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Modelling the Costs and Benefits of Delayed Product Differentiation

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Expanding product variety and high customer service provision are both major challenges for manufacturers to compete in the global market. In addition to many ongoing programs, such as lead-time reduction, redesigning products and processes so as to delay the point of product differentiation is becoming an emerging means to address these challenges. Such a strategy calls for redesigning products and processes so that the stages of the production process in which a common process is used are prolonged. This product/process redesign will defer the point of differentiation (i.e., defer the stage after which the products assume their unique identities). In this paper, we develop a simple model that captures the costs and benefits associated with this redesign strategy. We apply this simple model to analyze some special cases that are motivated by real examples. These special cases enable us to formalize three different product/process redesign approaches (*standardization, modular design, and process restructuring*) for delaying product differentiation that some companies are beginning to pursue. Finally, we analyze some special theoretical cases that enable us to characterize the optimal point of product differentiation and derive managerial insights.

(*Product / Process Redesign; Product Differentiation; Mathematical Models*)

1. Introduction

Companies with expanding product varieties are faced with increasing problems in getting accurate demand forecasts for different products, controlling the proliferation of inventory, and providing high service for the customers. To compete in the world market, product variety is needed for marketing and sales promotions. However, product variety has a significant impact on inventory level and service performance. In this paper, we are not concerned with the marketing issue of product portfolio (We refer the reader to Fisher (1992) for an excellent overview on product portfolio and to Child et al. (1991) for a comprehensive discussion of how a company can reorganize its operations and strategies to address the issue of product variety.) Instead, we consider a situation in which the company has determined its product portfolio, and we focus on the evaluation of a powerful concept that some companies are beginning

to pursue (Lee (1993 and 1996)). This concept is to redesign the product or the production process so that the point of differentiation (i.e., the stage after which the products assume their unique identities) is delayed as much as possible. (The concept of delayed product differentiation was first introduced in the marketing literature by Alderson (1950).)

Delaying the point of differentiation implies that the process would not commit the work-in-process into a particular finished product until a later point. This product/process redesign would increase the "flexibility" of the process to cope with the market uncertainties and lower the inventory for the same target service level. Delayed product differentiation through product/process redesign offers such an opportunity. In many cases, multiple end-products may share some common components or processes at the initial stages of the production process. At some point in the process,

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specialized components or processes are then used to customize the work-in-process (which was a generic product up to that point in the process) into the different end-products. Such a point is usually known as the point of product differentiation. The concept of delayed product differentiation has been used quite extensively in the logistics and distribution side of business. The reader is referred to Shapiro and Heskett (1985), van Doremalen and Fleuren (1991), Zinn and Bowersox (1988), and Zinn (1990) for various examples in which the point of product differentiation is deferred.

There would, of course, be some fixed and variable costs associated with product/process redesign for delayed product differentiation. Depending on the situation, the unit processing cost and the unit inventory holding cost may increase as a result of such redesign. The reader is referred to Lee and Billington (1994) for a detailed discussion on various cost drivers associated with delayed product differentiation. The benefit of delayed product differentiation in the form of inventory reduction or service improvement is actually quite similar in many aspects to the pooling effect of multiechelon inventory systems (Eppen and Schrage (1981), Federgruen and Zipkin (1984), and Schwarz (1989)). In the context of multiechelon inventory systems, a central warehouse procures products and in turn supplies multiple retailers. Lee (1996) described a model that captures the inventory reduction of delayed product differentiation that is essentially an adaptation of the multiechelon inventory system. The model, however, assumes that no buffer stocks are held until the end of production process. Hence, the only possible inventory savings of delayed product differentiation would come from the reduction of finished goods inventory. Indeed, Lee (1994) showed that this is always the case, and that the savings are greater in cases where the demands for the different products are negatively correlated. A real example of delayed product differentiation by means of deferring the localization process for the deskjet printer at Hewlett-Packard (HP), where inventories are kept in finished goods form, is reported in Lee et al. (1993).

The current paper complements the model developed by Lee (1996) in the following ways. First, Lee's model assumes that no buffer stocks are held until the end of the production process (i.e., only finished goods inven-

tories are allowed), while ours allows for holding inventories at different points of the process. Second, the focus of Lee's paper is to show that the finished goods inventory can be reduced as a result of delayed product differentiation. In this paper, we develop a model that incorporates other factors that would normally be affected by delayed product differentiation. These factors include the design cost, processing cost, inventory cost at intermediate stages, lead times, etc. In addition, we discuss cases in which these cost factors may outweigh the benefit of delayed product differentiation, and hence, it may not be worthwhile to delay the product differentiation. Third, Lee's model is based on a general concept related to delayed product differentiation, while this paper offers specific approaches (such as standardization, modular design, and process restructuring) that serve as enablers for delayed product differentiation that some companies have used. Moreover, we present examples based on these approaches and we develop a simple model for generating some basic insights regarding the conditions under which delayed product differentiation could be effective.

In this paper, we formalize three basic approaches for delayed product differentiation that some companies have used. These approaches are (1) *standardization*; (2) *modular design*; and (3) *process restructuring*. First, standardization refers to using common components or processes. It has many advantages. It reduces the complexity of the manufacturing system, it increases the "flexibility" of use for the work in process inventories, and it improves the service level of the system (due to risk-pooling). Next, modular design refers to decomposing the complete product into submodules that can be easily assembled together. It enables the manufacturer to delay the assembly operation of certain "product-specific modules" so that the point of product differentiation is delayed. The current trend towards standardization and modular design makes such redesign process much more feasible (Ulrich (1991), and Ulrich and Tung (1991)). Third, process restructuring refers to resequencing process steps in making a product. In some cases, it is quite possible to delay the assembly operation of certain "product-specific components" by restructuring the manufacturing process so that common pro-

cess steps shared by multiple products are performed before the product specific process steps.

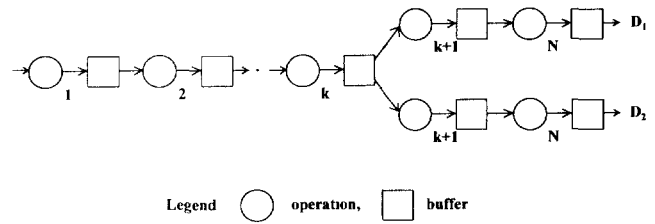
This paper is organized as follows. In the next §, we first develop a simple model to capture the costs and benefits associated with this process redesign strategy. Our model is intended for strategic planning rather than tactical planning. For this reason, we shall not model the detailed issue related to production planning and inventory control. Section 3 illustrates how our model can be applied to analyze some special cases motivated by real examples. These examples enable us to formalize three different product/process redesign approaches (standardization, modular design, and process restructuring) that some companies are beginning to pursue. In §4, we consider some special theoretical cases by imposing additional assumptions to our model. Then we analyze the optimal point of product differentiation, and derive managerial insights from the properties of the optimal point of product differentiation. This is followed by some concluding discussions and suggestions for future research.

2. The Model

To simplify the exposition of our model, we shall consider an existing manufacturing system that produces two end-products, where each end-product requires processes performed in N stages. (The same approach can be extended to the case of multiple products.) The manufacturing system has a buffer that stores the work-in-process inventory after each operation (or each stage). Throughout this paper, we shall use the term "stage" and the term "operation" interchangeably. The operations are numbered in ascending order. To focus on the issue of delayed product differentiation, we shall focus on a system in which the first k operations are *common operations* to both products (Figure 1). We shall refer to operation k as the *last common operation*. Throughout this paper, the products are considered to be "distinct" after the last common operation k (in the sense that the products require distinct operations after operation k). These distinct operations customize the products for different market segments.

As described in the introduction, the intent of this paper is to model a design concept that is known as

Figure 1 Products 1 and 2 Assume Their Identity After Operation k



delayed product differentiation (Lee (1993)). While there are many ways to delay product differentiation, we shall restrict our attention to the case in which one can delay the product differentiation by deferring the last common operation k (increasing the value of k). Thus, we shall use the expression "delay the product differentiation" and the expression "defer the last common operation k " interchangeably.

Consider the following extreme cases. First, suppose the decision maker decided to have specialized product line for each product. Then there is no common operation ($k = 0$). Next, because the products are different, one can delay the product differentiation by having at most $N - 1$ common operations. Therefore, the last common operation k can vary between 0 and $N - 1$, i.e., $0 \leq k \leq N - 1$. The extreme case where there is no more distinction between the products, i.e., $k = N$ (universal product), is not considered here.

Our model is a discrete time model. For each of the products i , where $i = 1, 2$, we assume that the demand of product i at the end of period t is denoted by an i.i.d. random variable $D_i(t)$, where $D_i(t)$ is normally distributed with

$$E(D_i(t)) = \mu_i, \quad \text{Var}(D_i(t)) = \sigma_i^2, \quad \text{for } i = 1, 2, \text{ and} \quad (2.1)$$

$$\text{Cov}(D_1(t), D_2(t)) = \rho \sigma_1 \sigma_2. \quad (2.2)$$

The normal assumption seems to be appropriate for products with high volumes. Notice that ρ represents the correlation of $D_1(t)$ and $D_2(t)$, where $-1 \leq \rho \leq 1$. For notational convenience, we let:

$$\sigma_{12} = \sqrt{\text{Var}[D_1(t) + D_2(t)]} = \sqrt{[\sigma_1^2 + 2\rho\sigma_1\sigma_2 + \sigma_2^2]}. \quad (2.3)$$

It is easy to check that $\sigma_{12} \leq \sigma_1 + \sigma_2$ for any ρ and that σ_{12} decreases as ρ decreases.

In order to evaluate the implications of delaying product differentiation; i.e., deferring the last common operation k , we shall develop a simple expression for the total relevant cost. The total relevant cost for a given system that has operation k as the last common operation includes the total investment cost for redesigning the product/process such that the manufacturing system has k common operations, the total processing costs, and the total inventory costs.

To develop a simple expression for the total relevant cost, we first assume that "sufficient" buffer inventories are held at each buffer located immediately after each operation (i.e., high service level at each stage) so that the entire system can be "decoupled" into N single-stage systems. Because high service level at each buffer reduces the interaction between stages (due to the rare occurrence of starvation or stockouts), it is adequate to control the production "locally" for each of the N stages. Such assumption is reasonable when the service level at each intermediate stage is sufficiently high, say, 90 percent or higher. We feel that high service level at each intermediate stage, say, 90 percent or higher, is a reasonable assumption for the following cases. First, high service level at each intermediate stage is called for when substantial value is added in each stage; for instance, aircraft manufacturing, jewelry manufacturing, and many other high technology manufacturing processes (The reader is referred to van Houtum and Zijm (1991) and Lagodimos (1992 and 1993) for detailed discussion on the impact of value added operations on the optimal service level.) Second, high service level at each intermediate stage has now been viewed as an "internal" quality measure, which is known as "internal guarantee" (Hart 1995).

High service level at each intermediate stage justifies the local production control of each stage. As a simple way to approximate the average inventory level at each buffer, we also assume that the safety stock at each buffer is replenished each period according to an "order-up-to level" policy (Silver and Peterson 1979). Better approximations of average inventory level can be obtained by using other methods presented in van Donsselaar (1989), de Kok (1990), Lee and Tang (1993), and Wijngaard and Wortmann (1985). However, these methods do not result in close-form expressions and are

more suitable for production planning. Because the focus of our paper is to analyze the managerial implications of three different product/process redesigns for delaying product differentiation, we shall keep our model simple. For this reason, we assume that the safety stocks are replenished according to an order-up-to level policy. For simplicity, we shall restrict our attention to the case in which the service levels for different buffers are the same. This assumption is reasonable when management is concerned about stockout at each buffer. In any event, the same approach can be applied to the case in which the service levels are different for different buffers. Finally, we assume that the quality of the output at each stage will not be affected by delayed product differentiation. This assumption is not uncommon, especially when management has strong commitment in quality and would only consider delayed product differentiation if the quality would not suffer. It is possible, however, that processing costs would be higher with delayed product differentiation so as to ensure the same quality standard of the outputs.

Let S_i be the average investment cost per period (amortized) if operation i became a common operation. Note that it is possible to have $S_i < 0$. For instance, consider the case when standardization of operation i means that a common part is used for both products and that we order this common part from a single vendor. Then the overhead cost could be reduced and hence $S_i < 0$.

Let $n_i(k)$ be the lead time of operation i when operation k is the last common operation. Let $p_i(k)$ be the processing cost per unit associated with operation i when operation k is the last common operation. Let $h_i(k)$ be the inventory holding cost for holding one unit of inventory at buffer i for one period when operation k is the last common operation.

Let z be the "safety factor" associated with the service level for each buffer. (Suppose that a buffer faces normal demand with mean μ and standard deviation σ and that the buffer replenishes its stock by following the order-up-to-level policy. Then the average on-order (WIP or "in-transit") inventory is equal to $n\mu$ and the average "buffer" inventory is equal to $\mu/2 + z\sigma\sqrt{n+1}$, where n is the lead time of this stage (Silver and Peterson 1979). To simplify the exposition of the model, we shall

assume that the (WIP or in-transit) inventories are valued as the same as the output of each stage. For this reason, we shall apply the same safety factor z for each of the buffers in the system.

Let $Z(k)$ be the total relevant cost per period for the case when operation k is the last common operation. It follows from (2.1)–(2.3) that $Z(k)$ can be expressed as:

$$\begin{aligned} Z(k) = & \sum_{i=1}^k S_i + \sum_{i=1}^N p_i(k)(\mu_1 + \mu_2) \\ & + \sum_{i=1}^N h_i(k)[n_i(k)(\mu_1 + \mu_2)] \\ & + \sum_{i=1}^k h_i(k)[(\mu_1 + \mu_2)/2 + z\sigma_{12}\sqrt{(n_i(k) + 1)}] \\ & + \sum_{i=k+1}^N h_i(k)[(\mu_1 + \mu_2)/2 \\ & + z(\sigma_1 + \sigma_2)\sqrt{(n_i(k) + 1)}]. \end{aligned} \quad (2.4)$$

Notice that the first term on the right hand side represents the total average investment cost per period, the second term corresponds to the total processing cost per period, the third term represents the total “WIP or in-transit” inventory cost, and the fourth and fifth terms represent the total “buffer” inventory cost per period when operation k is the last common operation. In this case the optimal last common operation $k^* = \arg\min\{Z(k): 0 \leq k \leq N - 1\}$.

3. Three Product/Process Redesign Approaches for Delaying Product Differentiation

In this § we shall use the expression for the total relevant cost $Z(k)$ to analyze some special cases that are motivated by real examples. These examples enable us to formalize three different product/process redesign approaches (standardization, modular design, and process restructuring). These approaches are illustrated by examples that are motivated by real cases observed in industry. However, these examples have been simplified to highlight the key tradeoffs.

3.1. Standardization of Components

This example is motivated by our industrial experience at a major computer manufacturer that manufactures two type of printers: black ink (mono) and multicolor ink (color) printers. (Due to the functionality of these two products, the demands for the two products in each period are negatively correlated.) The manufacturing process of the mono and color printers consists of three major steps: printed circuit board assembly (PCA), final assembly and test (FA&T), and final customization (Customization). At each step, different components are used for different end-products. Hence, we can view the manufacturing of mono and color printers as two distinct processes. Thus, in our notation, $N = 3$, and $k = 0$ for the case when none of the processes is standardized.

Delayed product differentiation could be achieved by either standardizing the PCA stage, or standardizing both the PCA and the FA&T stages. Standardizing the PCA stage requires the standardization of a key component, known as the head driver board, for both the mono and the color printer. Due to technical difficulties, the investment and the unit cost for the “common” head driver boards are relatively high. However, the lead time is not affected when the head driver board is standardized. Next, standardizing the FA&T stage requires also the standardization of a key component at that stage, namely, the print mechanism interface. This is a relatively simple task. Because the company manufactures the print mechanism interface inhouse, there is actually a strong incentive to standardize the print mechanism interface so as to exploit the benefits of economies of scale. In addition, the lead time is not affected with the FA&T stage standardized.

In this case, $S_1 \gg S_2 > 0$, $n_i(k) = n_i$ for each i , and the processing cost can be specified as:

$$p_i(k) = p_i \quad \text{for all } i \text{ when } k = 0 \quad (3.1)$$

In addition,

$$p_i(k) = p_i + \beta_i \quad \text{if } i \leq k \text{ and } k \geq 1, \quad (3.2)$$

$$p_i(k) = p_i \quad \text{if } i > k \text{ and } k \geq 1. \quad (3.3)$$

The term $\beta_i \geq 0$ captures the additional material and processing costs when operation i is standardized. Because the common head driver board is much more

difficult to develop and to process, $\beta_1 > \beta_2 \geq 0$. Next, the unit inventory holding cost can be specified as:

$$h_i(k) = b_i \quad \text{for all } i \text{ when } k = 0. \quad (3.4)$$

In addition,

$$h_i(k) = b_i + (\delta_1 + \delta_2 + \dots + \delta_i) \quad \text{for } i \leq k \text{ and } k \geq 1, \quad (3.5)$$

$$h_i(k) = b_i + (\delta_1 + \delta_2 + \dots + \delta_k) \quad \text{for } i > k \text{ and } k \geq 1. \quad (3.6)$$

Notice that b_i captures the *cumulative* value added at each operation i . Therefore, it is reasonable to assume that b_i is nondecreasing in i . In addition, we let $\delta_i \geq 0$ represent the "additional value added" at stage i when it is standardized. In our example, the value of the common head driver board is high. Therefore, $\delta_1 \gg 0$. Because it does not require significant effort to standardize the print mechanism interface, $\delta_2 \approx 0$. By substituting (3.1)–(3.6) into (2.4), we can evaluate the total relevant costs $Z(0)$, $Z(1)$ and $Z(2)$. Let us compare $Z(1)$ and $Z(0)$. It can be shown that:

$$\begin{aligned} Z(1) - Z(0) &= S_1 + \beta_1(\mu_1 + \mu_2) + 3\delta_1((\mu_1 + \mu_2)/2) \\ &\quad + \delta_1(\mu_1 + \mu_2) \left(\sum_{i=1}^3 n_i \right) \\ &\quad + z\delta_1(\sigma_1 + \sigma_2)[\sqrt{n_2 + 1} + \sqrt{(n_3 + 1)}] \\ &\quad + z[\sqrt{n_1 + 1}][(b_1 + \delta_1)\sigma_{12} - b_1(\sigma_1 + \sigma_2)]. \quad (3.7) \end{aligned}$$

The first five terms on the right-hand side represent the incremental cost incurred when we standardize the first stage (PCA stage). The sixth term corresponds to the potential savings due to reduction of inventory at the buffer located immediately after the first stage. However, the fifth term $[(b_1 + \delta_1)\sigma_{12} - b_1(\sigma_1 + \sigma_2)]$ could be positive when δ_1 is large enough. In this case, the incremental costs clearly outweigh the potential savings. Hence, we have $Z(1) > Z(0)$. Hence, it may not pay to delay product differentiation when the standardization of parts is costly. In addition, since $Z(1) > Z(0)$, the optimal $k^* = 0$ or $k^* = N - 1 = 2$, i.e., one should either

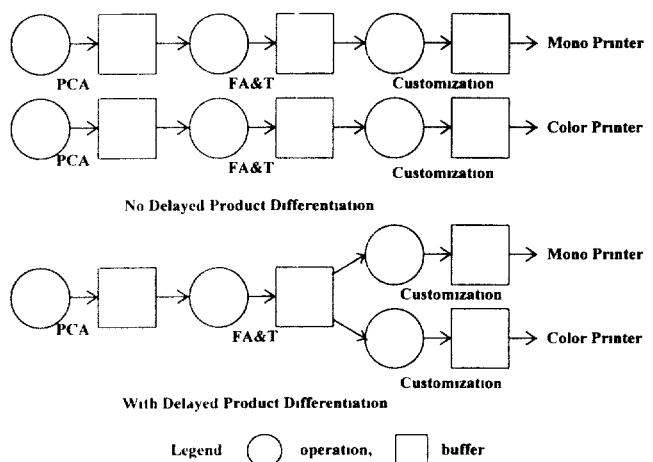
standardize both PCA and FA&T stages or none. (Figure 2 depicts these two scenarios.) However, in the event when $Z(1) < Z(0)$, then one needs to compare $Z(1)$ with $Z(2)$ to determine the optimal point of differentiation.

3.2. Modular Design

This example is motivated by our interaction with a manufacturer that manufactures dishwashers in different colors. For illustrative purposes, we consider the case in which the manufacturer produces two types of dishwashers: black or white. In addition, the manufacturing process can be grouped into three major steps: (1) fabrication; (2) integration and shipping; and (3) distribution. Basic components such as motors, basic circuit boards, racks, plastic parts, chassis, and metal frames (black or white) are produced at the fabrication stage. At the integration and shipping stage, motors, cables, control panels, and the basic components are assembled to produce working dishwashers. Then these dishwashers are shipped to different distribution centers. At the distribution stage, the dishwashers are distributed to different warehouses. Because the metal frames with different colors are assembled in the second stage, the dishwashers are differentiated after the first stage. In our notation, $N = 3$, $k = 1$.

Delayed product differentiation could be achieved by modular design of the metal frames. Specifically, consider the case when the metal frame is divided into 2

Figure 2 Standardization of Parts



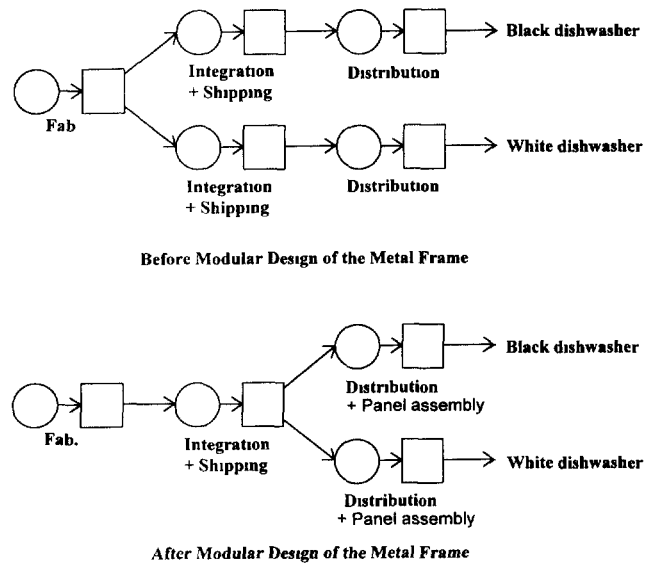
“modules”: a generic metal frame, and a plastic panel that specifies the color of the dishwasher. With this modular design, the manufacturer would produce and ship a generic dishwasher without the plastic panel. The distribution centers hold the plastic panels of different colors, and are responsible for “inserting” the plastic panels to the generic dishwashers to specify the color of each dishwasher. In this case, by letting the distribution centers handle an “additional” operation, we can delay the point of differentiation from $k = 1$ to $k = 2$. (Figure 3 depicts these two scenarios.)

For the base case (i.e., when $k = 1$), let $n_i(1) = n_i$, $p_i(1) = p_i$, $h_i(1) = b_i$ for each i . Since stage 2 includes both the assembly operation and the shipping operation, $n_2 \gg n_3$. Next, when $k = 2$, let S_2 be the investment cost when the metal frame is divided into two modules. Since this design is relatively simple, S_2 is relatively small. Since there is no significant change in the operations at the first two stages, the only change in the cost parameters occurs at the third stage. Specifically, the only changes are: $n_3(2) = n_3 + \alpha$, $p_3(2) = p_3 + \beta$, and $h_3(2) = b_3 + \delta$, where $\alpha \approx 0$ represents the “additional time” for inserting the plastic panels at the distribution centers, $\beta \approx 0$ represents the additional material costs associated with the plastic panels, and $\delta \approx 0$ represents the additional inventory holding cost per unit due to the plastic panels at the distribution centers. By substituting these cost parameters into (2.4), we can compare the total relevant costs associated with the modular design $Z(2)$ to that of the base case $Z(1)$. It can be shown that.

$$\begin{aligned} Z(2) - Z(1) &= S_2 + \beta(\mu_1 + \mu_2) + \delta(\mu_1 + \mu_2)/2 \\ &\quad + [(n_3 + \alpha)(b_3 + \delta) - n_3 b_3](\mu_1 + \mu_2) \\ &\quad + z b_2 [\sigma_{12} - (\sigma_1 + \sigma_2)] \sqrt{(n_2 + 1)} + z(\sigma_1 + \sigma_2) \\ &\quad \times [(b_3 + \delta) \sqrt{(n_3 + \alpha + 1)} - b_3 \sqrt{(n_3 + 1)}] \quad (3.8) \end{aligned}$$

The first four terms and the sixth term on the right represent the cost incurred due to the modular design of the metal frame. However, the fifth term $z b_2 [\sigma_{12} - (\sigma_1 + \sigma_2)] \sqrt{(n_2 + 1)}$ highlights the savings in inventory cost due to modular design. In our example, as $\alpha \approx 0$, $\beta \approx 0$, $\delta \approx 0$, S_2 is small, and n_2 is large, it is quite likely that

Figure 3 Modular Design of Parts



$Z(2) < Z(1)$. This implies that it pays to delay the product differentiation (from stage 1 to stage 2) when the lead time of stage 2 is long (i.e., when $n_2 \gg 0$), or when the additional module is simple to handle (i.e., when $\alpha \approx 0$ and $\beta \approx 0$), or when the modular design of the parts is relatively inexpensive (i.e., when $\delta \approx 0$ and $S_2 \approx 0$).

3.3 Process Restructuring

We present two examples. The first example deals with a situation in which delayed product differentiation is achieved by “postponing” an operation downstream. The second example is based on a case in which delayed product differentiation is achieved by “reversing” the order of two operations.

A. Postponement of Operation. This example is drawn from the manufacturing and distribution of a family of electronic devices as described in van Doremalen and Fleuren (1990). (Notice that van Doremalen and Fleuren (1990) present an approach for evaluating different designs of distribution network. Their analysis concentrates on reducing both time and inventories by delaying the product differentiation and by selecting other modes of transport.) For illustrative purposes, we shall consider the case of two end-products. The electronic devices are composed of four basic components.

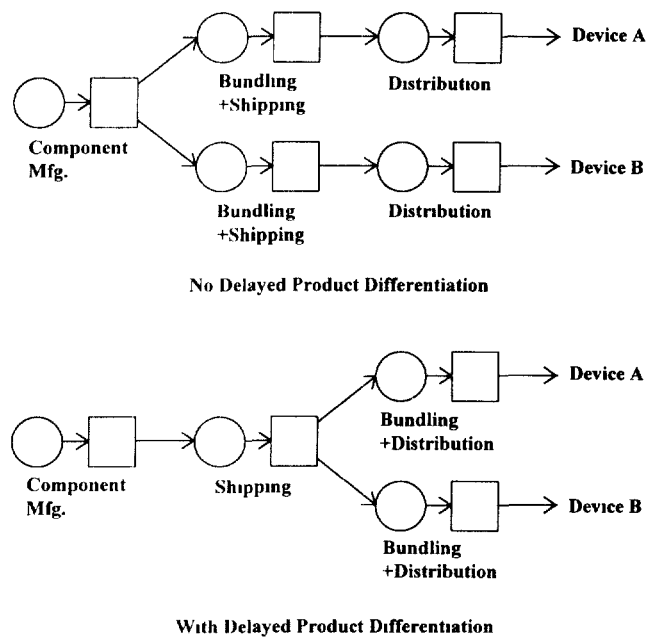
body, control panel, monitor, and technical manual. Currently, body and control panel are manufactured at a **central production center**, where these components are then bundled with the monitors and technical manuals to form different end-products. The end-products are then shipped to a **central supply center**, from where the end-products are distributed to the sales organizations

The supply chain of these products consists of three stages (1) manufacturing of components at the production center, (2) bundling of components into end-products at the production center, and (3) distribution at the supply center. Because the bundling is performed at the second stage, the products are differentiated after the first stage. In our notation, $N = 3$, and $k = 1$

An alternative is to "postpone" the bundling operation downstream by bundling the components into different end-products at the supply center instead of the production center. In this case, generic products are shipped from the production center to the supply center for bundling. Hence, the product differentiation is delayed from $k = 1$ to $k = 2$. We can view this alternative as a simple way to delay product differentiation (as illustrated in Figure 4). In this case, the first stage remains the same (i.e., manufacturing of the components). However, the second stage becomes a non-value-added activity (shipping activity), adding no value to the products. The third stage now consists of both bundling and distribution at the supply center.

For the base case (i.e., when $k = 1$), let $n_i(1) = n_i$, $p_i(1) = p_i$, $h_i(1) = b_i$ for each i . Clearly, $b_1 < b_2 < b_3$. Because stage 2 includes both bundling and shipping operations, $n_2 \geq n_3$. Next, when $k = 2$, let S_2 associated with the investment cost when the bundling operation is "postponed" to stage 3. Because bundling of components into an end-product is a relatively simple activity (van Doremalen and Fleuren 1990), S_2 is relatively small. Because there is no significant change in the operation in the first stage, the only change in the cost parameters occurs at the second and third stages. Specifically, when $k = 2$, stage 2, as a shipping activity, becomes a non-value-added activity. For this reason, we have $n_2(2) = n_2 - \alpha$, $p_2(2) = p_2 - \beta$, and $h_2(2) = b_1$. Notice that α represents the lead time for the bundling operation and β corresponds to the processing cost associated with the

Figure 4 Process Restructuring: Postponement of Operation



bundling operation. When postponing the bundling operation to stage 3, we have $n_3(2) = n_3 + \alpha$, $p_3(2) = p_3 + \beta$, and $h_3(2) = b_3$. By substituting these cost parameters into (2.4), we can compare the total relevant costs $Z(2)$ and $Z(1)$. It can be shown that:

$$Z(2) - Z(1)$$

$$\begin{aligned} &= S_2 + [(b_1 - b_2)n_2 + (b_3 - b_1)\alpha](\mu_1 + \mu_2) \\ &\quad + (b_1 - b_2)(\mu_1 + \mu_2)/2 \\ &\quad + z[b_1\sigma_{12}\sqrt{(n_2 - \alpha + 1)} - b_2(\sigma_1 + \sigma_2)\sqrt{(n_2 + 1)}] \\ &\quad + zb_3(\sigma_1 + \sigma_2)[\sqrt{(n_3 + \alpha + 1)} - \sqrt{(n_3 + 1)}]. \end{aligned} \quad (3.9)$$

The first term and the fifth term on the right hand side represent the cost incurred when the bundling operation is postponed to stage 3. We can interpret the second, third, and fourth terms as the savings in the "in-transit" and "buffer" inventory costs when the bundling operation is postponed to stage 3. It can easily be seen that this savings increases as b_2 increases. This implies that one can obtain a bigger savings from inventory cost if one can delay a high value-added activity. In our example, as $b_1 < b_2$, and S_2 is relatively small,

hence, it is quite likely that $Z(2) < Z(1)$ when α is small enough. This implies that it pays to delay the bundling operation from stage 2 to stage 3. Asanuma (1991) reports that some Japanese automakers have developed similar strategies by delaying the bundling of "optional packages" from the factory to the car dealers.

B. Reversal of Operations. This example is drawn from Dapiran (1992) and Harvard Business School Note (1991), which report ways in which Benetton, an Italian apparel manufacturer, addresses the problem of product proliferation. The traditional process of sweater making consists of three major stages. (1) dyeing; (2) knitting; and (3) distribution. At the dyeing stage, raw materials of white yarns are dyed into different colors. The dyeing operation is a batch operation in which a large quantity of white yarns is submerged into different pools of dyes. After the dyeing stage, the colored yarns are then knitted into sweaters at the knitting stage. At the knitting stage, certain operations such as cutting or trimming are performed by machines and others such as inspection, labeling, and packaging, etc., are performed manually. Relatively speaking, the processing time per sweater required by the dyeing operation is much shorter than that required by the knitting operation, because the dyeing operation is a batch process. The finished sweaters are shipped to different distribution centers. In our notation, $N = 3$ and $k = 0$.

An alternative is to reverse the operations in the first two stages. For this alternative, all sweaters are knitted out of a white yarn at the first stage. Then the sweaters are dyed into different colors in the second stage. In this case, the color of the sweater is not specified until the second stage. Hence, we can view this alternative as a simple way to delay product differentiation from $k = 0$ to $k = 1$ (as illustrated in Figure 5)

For the base case (i.e., when $k = 0$), let

$$n_1(0) = n_1, \quad p_1(0) = p_1, \quad h_1(0) = b + \delta_1,$$

$$h_2(0) = b + \delta_1 + \delta_2,$$

$$h_3(0) = b + \delta_1 + \delta_2 + \delta_3.$$

In this case, b represents the inventory cost for the raw material and δ_i represents the value added at stage i . Because the lead time of the dyeing operation is much shorter than the knitting operation, $n_1 < n_2$. However,

the value added during the dyeing stage is higher than that of the knitting stage because of the large capital investment required by the dyeing machines. In this case, we have $\delta_1 > \delta_2$.

Next, for the case when $k = 1$, let S_1 be the investment cost when the dyeing operation and knitting operation are reversed. Because the reversal of these two operations is relatively simple, S_1 is relatively small. Because there is no significant change in the operation in the third stage, the only change in the cost parameters occurs at the first and second stages, which have been reversed. In our example, we have

$$n_1(1) = n_2, \quad n_2(1) = n_1, \quad p_1(1) = p_2, \quad p_2(1) = p_1,$$

$$h_1(1) = b + \delta_2, \quad h_2(1) = b + \delta_2 + \delta_1,$$

$$h_3(1) = b + \delta_2 + \delta_1 + \delta_3.$$

By substituting these cost parameters into (2.4), we can compare the total relevant costs $Z(1)$ and $Z(0)$. It can be shown that:

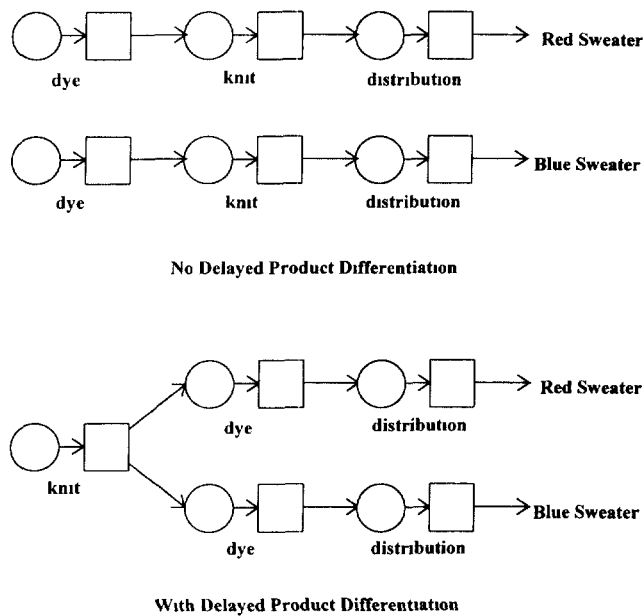
$$\begin{aligned} Z(1) - Z(0) &= S_1 + (\delta_2 n_1 - \delta_1 n_2)(\mu_1 + \mu_2) \\ &\quad + (\delta_2 - \delta_1)(\mu_1 + \mu_2)/2 \\ &\quad + z[(b + \delta_2)\sigma_{12}\sqrt{(n_2 + 1)} \\ &\quad \quad - (b + \delta_1)(\sigma_1 + \sigma_2)\sqrt{(n_1 + 1)}] \\ &\quad + z(b + \delta_1 + \delta_2)(\sigma_1 + \sigma_2) \\ &\quad \times [\sqrt{(n_1 + 1)} - \sqrt{(n_2 + 1)}]. \end{aligned} \quad (3.10)$$

By rearranging the terms, it can be shown that

$$\begin{aligned} Z(1) - Z(0) &= S_1 + (\delta_2 n_1 - \delta_1 n_2)(\mu_1 + \mu_2) \\ &\quad + z\delta_2(\sigma_1 + \sigma_2)\sqrt{(n_1 + 1)} + (\delta_2 - \delta_1)(\mu_1 + \mu_2)/2 \\ &\quad + z\sqrt{(n_2 + 1)}[(b + \delta_2)\sigma_{12} \\ &\quad \quad - (b + \delta_1 + \delta_2)(\sigma_1 + \sigma_2)]. \end{aligned} \quad (3.11)$$

The first term and third term on the right represent the cost incurred when the dyeing operation and the knitting operation are reversed. Since $\delta_2 < \delta_1$, $n_1 < n_2$, and $\sigma_{12} \leq (\sigma_1 + \sigma_2)$, we can interpret the second term,

Figure 5 Process Restructuring: Reversal of Operations



fourth term, and fifth term as the savings in “WIP” and “buffer” inventory costs when the dyeing operation and the knitting operation are reversed. These savings in-

creases as n_2 increases or as $(\delta_1 - \delta_2)$ increases. This implies that one can obtain a bigger savings from inventory cost if one can reverse a short operation at an early stage (n_1) with a long operation at a later stage (n_2). One can obtain additional savings in inventory cost if one can reverse a high valued-added operation at an early stage (δ_1) with a low value-added operation at a later stage (δ_2). In our example, as $\delta_1 > \delta_2 \approx 0$ and S_2 is small. Hence, it is quite likely that $Z(1) < Z(0)$ when n_2 is large enough. This implies that it pays to reverse the operations that result in delayed product differentiation.

In this §, we have examined different examples that utilize three different approaches for delaying product differentiation. In addition, we have used the simple expression of the total relevant cost to analyze the implications of each of these approaches. These implications can be summarized in Table 1.

4. Special Cases

We now consider some special theoretical cases in which the lead time and the unit inventory holding cost will not be affected when the point of differentiation is delayed. However, there would be an investment cost

Table 1 The Implications of Three Basic Approaches for Delayed Product Differentiation

Basic Approach	Redesign Process for Delaying Product Differentiation	Conditions for Effectiveness
Standardization	Design a <i>part</i> that is common to all products	Effective when the investment cost and incremental processing cost required for standardization are low
Modular Design	Divide a <i>part</i> into 2 modules—the first module is a common part and the assembly operation of the second module is deferred	The number of modules increases. However, this approach is effective when the incremental lead time, incremental processing cost and unit inventory holding cost are low
Process Restructuring Postponement of Operation	Divide an <i>operation</i> into 2 steps—the first step is common to all products and the execution of the second step is postponed	Effective when the lead time of the common step is significantly longer than the second step that is being delayed. In addition, this approach is effective when the second step is a high value-added activity
Process Restructuring Reversal of Operations	Reverse the order of 2 <i>operations</i> . As a result, the first operation is common to all products	Effective when deferring the high valued-added operation by reversing the operations

S_i incurred if operation i became a common operation. In addition, there would be incremental processing/material costs β_i if operation i became a common operation. In such a case, the cost parameters are given as follows: $n_i(k) = n_i$ and $h_i(k) = h_i$ for all i , and $p_i(k) = p_i + \beta_i$ for $i \leq k$ and $p_i(k) = p_i$ for $i > k$. Substituting these cost parameters into (2.4), the total relevant cost associated with the last common operation k , $Z(k)$, can be expressed as

$$\begin{aligned} Z(k) = & \sum_{i=1}^k S_i + \sum_{i=1}^N p_i(\mu_1 + \mu_2) + \sum_{i=1}^k \beta_i(\mu_1 + \mu_2) \\ & + \sum_{i=1}^N h_i n_i(\mu_1 + \mu_2) \\ & + \sum_{i=1}^k h_i[(\mu_1 + \mu_2)/2 + z\sigma_{12}\sqrt{(n_i + 1)}] \\ & + \sum_{i=k+1}^N h_i[(\mu_1 + \mu_2)/2 \\ & + z(\sigma_1 + \sigma_2)\sqrt{(n_i + 1)}]. \end{aligned} \quad (4.1)$$

In this case, it is easy to see that

$$\begin{aligned} Z(k+1) - Z(k) &= S_{k+1} + \beta_{k+1}(\mu_1 + \mu_2) \\ &+ z(\sigma_{12} - (\sigma_1 + \sigma_2))[h_{k+1}\sqrt{(n_{k+1} + 1)}]. \end{aligned} \quad (4.2)$$

Notice that the first two terms on the right represent the incremental cost when we delayed the point of differentiation from k to $k+1$. However, the third term $z(\sigma_{12} - (\sigma_1 + \sigma_2))[h_{k+1}\sqrt{(n_{k+1} + 1)}]$ highlights the "savings" in inventory cost due to delayed product differentiation. Let

$$G(k) = [Z(k+1) - Z(k)] - [Z(k) - Z(k-1)]$$

Equation (4.2) enables us to express $G(k)$ as

$$\begin{aligned} G(k) = & [S_{k+1} - S_k] + (\mu_1 + \mu_2)[\beta_{k+1} - \beta_k] \\ & + z(\sigma_{12} - (\sigma_1 + \sigma_2)) \\ & \times [h_{k+1}\sqrt{(n_{k+1} + 1)} - h_k\sqrt{(n_k + 1)}] \end{aligned} \quad (4.3)$$

It follows from the definition of $G(k)$ and equation (4.2) that the total relevant cost $Z(k)$ is *concave* if $G(k) \leq 0$ for

$1 \leq k \leq N-1$, and $Z(k)$ is *convex* if $G(k) \geq 0$ for $1 \leq k \leq N-1$. It is difficult to characterize the optimal last common operation in general. However, consider the following cases:

Case A

$$\begin{aligned} & [S_{i+1} - S_i] + (\mu_1 + \mu_2)[\beta_{i+1} - \beta_i] \\ & \leq -z(\sigma_{12} - (\sigma_1 + \sigma_2)) \\ & \times [h_{i+1}\sqrt{(n_{i+1} + 1)} - h_i\sqrt{(n_i + 1)}] \quad \text{for all } i \end{aligned}$$

In this case, $Z(k)$ is *concave*. Hence, the optimal last common operation is located at one of the extreme points, i.e., $k^* = 0$ or $k^* = N-1$, depending on the value of $Z(0)$ and $Z(N-1)$.

Case B:

$$\begin{aligned} & [S_{i+1} - S_i] + (\mu_1 + \mu_2)[\beta_{i+1} - \beta_i] \\ & \geq -z(\sigma_{12} - (\sigma_1 + \sigma_2)) \\ & \times [h_{i+1}\sqrt{(n_{i+1} + 1)} - h_i\sqrt{(n_i + 1)}] \quad \text{for all } i. \end{aligned}$$

In this case, $Z(k)$ is *convex* in k . Hence, the optimal last common operation k^* is given by:

$$k^* = \text{Min}\{N-1, \text{Min}\{k: Z(k+1) - Z(k) \geq 0, k \geq 0\}\}. \quad (4.4)$$

The following Proposition summarizes the properties of k^* when $Z(k)$ is *convex* in k :

PROPOSITION 1. Suppose that S_i , β_i , and $h_i \geq 0$ for all i and that $Z(k)$ is *convex* in k . Then (a) k^* increases as ρ decreases; (b) k^* increases as $h_i\sqrt{(n_i + 1)}$ increases for all i ; (c) k^* decreases as S_i increases for all i ; (d) k^* decreases as β_i increases for all i ; and (e) k^* increases as $(\mu_1 + \mu_2)$ decreases.

PROOF. The proof is given in Appendix A.

Proposition 1 has the following interpretation. Since the term $(\sigma_{12} - (\sigma_1 + \sigma_2)) \leq 0$ and it decreases as ρ decreases, the "savings" in inventory cost $z(\sigma_{12} - (\sigma_1 + \sigma_2))[h_{i+1}\sqrt{(n_{i+1} + 1)}]$ increases as ρ decreases (i.e., as the demands of the end-products become more "negatively correlated"). To take advantage of the increase in the inventory savings, it is desirable to defer the last common operation. This observation explains property (a). Next, as the term $h_i\sqrt{(n_i + 1)}$ increases for all i , one can obtain a higher savings in inventory cost by

deferring the last common operation. This is reflected in property (b).

On the other hand, when the cost of standardizing an operation S_i increases, property (c) implies that it is not cost effective to invest in delaying the differentiation of the products. Similarly, when β_i increases (or when the incremental processing cost associated with delayed product differentiation increases), property (d) suggests that it is not cost effective to invest in delaying the differentiation of the products. Finally, as the total average demand $(\mu_1 + \mu_2)$ decreases and the variances and covariances of the demand remain the same, the effective demand is more variable. In this case, property (e) suggests that it is more cost effective to defer the last common operation.

Let us consider the three additional special cases:

Case 1: $S_i = S$, $\beta_i = \beta$, and $h_i = h$ for $1 \leq i \leq N - 1$, and $n_i = n$ for all i .

In this case, the cost of standardizing an operation S_i , the incremental processing cost β_i , and the inventory cost h_i , are constant for operations 1 through $N - 1$. However, it is possible to have $h_N > h_i$ for $i = 1, \dots, N - 1$. This situation can occur when the first $N - 1$ operations are of inspection nature that do not add any significant value to the products. However, the last operation N could be an assembly operation that adds significant value to the unit at buffer N .

When $S_i = S$, $\beta_i = \beta$, $h_i = h$, and $n_i = n$ for $1 \leq i \leq N - 1$, it can be easily checked from equation (4.3) that $G(k) = 0$ for all k . Hence, $Z(k)$ is concave and the optimal last common operation k^* is either 0 or $N - 1$. From (4.1), it can be shown that

$$\begin{aligned} Z(N - 1) - Z(0) \\ = (N - 1)[S + (\mu_1 + \mu_2)\beta \\ + zh[(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}]]. \end{aligned} \quad (4.5)$$

Equation (4.5) has the following implications:

- (a) $k^* = 0$ when $S + (\mu_1 + \mu_2)\beta \geq -zh[(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}]$, and
- (b) $k^* = N - 1$ when $S + (\mu_1 + \mu_2)\beta < -zh[(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}]$.

The term $S + (\mu_1 + \mu_2)\beta$ corresponds to the additional investment cost when we defer the last common opera-

tion by an additional operation, and the term $-zh[(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}]$ can be interpreted as the savings in inventory cost when we defer the last common operation by an additional operation. In this case, implications (a) and (b) have the following interpretation: if the additional investment cost exceeds the savings in inventory cost, then it is optimal not to standardize any operation; i.e., $k^* = 0$. Otherwise, it is optimal to defer the last common operation as late as possible; i.e., $k^* = N - 1$.

Case 2: $S_i = S$, $\beta_i = \beta$, $h_i = ih$, and $n_i = n$ for all i .

This is another special case in which the cost of standardizing an operation S_i and the incremental processing cost β_i are constant. In addition, the inventory cost h_i increases linearly in the number of operations. This situation captures the case in which the same incremental value is added to the product at each operation.

In this case, it can be easily checked from equation (4.3) that $G(k) \leq 0$ for all k . Hence, $Z(k)$ is concave and the optimal last common operation k^* is either 0 or $N - 1$. It follows from the definition of $Z(k)$ in (4.1),

$$\begin{aligned} Z(N - 1) - Z(0) \\ = (N - 1)[S + (\mu_1 + \mu_2)\beta + zh(N/2) \\ \times (\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}] \\ = (N - 1)[S + (\mu_1 + \mu_2)\beta + z(h_N/2) \\ \times (\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}]. \end{aligned} \quad (4.6)$$

Equation (4.6) has the following implications:

- (a) $k^* = 0$ when $S + (\mu_1 + \mu_2)\beta \geq -z(h_N/2)(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}$; and
- (b) $k^* = N - 1$ when $S + (\mu_1 + \mu_2)\beta < -z(h_N/2)(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}$.

Notice that the term $S + (\mu_1 + \mu_2)\beta$ corresponds to the additional investment cost when we defer the last common operation by one additional operation and the term $-z(h_N/2)(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}$ can be interpreted as the savings in inventory cost when we defer the last common operation by an additional operation. Notice that when we defer the last common operation by an additional operation, the savings in inventory cost $-z(h_N/2)(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{n + 1}$ is measured at half of the finished goods inventory cost at buffer N . Implications (a) and (b) can be interpreted as follows: if the additional investment cost exceeds the savings in inventory cost,

then it is optimal not to standardize any operation; i.e., $k^* = 0$. Otherwise, it is optimal to defer the last common operation as much as possible; i.e., $k^* = N - 1$.

Case 3: $S_i = \pi_1 h_i$, $\beta_i = \pi_2 h_i$, and $n_i = n$ for all i .

This is another special case in which the investment cost S_i and the incremental processing cost β_i can be expressed as constant multipliers of the inventory holding cost of that operation. In this case equation (4.3) implies that

$$G(k) = (h_{k+1} - h_k)[\pi_1 + (\mu_1 + \mu_2)\pi_2 + z(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{(n+1)}]. \quad (4.7)$$

Consider the following situations:

(a) Suppose that

$$[\pi_1 + (\mu_1 + \mu_2)\pi_2 + z(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{(n+1)}] \geq 0.$$

Then one can check from (4.2) that the total relevant cost $Z(k)$ is an increasing and convex function in k . Therefore, it is optimal to have $k^* = 0$.

(b) Suppose that

$$[\pi_1 + (\mu_1 + \mu_2)\pi_2 + z(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{(n+1)}] < 0$$

Then it follows from the definition (4.7) that $Z(k)$ is concave. In addition, it can be easily shown that

$$\begin{aligned} Z(N-1) - Z(0) &= \left(\sum_{k=1}^{N-1} h_k \right) [\pi_1 + (\mu_1 + \mu_2)\pi_2 \\ &\quad + z(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{(n+1)}] \end{aligned} \quad (4.8)$$

Since

$$[\pi_1 + (\mu_1 + \mu_2)\pi_2 + z(\sigma_{12} - (\sigma_1 + \sigma_2))\sqrt[n]{(n+1)}] < 0$$

and since $Z(k)$ is concave, we can conclude from (4.8) that it is optimal to defer the last common operation as late as possible, i.e., $k^* = N - 1$.

5. Conclusion

We have developed a model for evaluating the costs and benefits associated with a situation in which the product differentiation can be delayed through product/process redesign. Such redesign may incur some investment cost and additional processing cost; however, delayed product differentiation lowers the buffer inventories. The ben-

efits of such redesign include reducing the complexity of the manufacturing process, increasing the 'flexibility' of use for the buffer inventories, and improving the service level of the system (due to risk-pooling). Although our model focuses on the two-product case, our analysis highlights the fact that delayed product differentiation can be viewed as a strategy for a company to improve the service level and reduce inventories when dealing with product proliferation. Our model could be more accurate if one improved the analysis of the average inventory level at each buffer. However, such analysis is complex and beyond the scope of this paper.

We have evaluated each of the redesign approaches by considering the total relevant cost per period. In our future studies, we plan to analyze the total relevant cost for the entire product life cycle by considering the dynamics of the demands throughout the product life cycle. In the current model, buffer stocks are kept after every stage. We plan to model the issue of selecting the stocking points, and to analyze the managerial implications of different selections of stocking points. Moreover, we plan to model the issue of grouping product attributes into different "levels" of specifications, and to analyze the strategic implications of this grouping concept in the context of product/process redesign and delayed product differentiation. This issue is motivated by the example described by Asanuma (1991), who reports that Toyota and Nissan have grouped the product attributes (body type, engine type, and transmission type) into the "first-level" specification, and grouped the remaining product attributes as the "second-level" specification. The second level significantly proliferates product variety. By separating the attributes into two groups, product variety is limited at the first level. Having the manufacturing process sequenced by these two levels results in delayed product differentiation.¹

¹ The authors would like to thank the associate editor and two anonymous referees for their valuable comments on an earlier version of this paper.

Appendix A. Proof of Proposition 1

Let $\theta = \sigma_{12} - (\sigma_1 + \sigma_2)$. It can be easily checked from (2.3) that $\theta \leq 0$ for all ρ and θ decreases as ρ decreases.

PROOF OF PART (a) As ρ decreases to ρ' , θ decreases to θ' , where $\theta' = \theta - \epsilon$. Let k^* and k' be the optimal last common operation associated with ρ and ρ' , respectively.

Suppose $k' < k^*$. Since k' is optimal and since $Z(k)$ is convex, k' must be the smallest integer that satisfies the optimality condition (4.4). From (4.2), the optimality condition (4.4) can be expressed as

$$S_{k+1} + (\mu_1 + \mu_2)\beta_{k+1} + zh_{k+1}\sqrt{(n_{k+1} + 1)\theta'} \geq 0 \quad (A.1)$$

Since $\theta' = \theta - \epsilon$, we have

$$S_{k+1} + (\mu_1 + \mu_2)\beta_{k+1} + zh_{k+1}\sqrt{(n_{k+1} + 1)\theta} - zh_{k+1}\sqrt{(n_{k+1} + 1)\epsilon} \geq 0 \quad (A.2)$$

Recall from the definition of k^* in (4.4) that

$$k^* = \min\{N - 1, \min\{k : Z(k + 1) - Z(k) \geq 0\}\}$$

Since $k' < k^*$, we have

$$Z(k' + 1) - Z(k') = S_{k+1} + (\mu_1 + \mu_2)\beta_{k+1} + zh_{k+1}\sqrt{(n_{k+1} + 1)\theta} < 0 \quad (A.3)$$

This implies that

$$S_{k+1} + (\mu_1 + \mu_2)\beta_{k+1} + zh_{k+1}\sqrt{(n_{k+1} + 1)\theta} - zh_{k+1}\sqrt{(n_{k+1} + 1)\epsilon} < 0,$$

which contradicts that k' satisfies the optimality condition (A.2).

Therefore, $k' \geq k^*$.

Similarly, one can apply the same argument to prove the statements (b) through (e). We omit the details.

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