IE 443
MULTI-OBJECTIVE DECISION ANALYSIS

Some Motivating Examples

Choosing a university

- Decision maker:
- Alternatives:
- Decision criteria:

Choosing a university

- The alternatives are **explicitly known.**
- Each alternative is evaluated with respect to a set of criteria.
- Each alternative is associated with a **vector** showing its performance with respect to these criteria.
- This is a **multicriteria choice** problem.
- Why is this problem challenging?

Choosing a university

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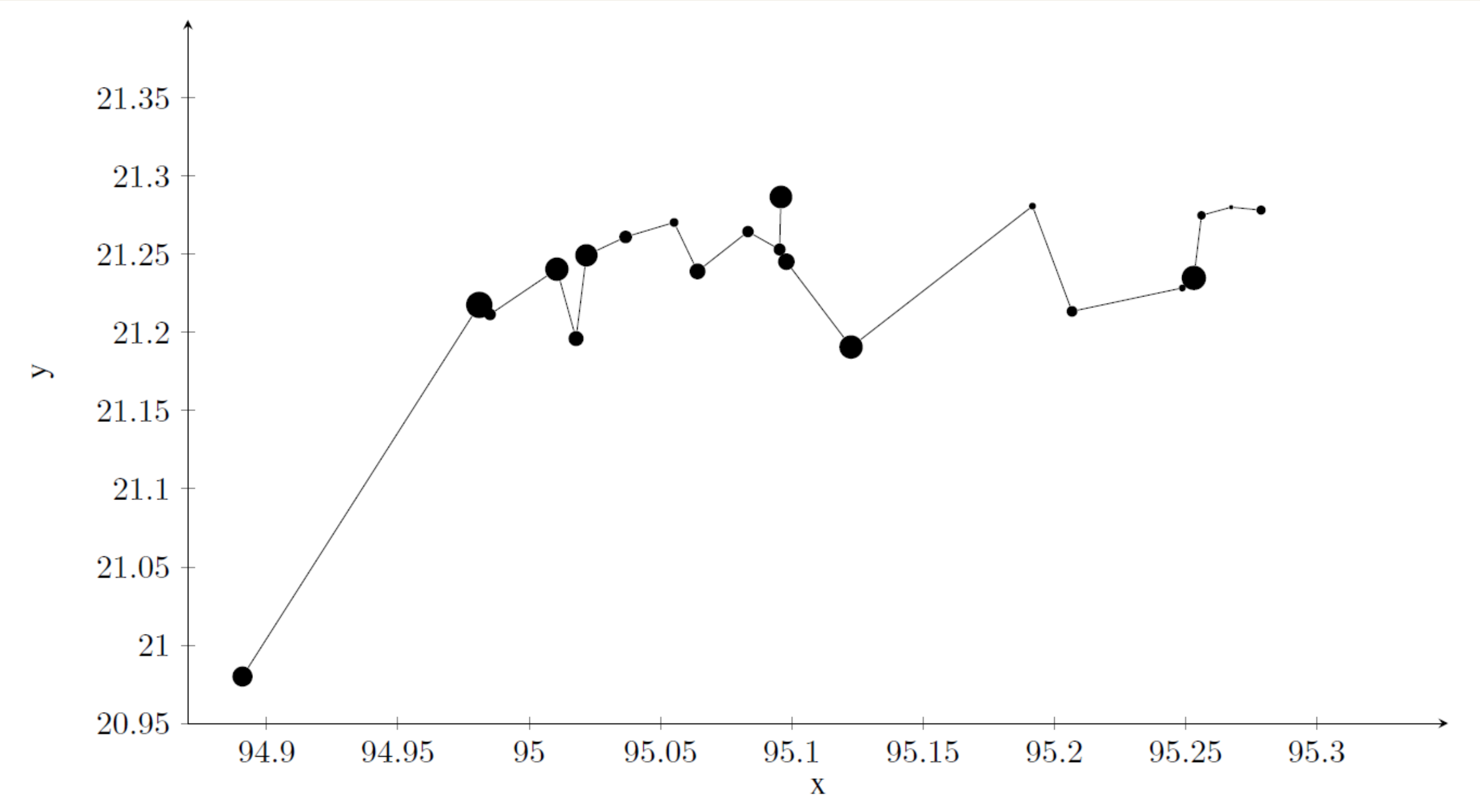
- Each alternative is evaluated with respect to a set of criteria.
- Each alternative is associated with a **vector** showing its performance with respect to these criteria.
- This is a **multicriteria choice** problem.
- Why is this problem challenging?
  - *No alternative is the best with respect to all criteria.*
  - *Dominance helps but not sufficient.*
  - *Trade-off. It is a matter of preference*

Energy Systems Planning

- A set of demand nodes (cities, towns, villages)
- Each will be served electricity via one of the two options:
  - *Connect to a grid network (less costly, higher emission)*
  - *Decentralized system (more costly, lower emission)*

- Decision maker: Ministry of Energy
- Decision criteria (objectives): minimizing cost, minimizing CO_2 emission
- Alternatives: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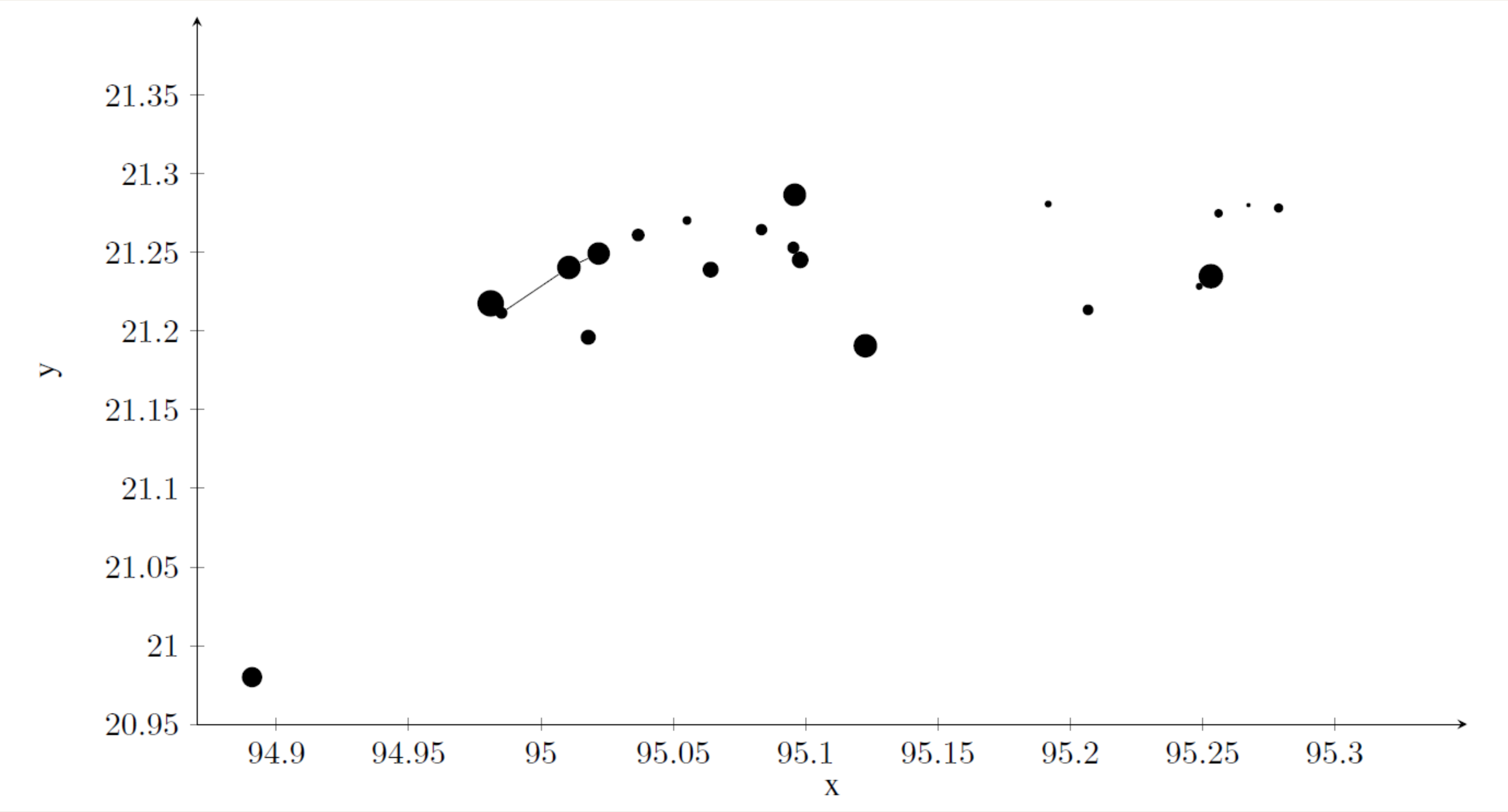
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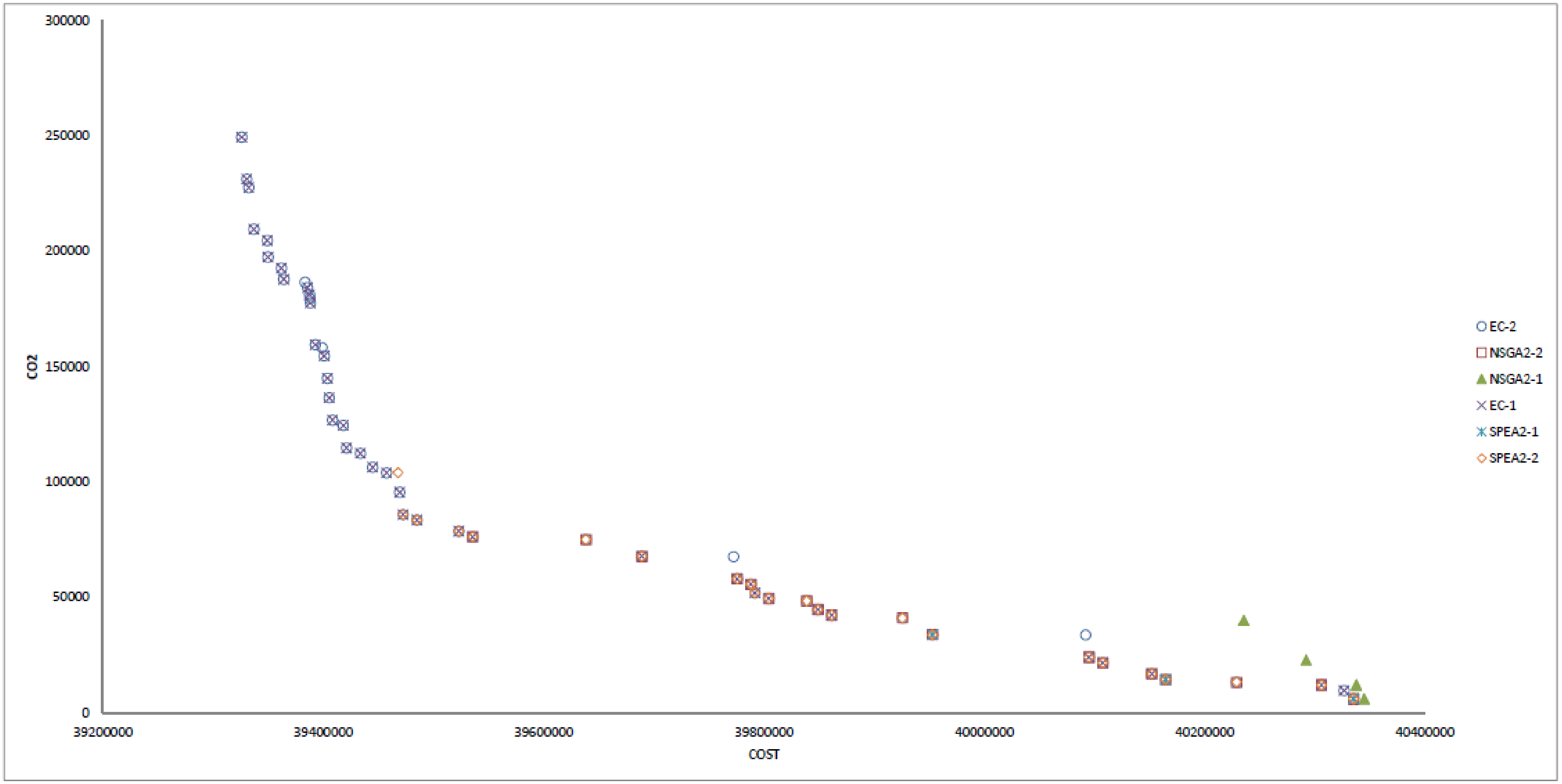
Energy Systems Planning

- The alternatives are not **explicitly known**, implicitly defined by constraints.
- Each alternative is associated with a **vector** showing its performance with respect to these objective functions.
- This is a **biobjective optimization problem**.
- Why is this problem challenging?
  - *How do we solve a multi (bi)objective optimization problem?*

Energy Systems Planning

- The alternatives are not **explicitly known**, implicitly defined by constraints.
- Each alternative is associated with a **vecto**r showing its performance with respect to these objective functions.
- This is a **biobjective optimization problem**.
- Why is this problem challenging?
  - *How do we solve a multi (bi)objective optimization problem?*
    - Determine the nondominated points (Pareto points).



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EC-2
NSGA2-2
NSGA2-1
EC-1
SPEA2-1
SPEA2-2

Decisions

- Multiple competing objectives
- Both personal and organizational

- Any personal examples?

Why use a theoretical approach?

- Less stressful
- The reasons to offer justifications for your decision become readily available
- You can be more persuasive…

What to start with?

- Alternatives (including status quo) leading to different outcomes
  - *These outcomes will have different aspects*
  - *You need to first decide on the relative importance of these different aspects*
  - *Usually there will be uncertainty about the consequences to be produced by each alternative*
  - *(We will deal with this later using utility theory and stochastic optimization)*

General Approach

- Specify objectives and their respective scales
- Develop alternatives to achieve these objectives
- Determine the performance of each alternative for each objective
- Weigh the objectives against one another, consider tradeoffs
- Select the ‘best’ performing alternative by properly accounting for uncertainties

Some Terminology

- Objective – preferred direction of movement
  - *Assumption of monotonic behavior with respect to the evaluation measure*

  (i.e., either more is better or less is better)
- Goal – desired threshold
- Evaluation measure – the scale
- Level or score – numerical rating of an alternative with respect to an evaluation measure
- Value hierarchy – “treelike” structure for evaluation considerations

Obtaining an Additive Value Function

- Use an appropriate **single dimensional (attribute) value** function for each evaluation measure
- Assign a **weight** to each single attribute value function

Example: Networking Strategy

- An engineering design company networking its computers
- Evaluation measures
  - *Productivity enhancement*
  - *Cost increase*
  - *Security*

What kind of a scale would be appropriate for each one of these evaluation measures?

Scales

- Productivity enhancement
  - *-1: significantly diminished*
  - *0: no change*
  - *1: improvement*
  - *2: significant improvement*
- Security:
  - *-2: serious decrease*
  - *-1: noticible but acceptable decrease*
  - *0: no change*
  - *1: security is enhanced*
- Cost increase:
  - *net present value of the increased cost in $1000's.*

What is the score of ***status quo*** in each case?

Alternatives & Scores/Levels with respect to Evaluation Measures

Alternative	Evaluation Measure		
	Productivity	Cost	Security
Status Quo	0	0	0
1	2	125	0.5
2	1	95	0
3	0.5	65	-1

How would you rank these alternatives?

Considering the Tradeoffs

- Combine the three evaluation measures and obtain a single rating/value for each alternative.
- A higher score is preferred in productivity and security but a lower score is better in cost…

Averaging

Alternative	Score
Status Quo	0
1	(2-125+0.5)/3 = -40.83
2	-31.33
3	-21.83

It seems that doing nothing is the best option… Is it really?

Normalize the ratings

- Proportion of the way along the allowed range
- If a higher score is more preferred

$$\text{Rating} = \frac{\text{Score} - \text{Lowest Level}}{\text{Highest Level} - \text{Lowest Level}}$$

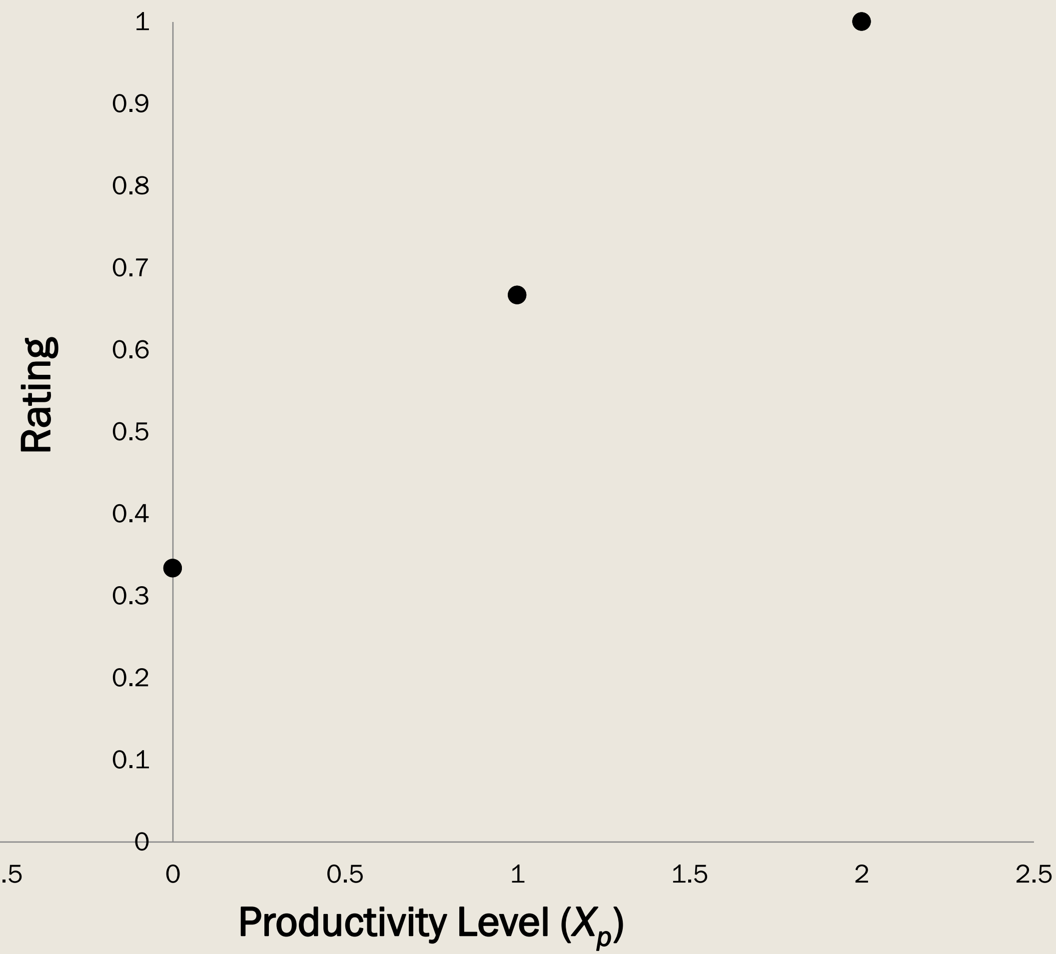
- If a lower score is more preferred

$$\text{Rating} = \frac{\text{Highest Level - Score}}{\text{Highest Level - Lowest Level}}$$

Productivity

- Productivity enhancement
  - *-1: significantly diminished*
  - *0: no change*
  - *1: improvement*
  - *2: significant improvement*

$$\text{Rating} = \frac{\text{Prod.Score} - \text{Lowest Level}}{\text{Highest Level} - \text{Lowest Level}}$$



Ratings Now

- Status Quo:

$$\frac{1}{3}\left\{\frac{0-(-1)}{2-(-1)}+\frac{125-0}{125-0}+\frac{0-(-2)}{1-(-2)}\right\}=0.67$$

- Alternative 1: 0.61
- Alternative 2: 0.52
- Alternative 3: 0.44

Now the units have no impact on the ratings and hence the ranking. Any other issues?

Other issues

- The ranges within which the evaluation measures can take on values have an impact on the ratings
  - *Example: Consider two alternatives with scores*

    (2, 125, 1) and (-1, 0, 1)
- Each evaluation measure is given **equal** importance
- For example, security problems are equally as important as cost issues

Alternative	Evaluation Measure		
	Productivity	Cost	Security
Status Quo	0	0	0
1	**2**	125	0.5
2	1	95	0
3	0.5	65	-1

Using Weights

- Account for both
  - *The effects of the ranges and*
  - *The impacts of different ranges*
- Example: assign 0.5 to productivity, 0.3 to cost and 0.2 to security.

New Ratings with the Assigned Weights

- Status Quo:

$$0.5\frac{0-(-1)}{2-(-1)}+0.3\frac{125-0}{125-0}+0.2\frac{0-(-2)}{1-(-2)}=0.60$$

- Alternative 1: ~0.67
- Alternative 2: ~0.54
- Alternative 3: ~0.32

Example

Let the evaluation measures for productivity, cost and security be

$$X_p, X_c \text{ and } X_s$$

The value functions be

$$\mathrm{v_p}\left(X_p\right), \mathrm{v_c}\left(X_c\right) \text{and } \mathrm{v_s}\left(X_s\right)$$

The weights to be

$$\mathrm{w_p}, w_c, \text{and } w_s.$$

The overall value function is

$$\mathrm{v}\left(X_p, X_c, X_s\right) = \mathrm{w_p v_p}\left(X_p\right) + \mathrm{w_c v_c}\left(X_c\right) + \mathrm{w_s v_s}\left(X_s\right)$$

Conventions

- Weights sum to one

$$w_p + w_c + w_s = 1$$

- Value functions take on values between 0 and 1

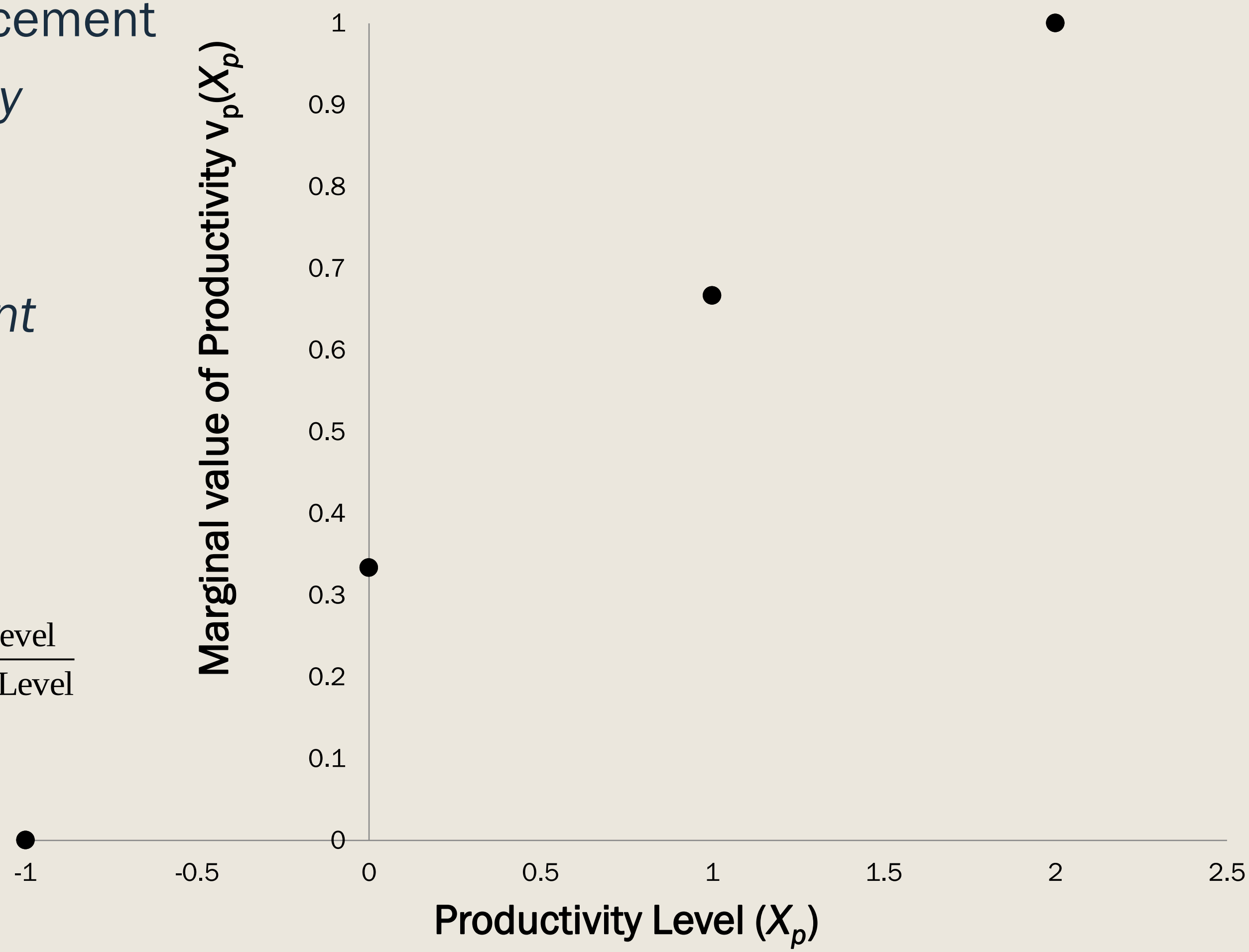
$$v_p(X_p), v_c(X_c), v_s(X_s) \in [0,1]$$

- The least preferred alternative has a rating of 0.
- The most preferred alternative has a rating of 1.

In our previous approach: Productivity

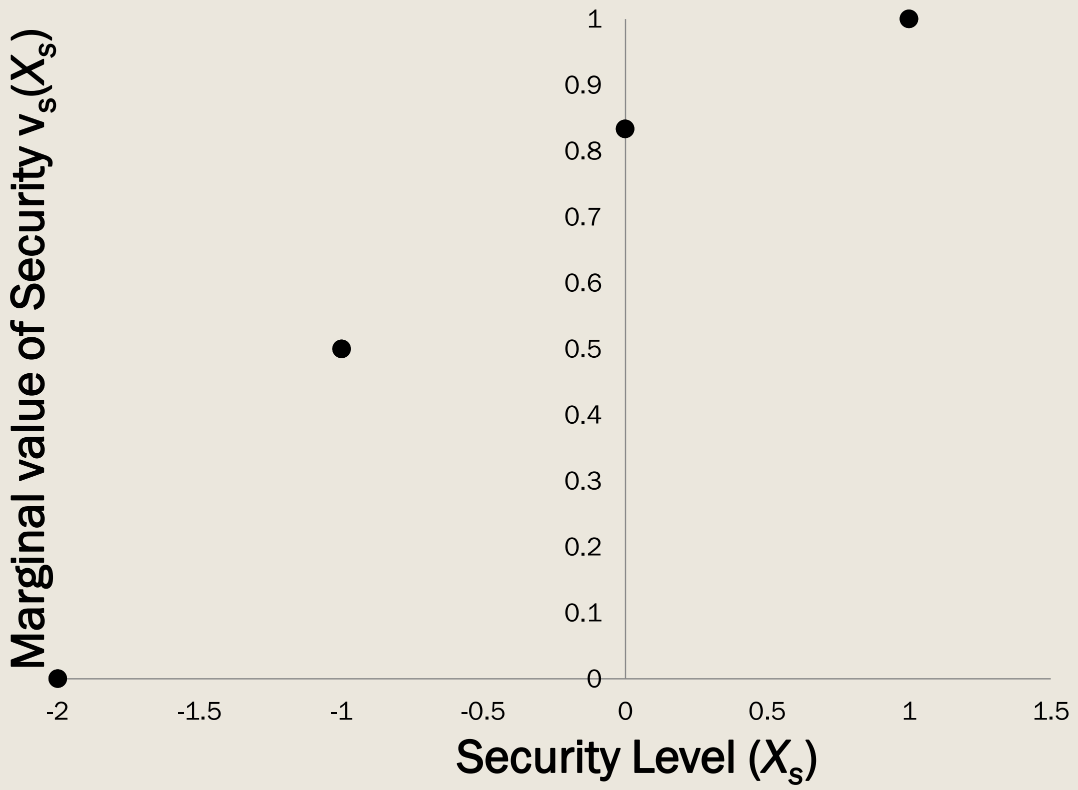
- Productivity enhancement
  - *-1: significantly diminished*
  - *0: no change*
  - *1: improvement*
  - *2: significant improvement*

$$\text{Rating}\,(V_p) = \frac{\text{Prod.Score} - \text{Lowest Level}}{\text{Highest Level} - \text{Lowest Level}}$$



A Further Concern

- **Returns to scale** might be different
- For example
  - *A deterioration of security of from 0 to –1 may be insignificant*
  - *Whereas one from –1 to –2 might have severe effects*
- There can be both increasing and decreasing returns to scale



Solution to account for returns to scale

- Determine a **single dimensional value function** for each evaluation measure
- Put the evaluation score into the respective value function for each measure
- Then apply the weights to obtain a rating for each alternative

Single-attribute Value Functions

- There are different forms for value functions:
  - *Piecewise linear*
  - *Exponential*
  - *…*

Piecewise Linear Single Attribute Value Function

- Value increments between possible scores of the evaluation measure must be specified
  - *For example,*
    - a productivity improvement from 0 to 1 may be as good as that from 1 to 2
    - But that from -1 to 0 may result in twice as much as that resulting from an improvement from either 0 to 1 or 1 to 2
    - Can you determine the corresponding value function

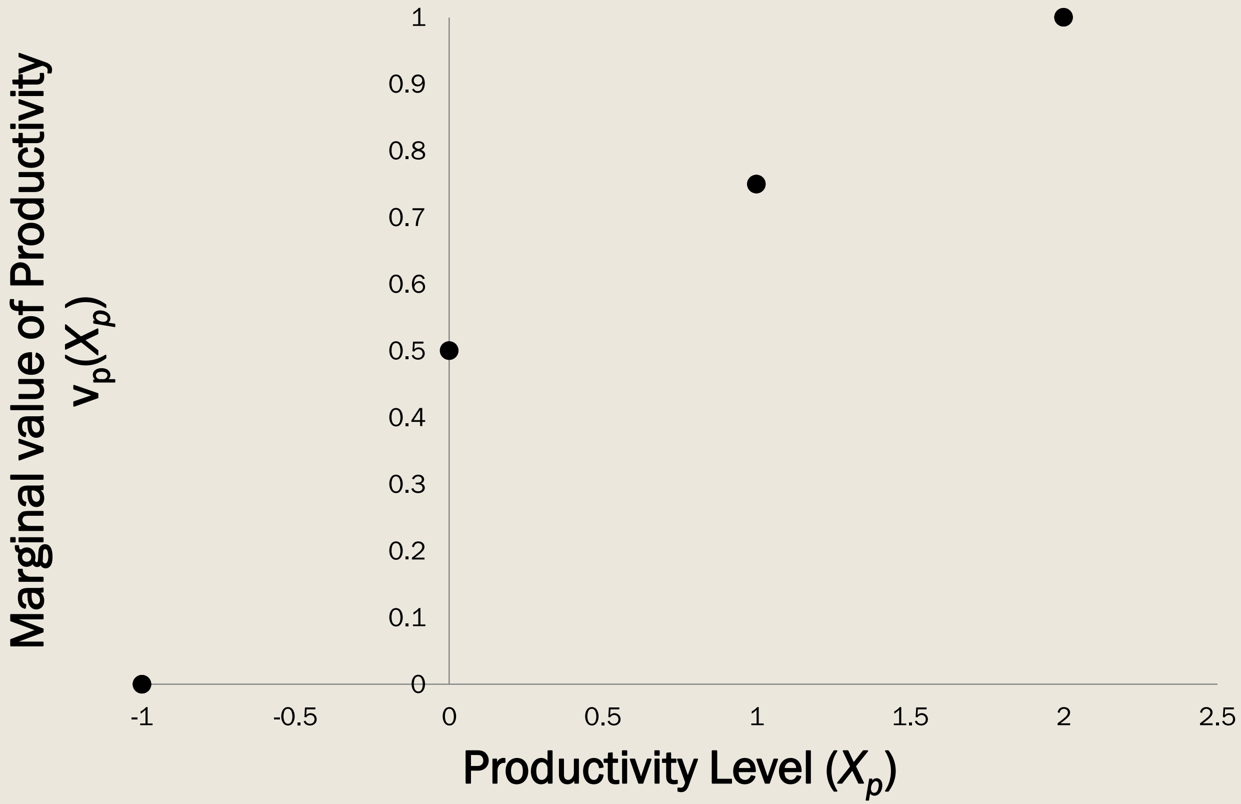
Single attribute value function for productivity improvement

$$v_p(-1) = 0;$$

$$v_p(0) = 0.5;$$

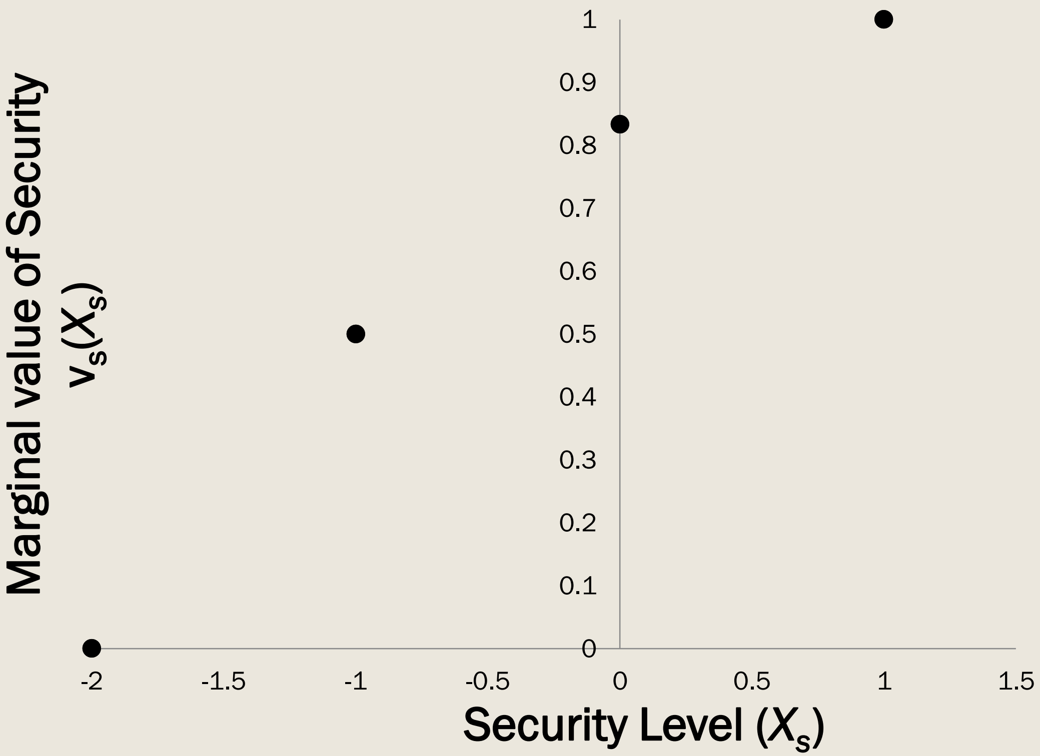
$$v_p(1) = 0.75;$$

$$v_p(2) = 1.$$



Single attribute value function for security

$$v_s(-2) = 0;$$
$$v_s(-1) = 1/2;$$
$$v_s(0) = 5/6;$$
$$v_s(1) = 1.$$



What about the value function for cost…

- We can use a similar method but the value function should be decreasing.
- Assume the range of the scale is [0,150].
- If we are given some fixed values in the range and preferences among them, then we could fit a piecewise linear function.
- Or, we could fit a strictly concave/convex function!

Exponential Single Attribute Value Functions

$$v(x) = \begin{cases} \dfrac{1 - e^{-\frac{H-x}{\rho}}}{1 - e^{-\frac{H-L}{\rho}}}, & |\rho| \neq \infty \\ \dfrac{H-x}{H-L}, & |\rho| = \infty \end{cases}$$

(decreasing)

(increasing)

$$v(x) = \begin{cases} \dfrac{1 - e^{-\frac{x-L}{\rho}}}{1 - e^{-\frac{H-L}{\rho}}}, & |\rho| \neq \infty \\ \dfrac{x-L}{H-L}, & |\rho| = \infty \end{cases}$$

How to select an appropriate rho value?

- Rule of thumb

Use a straight line otherwise

$$\frac{H-L}{10} < \rho < 10 \times (H-L)$$

- **Midvalue** (*m*) of a range [L,H] is:

$$v(H) - v(m) = v(m) - v(L) = 0.5$$

Now the equation can be solved for rho. Use Excel Goal Seek…
https://support.office.com/en-us/article/Use-Goal-Seek-to-find-the-result-you-want-by-adjusting-an-input-value-320cb99e-f4a4-417f-b1c3-4f369d6e66c7

When higher scores are more preferred?

- What are the required conditions for

$$\rho < 0$$

$$\rho > 0$$

$$\rho = \infty$$

What if lower scores are more preferred?

Cost Increase Evaluation Measure

- Consider a range between 0 and 150
- Suppose that the *midvalue* is 90

$$0.5 = \frac{1 - e^{-\frac{150-90}{\rho}}}{1 - e^{-\frac{150}{\rho}}} \quad \Rightarrow \quad \rho \approx 182.4$$

(If you use Goal Seek You will find $\rho \approx 182.225$)

$$v_c(0) = 1;\ v_c(65) = 0.6645;\ \ v_c(95) = 0.4644;\ \ v_c(125) = 0.2285$$

Determination of Weights

- Single attribute value functions take values between 0 and 1
- The weight of an evaluation measure can be taken as the increase in overall value obtained by changing the score of that evaluation measure from its least preferred level to its most preferred level.

** There are different ways to determine the weights – see page 71 of the coursebook*

Back to the Network Design Example

- Reducing cost from 150 to 0 is 1.5 times as important as a productivity increase from -1 to 2

$$w_c = 1.5 w_p$$

- A security increase from -2 to 1 is 1.25 times as important as a cost reduction of from 150 to 0

$$w_s = 1.25 w_c = 1.25 \times 1.5 w_p$$

$$w_p + w_c + w_s = 1.0$$

$$\Rightarrow w_p = 0.22; w_c = 0.35; w_s = 0.43$$

Summary of Values

Alternative	Evaluation Measure		
	Productivity	Cost	Security
Status Quo	0.50	1.00	0.83
1	1.00	0.23	0.92
2	0.75	0.46	0.83
3	0.63	0.66	0.50

Overall Ratings

Alternative	Rating
Status Quo	0.22x0.50+0.35x1.00+0.43x0.83=0.82
1	0.70
2	0.68
3	0.58

Interpretation

- Status Quo provides 82% of the value that would be obtained by moving
  - from the hypothetical worst alternative

    $(X_p=-1, X_c=150, X_s=-2)$

  - to the hypothetical best alternative

    $(X_p=2, X_c=0, X_s=1)$.

- The best and worst alternatives may be real in some problems…

VB Function for a Piecewise Linear Single Dimensional Value Function

```
Function ValuePL(x, Xi, Vi)
    i = 2
    Do While x > Xi(i)
       i = i + 1
    Loop
    ValuePL = Vi(i - 1) _
        + (Vi(i) - Vi(i - 1)) * (x - Xi(i - 1)) / (Xi(i) - Xi(i - 1))
End Function
```

VB Function for an Exponential Single Dimensional Value Function

```
Function ValueE(x, Low, High, Monotonicity, Rho)
  Select Case UCase(Monotonicity)
    Case "INCREASING"
      Difference = x - Low
    Case "DECREASING"
      Difference = High - x
  End Select
  If UCase(Rho) = "INFINITY" Then
    ValueE = Difference / (High - Low)
  Else
    ValueE = (1 - Exp(-Difference / Rho)) / (1 - Exp(-(High - Low) / Rho))
  End If
End Function
```

Sensitivity Analysis

- What if the weights are not determined properly?
- We may want to vary one of the weights (e.g., w_p) and see how the ranking of the alternatives would be affected.
- If we were to change w_p however, we would also need to change w_c and w_s as it must be that
  $w_p + w_c + w_s = 1.$

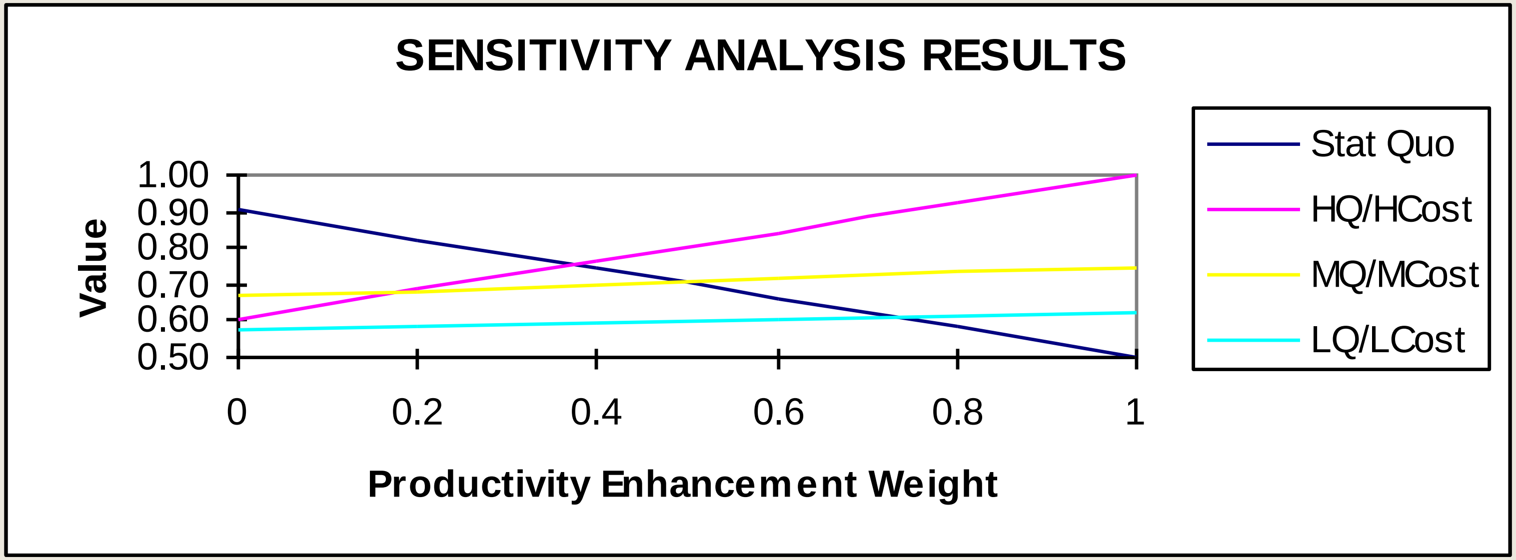
Maintain the Ratio of the Remaining Weights

Let w_c^o and w_s^o be the original weights. To maintain the ration we should enforce

$$w_c = \left(1 - w_p\right)\frac{w_c^o}{w_c^o + w_s^o}$$ and

$$w_s = 1 - w_p - w_c$$

Results of Varying w_p



Continuous Decision Variables

- Resource allocation
  - *You can allocate any percentage of a given budget within a certain range to a number of different activities.*
  - *How many alternatives do you have?*
- What are your objectives for this semester?
- How can you allocate your time to achieve these objectives?
- What are the expected consequences of these different allocations/alternatives?

Allocation of Engineering Budget

- Activities
  - *Cost improvement*
  - *Quality improvement*
  - *Development of new features and models*
- Our assumption is that performance with respect to each one of these activities is proportional to the budget allocated to that activity. *(no uncertainty for now)*

Evaluation Measures

- First year cost improvement, X_{fc}
- Carryover (cumulative) cost improvement, X_{cc}
- Quality – constructed scale $X_q \in \{0,1,2,3,4\}$
- New features and models – constructed scale $X_m \in \{0,1,2,3,4\}$

Single Attribute Value Functions and Weights

- First year cost improvement – linear, $w_{fc} = 0.10$
- Carryover (cumulative) cost improvement - linear , $w_{cc} = 0.15$
- Quality – constructed scale - piecewise linear , $w_q = 0.55$
- New features and models – exponential, *midvalue* = 3 , $w_m = 0.20$

Alternatives

Activity	Budget Allocation Range
Cost improvement	[0.20, 0.40]
Quality	[0.20,0.32]
New features and models	[0.30,0.60]

Fractions allocated to the three activities must sum to 1.
What is the number of alternatives/scores/single dimensional values?

Calculation of Scores

- Assumption – no abrupt jumps
  - *As the fraction of resources allocated to a given activity increases, performance wrt that activity improves in a smooth fashion.*
- Determine the scores corresponding to several budget fractions and fit a curve (a response function) to perform interpolation or extrapolation for other scores.