 major districts in the city (A…I). The exact locations of these districts are shown on the graph below, along with the travel time values between pairs (in minutes). The stations can only be located on the nodes, not on the edges.



a) Construct the travel time matrix.
b) What are the optimal fire station locations if we want to minimize the total time of reaching every district while locating two stations.
c) What is the minimum allowable time to cover every district in the shortest possible time using a single station? On which location(s) should the fire station be located at in order to obtain this minimum time?
d) Suppose that the allowable time is 7 minutes for full-coverage.
   i. Find the feasible solutions with two stations. Please indicate all the combinations satisfying the full-coverage of the city.
   ii. In order to evaluate each feasible location combination, find the values for the following criteria.
   • Total of time reaching every district
   • Number of twice covered districts
   iii. Scale the values you found on the previous part to common units. The range of each criterion should be between the minimum and maximum values found in part ii.
   iv. Suppose that the weight of each criterion is as shown below. List the location combinations in preference order according to their total scores.
   • Total sum of time reaching each district: 0.6
   • Number of twice covered districts: 0.4