

IE 443: Multiobjective Decision Analysis

Homework Assignment 1

Fall 2022-2023

Due: October 16, 2022 - 23:59/11:59 P.M.

NOTE: For the figures that you are asked to draw, be sure to provide a complete figure in the sense that the names of the axes, the coordinates of important points etc. are indicated. You may use a software of your choice in order to draw the figures if you think this would be easier but you are not required to do that.

Question 1. Read the article from S.S. Stevens titled *On the Theory of Scales of Measurement*.

- (a) Briefly explain the scales of measurement covered in the article.
- (b) Provide example use cases for each of the scales listed above.

Question 2. Consider the exponential marginal value function discussed in class. Apply the following analysis on $v(x)$ to be used for criterion for which the less the better.

- (a) Sketch the graphs of the function for $\rho < 0$ and $\rho > 0$. ($|\rho| \neq \infty$)
- (b) State if the function is convex or concave for each of the cases, explain the position of m values according to H and L , where H is the highest and L is the lowest values in the function.
- (c) In what case the function would be linear (state the values for ρ and m in that case)?
- (d) State if it is a decreasing or increasing scale of return for each case and briefly explain.

Question 3. Assume you are to select a new delivery provider for your company among a list of given ones. The management wants you to pay attention to four criteria among many others: expected average costs per product unit, service quality (the higher the quality, the less likely the products are to have defects when they reach the customers), expected order delivery time given in days, and average carbon emissions (amount of CO_2 emitted per kilometer kg/km) reported by the delivery firm. The list of 8 delivery contract offers is given in the table below.

Option	Cost (c) TL	Quality (q)	Delivery Time (d) days	Emitted CO_2 (e)
1	15	1	7	0.22
2	25	2	7	0.19
3	25	2	10	0.22
4	10	0	5	0.22
5	50	-1	2	0.24
6	21	2	11	0.22
7	70	-1	2	0.24
8	20	2	10	0.20

- (a) Write down the nondominated options.
- (b) After an initial discussion with management, you have decided to use an exponential marginal value function for cost with a midvalue of 30 TL. The value function for quality is piecewise linear with two different line segments joining at a quality score of 1, which has a corresponding value of 0.4 (i.e. $v_q(1) = 0.4$). Similarly for delivery time, the value function is piecewise linear with three line segments joining at delivery time of 4 and 7 days with corresponding value scores of 0.75 and 0.4 respectively. Finally, emitted CO_2 's value function was decided to be linear. Draw the corresponding marginal value functions and discuss the corresponding scale of returns for each one.
- (c) You showed the management the following pairs and obtained the following answers
- (1) They are indifferent between (70, 2, 7, 0.22) and (15, 1, 7, 0.22).

- (2) They are indifferent between $(70, 2, 7, 0.24)$ and $(70, -1, 4, 0.24)$.
- (3) They are indifferent between $(70, 2, 7, 0.19)$ and $(70, 2, 5, 0.24)$.

According to the answers of the DM, calculate the weights of each criterion.

- (d) Briefly comment on whether these weights directly show the importance of each criterion to the management.
- (e) Provide the full preference ranking of nondominated options, assuming the preferences can be reflected with an additive value function.

Question 4. Read the book excerpt from *Multiple Attribute Decision Making* by Yoon and Hwang. For the delivery provider selection problem described in *Question 3*, apply the TOPSIS method covered in the reading. Use the weight vector obtained from the management's preferences provided in Part (c). Provide full ranking of the preferences.