IE 443: Multiobjective Decision Analysis

Homework Assignment 3

Fall 2022-2023

Due: December 11th, 2022 - 23:59/11:59 **P.M.**

NOTE: For the figures that you are asked to draw, be sure to provide a complete figure in the sense that the names of the axes, the coordinates of important points etc. are indicated. You may use a software of your choice in order to draw the figures if you think this would be easier but you are not required to do that. Please email your answers to **rana.yoner@bilkent.edu.tr** by the deadline.

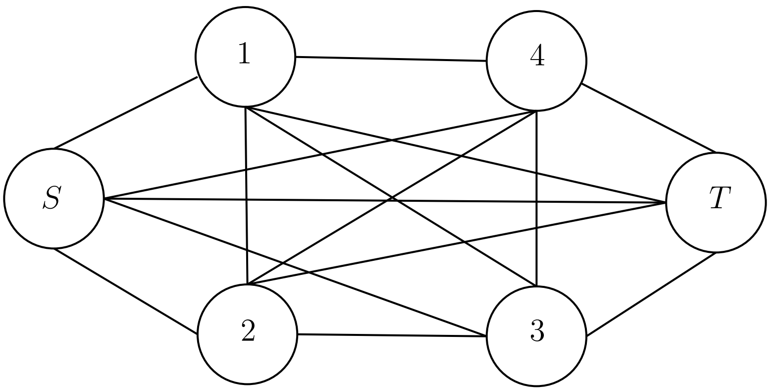
Consider the biobjective integer programming problem introduced below:
You and your friends are planning a road trip from S to T. As a part of the planning stage before the trip, you have identified 4 possible tourist attractions to visit on your way from S to T. As the road ahead is tricky due to terrain and weather conditions, you realize that the travel times from one location to another are not symmetric. Hence, it may be the case that returning back to a location may save you more time!
Moreover, you decide to assign utility scores for each location in accordance with how much fun you believe you are predicted to have as a group. Naturally, you wish to maximize total obtained utility while minimizing the total travel time from S to T.

***Table 1*: Travel Times Between Touristic Sites**

	S	**1**	**2**	**3**	**4**	**T**
S	0	10	10	7	14	1
1	7	0	6	10	5	15
2	3	2	0	14	10	3
3	15	1	15	0	6	6
4	2	2	6	11	0	10
T	15	1	4	12	10	0

***Table 2*: Utilities Obtained from Visiting Each Touristic Site**

Node	**Utility**
S	0
1	5
2	10
3	7
4	8
T	0



In the figure, the starting location (S) and the final location (T), are represented together with the four tourist attractions you have identified (1, 2, 3, 4) on a graph. As it could be seen, the graph is fully connected.

The travel times for each arc, and the utilities obtained by visiting each city are provided in tables 1 and 2. Given set N which is the set of all nodes, with parameters U_j representing utility of visiting j, and T_ij which is the duration of travel from i to j, the mathematical formulation of this problem is provided as follows:

$$\text{minimize} \quad Z_1 = \sum_{i \in N} \sum_{j \in N} T_{ij} x_{ij} \tag{1}$$

$$\text{maximize} \quad Z_2 = \sum_{j \in N} U_j z_j \tag{2}$$

$$\text{s.t.} \sum_{j \in N} x_{Sj} = 1 \tag{3}$$

$$\sum_{i \in N} x_{iT} = 1 \tag{4}$$

$$\sum_{i \in N} x_{ij} = \sum_{j \in N} x_{jk} \quad \forall j \in N \tag{5}$$

$$x_{ii} = 0 \quad \forall i \in N \tag{6}$$

$$\sum_{i \in N} z_j \leq \sum_{i \in N} x_{ij} \quad \forall j \in N \tag{7}$$

$$x_{ij}, z_j \in \{0, 1\} \qquad \forall i \in N, j \in N \tag{8}$$

Question 1. Apply the ϵ-constraint algorithm and find the set of all nondominated points. Write and solve each model with a solver of your choice. Provide the mathematical models and write the steps of the algorithm and your results (the solution of the model and its image) in each iteration. (Note: Providing only the output is not enough. You must clearly write the models and results of each step in a report.)

Question 2. Referring to the plot of the nondominated points that you obtained in Question 1, for each point, discuss whether it could be found by solving a weighted sum scalarization problem. If it is possible, give a weight vector λ such that the weighted sum scalarization problem WS(λ, 1 λ) would yield this nondominated point.