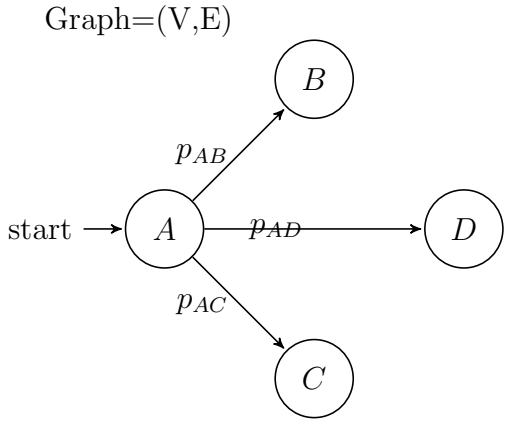
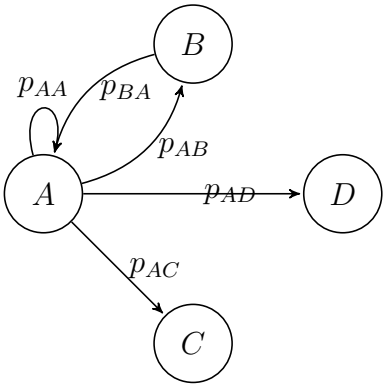
IE-452/IE-552: Algebraic and Geometric Methods in Data Analysis
Lecture Notes

April 28, 2020



$p_{AB} + p_{AD} + p_{AC} = 1$



$p_{AA} + \cdots + p_{AD} = 1$ and $p_{AB} = p_{BA}$

P is the probability matrix and P-AB=p_{AB}. $p_{AB} = 0.5$ and $p_{AA} = 0.5$ and $p_{BA} = 0.5$ and $p_{BB} = 0.5$. P=$\begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

Stochastic Version
X(t)=A or B and Q(X(t)=A) and X(2)=(0.52 0.48), X(0)=1 0 X(1)=X(0)P and X(2)=X(0)P^2

$$0.6 \times 0.6 + 0.4 \times 0.4 = 0.52 \qquad X(2) = (0.52 \;\; 0.48)$$

$$X(3) = X(2)P \qquad X(3) = (0.508 \;\; 0.492)$$

$$P = \begin{bmatrix} 0.6 & 0.4 \\ 0.6 & 0.4 \end{bmatrix} X(0) = (10) \qquad P = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}$$

$$X(3) = (0.444 \;\; 0.556) \qquad \lim_{t \to +\infty} X(t) = (0.4286 \;\; 0.5714)$$

$P^{t+1} = P^t P$ and $p_{11}(t+1) = 0.6p_{11}(t) + 0.3p_{21}(t)$, $p_{11}(t) + p_{21}(t) = 1$
$p_{11}(t+1) = (0.6 - 0.3)p_{11}(t) + 0.3$ and $p_{11}(0) = 1$

$$\begin{aligned}
y(t+1) &= a * y(t) + b \\
y(t+1) &= c, c = b/(1-a) \\
\frac{0.3}{0.7} &\overset{??}{=} 0.4286, \quad y(t) - \frac{b}{1-a} \\
z(t) &= y(t) - \frac{b}{1-a} \\
z(t+1) &= y(t+1) - \frac{b}{1-a}, \quad y(t+1) = a(z(t) + \frac{b}{1-a}) + b \\
z(t+1) &= a * z(t) + \frac{ab}{1-a} + b - \frac{b}{1-a} \\
z(t+1) &= z(t), z(t+1) = a * z(t), z(t) = a^t z(0) \\
y(t) &= a^t * z(0) + \frac{b}{1-a} \\
y(t) &= \big(1 - \frac{b}{1-a}\big)a^t + \frac{b}{1-a} \\
p_{11}(t) &= 0.5714(0.3)^t + 0.4286
\end{aligned}$$