

IE 443: Multiobjective Decision Analysis

Homework Assignment 2

Fall 2022-2023

Due: October 30, 2022 - 11:59/11:59 **A.M.**

NOTE: For the figures that you are asked to draw, be sure to provide a complete figure in the sense that the names of the axes, the coordinates of important points etc. are indicated. You may use a software of your choice in order to draw the figures if you think this would be easier but you are not required to do that. Please email your answers to rana.yoner@bilkent.edu.tr by the deadline.

Question 1. Assume the decision maker has indifference curves of form

$$U(x_1, x_2) = 2 \min(x_1, x_2) + x_2$$

$$U : \mathbb{R}^2 \rightarrow \mathbb{R}$$

- (a) Plot the indifference curves for $U(x_1, x_2) = 3$, $U(x_1, x_2) = 6$, and $U(x_1, x_2) = 9$.
- (b) Are the preferences of the decision maker convex in the weak sense? Please explain.

Question 2. Consider the following Multiobjective Linear Optimization Problem:

$$\text{minimize } [Z_1, Z_2] \quad (1)$$

$$\text{s.t. } x_2 - \frac{1}{2}x_1 \geq 0 \quad (2)$$

$$x_2 - 2x_1 \leq 4 \quad (3)$$

$$x_2 + \frac{1}{4}x_1 \geq 7.5 \quad (4)$$

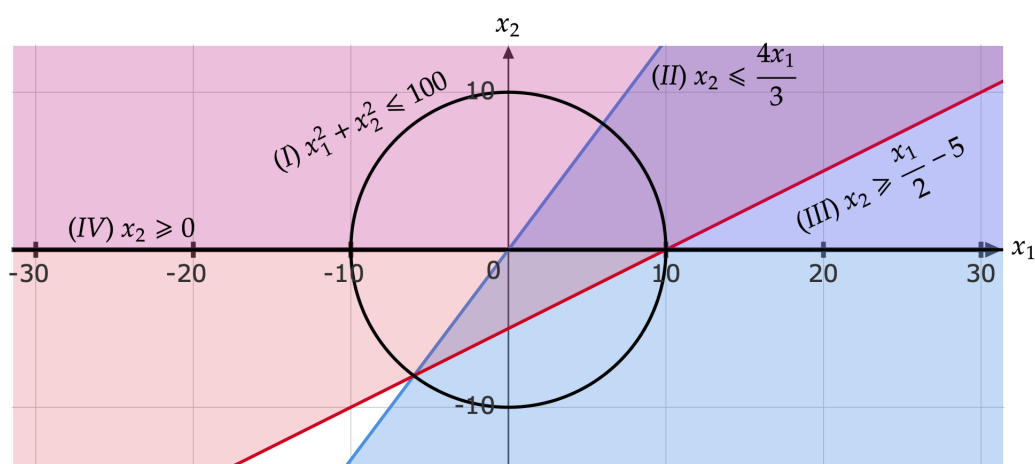
$$x_2 \geq 1 \quad (5)$$

$$x_1 \geq 0 \quad (6)$$

Where Z_1, Z_2 are functions of the form $\alpha x_1, \alpha x_2$ respectively for $\alpha \in \mathbb{R}$.

- Draw the feasible region (decision space).
- Draw the image of the feasible region for $\alpha = 1$. Determine the non-dominated points. Provide the set of efficient solutions. Do the set of efficient solutions change for other $\alpha > 0$?

Question 3. The decision space given by simultaneous satisfaction of all four constraints *I, II, III* and *IV* (feasible region) to the Multiobjective Optimization Problem $\max\{x_1, 2x_2\}$, is found below:



- Draw the image of the feasible region.
- Provide the supported nondominated points. Are there any unsupported ones?

Question 4. Read the article A Preference Ranking Organisation Method (*Brans and Vincke, 1985*). For the delivery provider selection problem described in *Question 3* of Homework 1, apply the PROMETHEE I method for the **5 Nondominated Alternatives**, namely 1, 2, 4, 5, and 8, for four criteria cost, quality, delivery time and CO_2 emission. Use the weight vector $(w_c, w_q, w_d, w_{co_2})$ where

- $w_c = 0.135$
- $w_q = 0.192$
- $w_d = 0.547$
- $w_{co_2} = 0.126$

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