# IE 443: Multiobjective Decision Analysis
# Homework Assignment 1

Spring 2022-2023

Due: March 15, 2023 - 23:59/11:59 **P.M.**

**NOTE:** For the figures that you are asked to draw, be sure to provide a complete figure in the sense that the names of the axes, the coordinates of important points etc. are indicated. You may use a software of your choice in order to draw the figures if you think this would be easier but you are not required to do that.

**Question 1.** Read the article from S.S. Stevens titled *On the Theory of Scales of Measurement*.

(a) Briefly explain the scales of measurement covered in the article.

(b) Provide example use cases for each of the scales listed above.

**Question 2.** Consider the exponential marginal value function discussed in class. Apply the following analysis on $v(x)$ to be used for the criterion for which the more the better.

(a) Sketch the graphs of the function for $\rho < 0$ and $\rho > 0$. ($|\rho| \neq \infty$)

(b) State if the function is convex or concave for each of the cases, explain the position of $m$ values according to H and L, where H corresponds to the highest criterion in the function's domain and L corresponds to the lowest criterion in the function's domain.

(c) In what case the function would be linear (state the values for $\rho$ and $m$ in that case)?

(d) State if it is a decreasing or increasing return to scale for each case and briefly explain.

**Question 3.** The company you work for decided to throw a banquet to celebrate the end of a successful financial quarter. They rely on your decision making framework to choose the best restaurant for the event. Your supervisors want you to pay attention to three criteria: meal cost per guest (in TL), average rating of the food by reviewers, and location points (decided based on how lavish the restaurant is). The list of 8 offers from different restaurants that were recommended is in the table below.

| Option | Cost (c) TL | Rating (r) | Location Points (l) |
|---|---|---|---|
| 1 | 150 | 4.1 | 2 |
| 2 | 250 | 3.2 | 6 |
| 3 | 250 | 3.5 | 6 |
| 4 | 100 | 2.5 | 3 |
| 5 | 500 | 4.5 | 7 |
| 6 | 210 | 3.1 | 6 |
| 7 | 700 | 4.8 | 10 |
| 8 | 200 | 3.1 | 6 |

(a) Write down the nondominated options.

(b) After an initial discussion with the finance department, you have decided to use an exponential marginal value function for cost ($v_c$) with a midvalue of 200 TL. Upon talking with your coworkers you approximate that the value function for review ratings ($v_r$) is linear. Likewise, for location points ($v_l$), the value function is piecewise linear with three line segments joining at points of 5 and 7 with corresponding value scores of 0.4 and 0.8 respectively. For all marginal value functions, use the highest and lowest values after eliminating the dominated options (as we did in class). Draw the corresponding marginal value functions $v_c$, $v_r$, $v_l$ and discuss the corresponding scale of returns for each one.

(c) You showed the management the following pairs and obtained the following answers

(1) They are indifferent between (700, 4.8, 5) and (200, 2.5, 5).

(2) They are indifferent between (700, 4.8, 2) and (700, 2.5, 10).

According to the answers of the DM, calculate the weights of each criterion (assuming the preferences can be represented with an additive value function).

(d) Provide the full preference ranking of nondominated options.

**Question 4.** Consider the following value function defined for the vector of criterion values $(x_1, x_2)$ such that $x_1, x_2 \in (0, 10]$ :

$$v(x_1, x_2) = x_1 x_2$$

(a) Plot the indifference curves for $v(x_1, x_2) = 1$, $v(x_1, x_2) = 4$, and $v(x_1, x_2) = 9$.

(b) For the function given above, is the corresponding preference relation convex? If so, why?

(c) According to the definition below, show using the indifference curves whether $v$ is quasiconcave or not.

A function $f : S \to R$ defined on a convex subset $S$ of a real vector space is quasiconcave if for all $x, y \in S$ and $\lambda \in [0, 1]$ we have:
$f(\lambda x + (1 - \lambda)y) \geq \min\{f(x), f(y)\}$