IE 443: Multiobjective Decision Analysis
Homework Assignment 1

Fall 2021

Due: October 15, 2021 - 12.30

NOTE: Please, e-mail your homeworks to your teaching assistant Egemen Elver (egemen.elver@bilkent.edu.tr). For the figures that you are asked to draw, be sure to provide a complete figure in the sense that the names of the axes, the coordinates of important points etc. are indicated. You may use a software of your choice in order to draw the figures if you think this would be easier but you are not required to do that.

Question 1. Consider the following marginal value function:

$$v(x) = \frac{1 - e^{-\frac{x-L}{\rho}}}{1 - e^{-\frac{H-L}{\rho}}}, x \in [L, H], \rho \neq 0 \tag{1}$$

(a) Sketch the graphs of the function for $\rho < 0$ and $\rho > 0$. ($|\rho| \neq \infty$)

(b) Show that $v(x)$ is increasing over [L,H].

(c) Show that $v(x)$ is convex if $\rho < 0$ and concave if $\rho > 0$.

(d) In what case the function would be linear (state the values for ρ and m (midvalue)) in that case)?

Question 2. A decision maker decides that it is time to change his home. He wants to buy a large home but it should be close to his work with a fair price. So he comes up with 10 options given in the table below.

Option	Price (p)($\times$1000) TL	Area (a) m^2	Distance to work (d) km
1	450	100	2
2	550	160	7
3	550	140	10
4	500	130	8
5	650	125	9
6	510	140	7
7	700	130	7
8	640	180	18
9	540	130	13
10	580	130	14

(a) Write down the nondominated options.

(b) After an initial discussion with the DM, you have decided to use an exponential marginal value function for the price with a midvalue of 581,288 TL. The value function for area is piecewise linear with two different line segments joining at the area of 130, which has a single dimensional value of 0.25 (i.e. $v_a(130) = 0.25$). For distance to work, the value function is linear. Draw the corresponding marginal value functions and discuss the corresponding scale of return.

(c) You asked the DM some questions and obtained the following answers:

(1) The DM is indifferent between (500000, 120, 9) and (450000, 100, 9).

(2) He is also indifferent between (600000, 160, 12) and (600000, 140, 4). According to the answers of the DM, calculate the weights of each criterion. Given these weights, provide the full preference ranking of nondominated solutions.

Question 3. Assume there are two evaluation criteria for a decision making problem. Consider the following (overall) value function over two attributes

$$v : \mathbb{R}^2_+ \to \mathbb{R}, \;\; v(x_1, x_2) := \max\{x_1, 2x_2\}.$$

a) Plot the indifference curves for $v(x_1, x_2) = 1$, $v(x_1, x_2) = 2$, $v(x_1, x_2) = 3$.

b) For the function given above, is the corresponding preference relation convex? (Show/explain your work and logic briefly and clearly).

c) Determine if v is quasiconvex or quasiconcave. Note that some functions can be both quasiconvex and quasiconcave.

A function $f : S \to R$ defined on a convex subset S of a real vector space is quasiconcave if for all $x, y \in S$ and $\lambda \in [0, 1]$ we have:

$$f(\lambda x + (1 - \lambda)y) \geq min\{f(x), f(y)\}$$

and it is quasiconvex if for all $x, y \in S$ and $\lambda \in [0, 1]$ we have:

$$f(\lambda x + (1 - \lambda)y) \leq max\{f(x), f(y)\}$$