2.3.5 TECHNIQUE FOR ORDER PREFERENCE BY SIMILARITY TO IDEAL SOLUTION (TOPSIS)

Yoon and Hwang [440a, 440b] developed the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) based upon the concept that the chosen alternative should have the shortest distance from the ideal solution and the farthest from the negative-ideal solution.

Assume that each attribute takes the monotonically increasing (or decreasing) utility; then it is easy to locate the "ideal" solution which is composed of all best attribute values attainable, and the "negative-ideal" solution composed of all worst attribute values attainable. One approach is to take an alternative which has the (weighted) minimum Euclidean distance to the ideal solution in a geometrical sense [381, 459]. It is argued that this alternative should be farthest from the negative-ideal solution at the same time. Sometimes the chosen alternative, which has the minimum Euclidean distance from the ideal solution, has the shorter distance (to the negative-ideal) than the other alternative(s). For example, in Fig. 3.6 an alternative A_1 has shorter distances (both to ideal solution A^* and to the negative-ideal solution A^-) than the other alternative A_2. Then it is very difficult to justify the selection of A_1. TOPSIS considers the distances to both the ideal and the negative-ideal solutions simultaneously by taking the relative closeness to the ideal solution. Dasarathy [66] used this similarity measure in clustering multidimensional data arrays. This method is simple and yields an indisputable preference order of solution.

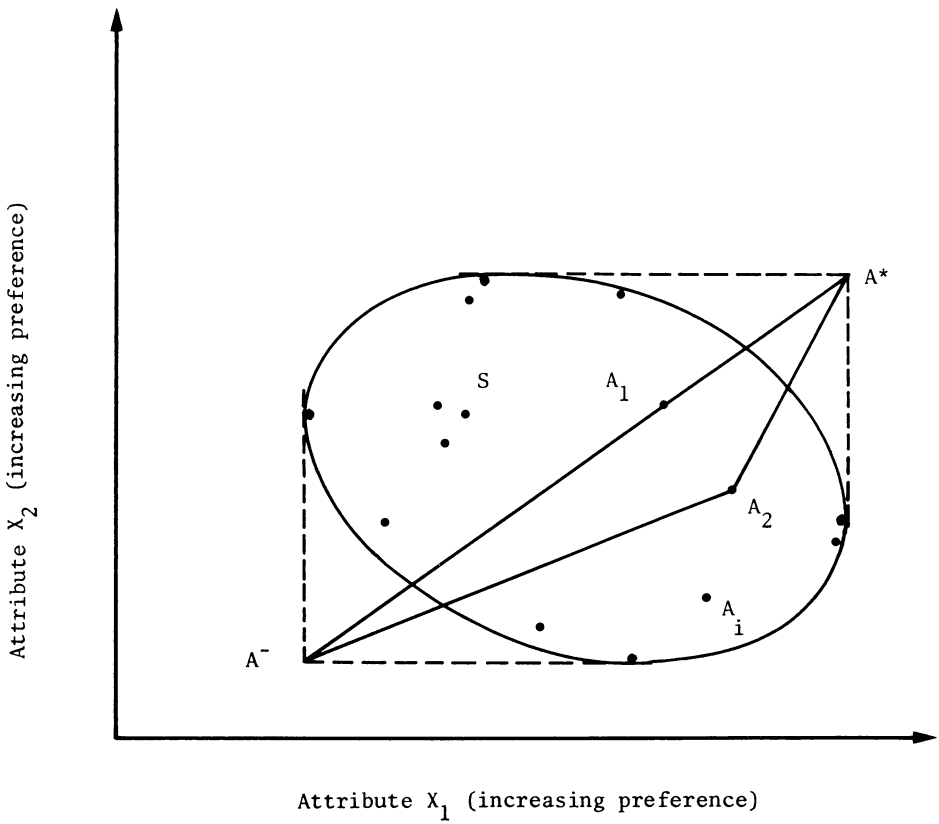

Fig. 3.6 Euclidean distances to the ideal and negative-ideal solutions in two dimensional space.

The Algorithm

The TOPSIS method evaluates the following decision matrix which contains m alternatives associated with n attributes (or criteria):

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccccc} X_1 & X_2 & & X_j & & X_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

where

A_i = the i^{th} alternative considered,

x_{ij} = the numerical outcome of the i^{th} alternative with respect to the j^{th} criterion.

TOPSIS assumes that each attribute in the decision matrix takes either monotonically increasing or monotonically decreasing utility. In other words, the larger the attribute outcomes, the greater the preference for the "benefit" criteria and the less the preference for the "cost" criteria. Further, any outcome which is expressed in a nonnumerical way should be quantified through the appropriate scaling technique. Since all criteria cannot be assumed to be of equal importance, the method receives a set of weights from the decision maker. For the sake of simplicity the proposed method will be presented as a series of successive steps.

Step 1. Construct the normalized decision matrix: This process tries to transform the various attribute dimensions into nondimensional attributes, which allows comparison across the attributes. One way is to take the outcome of each criterion divided by the norm of the total outcome vector of the criterion at hand. An element r_{ij} of the normalized decision matrix R can be calculated as

$$r_{ij} = x_{ij} \Big/ \sqrt{\sum_{i=1}^{m} x_{ij}^2}$$

Consequently, each attribute has the same unit length of vector.

Step 2. Construct the weighted normalized decision matrix: A set of weights $\underline{w} = (w_1, w_2, \ldots, w_j, \ldots, w_n)$, $\sum_{j=1}^{n} w_j = 1$, from the decision maker is accommodated to the decision matrix in this step. This matrix can be calculated by multiplying each column of the matrix R with its associated weight w_j. Therefore, the weighted normalized decision matrix V is equal to

$$V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1j} & \cdots & v_{1n} \\ \vdots & \vdots & & \vdots & & \vdots \\ v_{i1} & v_{i2} & \cdots & v_{ij} & \cdots & v_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ v_{m1} & v_{m2} & \cdots & v_{mj} & \cdots & v_{mn} \end{bmatrix} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_j r_{ij} & \cdots & w_n r_{1n} \\ \vdots & \vdots & & \vdots & & \vdots \\ w_1 r_{i1} & w_2 r_{i2} & \cdots & w_j r_{ij} & \cdots & w_n r_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_j r_{mj} & \cdots & w_n r_{mn} \end{bmatrix}$$

Step 3. Determine ideal and negative- ideal solutions: Let the two artificial alternatives A^* and A^- be defined as

$$A^* = \{(\max_i v_{ij} \mid j \in J), (\min_i v_{ij} \mid j \in J') \mid i = 1,2,\ldots,m\}$$

$$= \{v_1^*, v_2^*, \ldots, v_j^*, \ldots, v_n^*\} \tag{3.44}$$

$$A^- = \{(\min_i v_{ij} \mid j \in J), (\max v_{ij} \mid j \in J') \mid i = 1,2,\ldots,m\}$$

$$= \{v_1^-, v_2^-, \ldots, v_j^-, \ldots, v_n^-\} \tag{3.45}$$

where $J = \{j = 1,2,\ldots,n | j$ associated with benefit criteria$\}$

$J' = \{j = 1,2,\ldots,n | j$ associated with cost criteria$\}$

Then it is certain that the two created alternatives A^* and A^- indicate the most preferable alternative (ideal solution) and the least preferable alternative (negative-ideal solution), respectively.

4. Calculate the separation measure: The separation between each alternative can be measured by the n-dimensional Euclidean distance. The separation of each alternative from the ideal one is then given by

$$S_{i*} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}, \qquad i = 1,2,\ldots,m \tag{3.46}$$

Similarly, the separation from the negative-ideal one is given by

$$S_{i-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \qquad i = 1,2,\ldots,m \tag{3.47}$$

5. Calculate the relative closeness to the ideal solution: The relative closeness of A_i with respect to A^* is defined as

$$C_{i*} = S_{i-}/(S_{i*} + S_{i-}), \quad 0 < C_{i*} < 1, \qquad i = 1,2,\ldots,m \tag{3.48}$$

It is clear that $C_{i*} = 1$ if $A_i = A^*$ and $C_{i*} = 0$ if $A_i = A^-$. An alternative A_i is closer to A^* as C_{i*} approaches to 1.

6. Rank the preference order: A set of alternatives can now be preference ranked according to the descending order of C_{i*}.

Numerical Example (The Fighter Aircraft Decision Problem)

The decision matrix of the fighter aircraft problem after the quantification of nonnumerical attributes of X_5 and X_6 is:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccccc} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \end{array} \\ \begin{bmatrix} 2.0 & 1500 & 20000 & 5.5 & 5 & 9 \\ 2.5 & 2700 & 18000 & 6.5 & 3 & 5 \\ 1.8 & 2000 & 21000 & 4.5 & 7 & 7 \\ 2.2 & 1800 & 20000 & 5.0 & 5 & 5 \end{bmatrix} \end{array}$$

Note that all attributes except X_4 are the benefit criteria.

1. Calculate the normalized decision matrix

$$R = \begin{bmatrix} .4671 & .3662 & .5056 & .5063 & .4811 & .6708 \\ .5839 & .6591 & .4550 & .5983 & .2887 & .3727 \\ .4204 & .4882 & .5308 & .4143 & .6736 & .5217 \\ .5139 & .4392 & .5056 & .4603 & .4811 & .3727 \end{bmatrix}$$

2. Calculate the weighted decision matrix: Assume that the relative importance of attributes is given by the decision maker as $\underline{w} = (w_1, w_2, w_3, \ldots, w_6) = (.2, .1, .1, .1, .2, .3)$. The weighted decision matrix is then

$$V = \begin{bmatrix} .0934 & .0366 & .0506 & .0506 & .0962 & .2012 \\ .1168 & .0659 & .0455 & .0598 & .0577 & .1118 \\ .0841 & .0488 & .0531 & .0414 & .1347 & .1565 \\ .1028 & .0439 & .0506 & .0460 & .0962 & .1118 \end{bmatrix}$$

3. Determine the ideal and negative-ideal solutions:

$$A^* = (\max_i v_{i1}, \max_i v_{i2}, \max_i v_{i3}, \min_i v_{i4}, \max_i v_{i5}, \max_i v_{i6})$$

$= (.1168, \quad .0659, \quad .0531, \quad .0414, \quad .1347, \quad .2012)$

$$A^- = (\min_i v_{i1}, \min_i v_{i2}, \min_i v_{i3}, \max_i v_{i4}, \min_i v_{i5}, \min_i v_{i6})$$

$= (.0841, \quad .0366, \quad .0455, \quad .0598, \quad .0577, \quad .1118)$

4. Calculate the separation measures:

$$S_{i*} = \sqrt{\sum_{j=1}^{6} (v_{ij} - v_j^*)^2}, \qquad i = 1,2,3,4$$

$S_{1*} = .0545 \qquad S_{2*} = .1197$

$S_{3*} = .0580 \qquad S_{4*} = .1009$

$$S_{i-} = \sqrt{\sum_{j=1}^{6} (v_{ij} - v_j^-)^2}, \qquad i = 1,2,3,4$$

$S_{1-} = .0983 \qquad S_{2-} = .0439$

$S_{3-} = .0920 \qquad S_{4-} = .0458$

5. Calculate the relative closeness to the ideal solution:

$C_{1*} = S_{1-}/(S_{1*} + S_{1-}) = .643, \quad C_{2*} = .268,$
$C_{3*} = .613, \quad C_{4*} = .312$

6. Rank the preference order: According to the descending order of C_{i*}, the preference order is:

A_1, A_3, A_4, A_2