IE 443: Multiobjective Decision Analysis

Homework Assignment 3

Spring 2022-2023

Due: May 19th - 23:59/11:59 **P.M.**

This homework can be done in groups of 2 people. For any solvers/ software you use, be sure to send the necessary files documenting your work. Please report your answers in a written manner (do not just send the code).

Question 1. A formulation for the Biobjective Knapsack Problem (BOKP) is given below. This version modifies the knapsack problem to consider two objectives at the same time.

$$\begin{aligned}
\max \quad & f_1(\mathbf{x}) = \sum_{i=1}^{n} c_{1i} x_i && (1)\\
\max \quad & f_2(\mathbf{x}) = \sum_{i=1}^{n} c_{2i} x_i && (2)\\
\text{subject to} \quad & \sum_{i=1}^{n} w_i x_i \leq W && (3)\\
& x_i \in \{0,1\} \quad \text{for } i = 1, \ldots, n && (4)
\end{aligned}$$

Using the following instance of 10 items with weight vector w, cost vectors c_1, c_2 and capacity $W = 100$; run an ϵ-Constraint algorithm to find the Pareto-frontier. Plot the images of the set of efficient solutions. Classify them as supported and non-supported.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
w	67	17	25	1	16	16	90	45	38	90
c_1	1	4	8	7	6	2	6	7	1	8
c_2	10	9	10	5	2	4	8	10	10	3

In each iteration of the algorithm, be sure to indicate your ϵ value, objective, and constrained objective clearly. Also include the optimal solution of the model that is solved in the iteration.

Question 2. Consider the following Biobjective Integer Programming (BOIP) Problem:

$$\min f_1(x_1, x_2) = x_1 \tag{1}$$

$$\min f_2(x_1, x_2) = x_2 \tag{2}$$

$$\text{s.t.} \quad 2x_1 + 3x_2 \geq 12 \tag{3}$$

$$x_1 \leq 4 \tag{4}$$

$$x_2 \leq 4 \tag{5}$$

$$x_1, x_2 \geq 0 \qquad \text{and integer} \tag{6}$$

(I) Write the Pascoletti-Serafini (PS) and the Weighted Tchebycheff (WT) scalarizations to this problem in their general form, using reference vector v, direction vector d, and weight vector w.

(II) Indicate under which parameter choices of v and d, $PS(v, d)$ is equivalent to the WT scalarization with $w = (1, 1)$ and $v = Z^I$ (denoted as $WT(1, Z^I)$).

(III) Solve $PS(Z^I, (1, 1))$. What non-dominated point does it yield?

(IV) Now use the image vector $f_1(x_1, x_2) = 2x_1 + 4x_2$, $f_1(x_1, x_2) = 7x_1 - x_2$. Using the same problem formulation, solve $PS(v = (-8, -18), d = (4, 3))$. What is the optimal ρ and which non-dominated point does it find?

(V) Give another direction vector d for which we find another non-dominated point with $v = (-8, -18)$. What is the efficient solution corresponding to the image?