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This paper develops novel mathematical models to maximize the benefits derived from the distribution of critical supplies to populations in need after a disaster. The formulations are based on welfare economics and the use of social costs, which are incurred by the segments of society involved in, and impacted by, the relief distribution strategy. The costs to the relief group are assessed as logistical costs, whereas the impacts to the beneficiaries are measured as the effects that the relief distribution has on their deprivation costs, which are the economic value of human suffering resulting from the lack of access to a good or service. The impacts on beneficiaries take into account two components: the reduction in deprivation costs for the recipients of the aid and the increase in deprivation costs for those individuals who do not receive the aid at a delivery epoch (the opportunity costs of the delivery strategy). The paper develops an inventory-allocation-routing model for the optimal assignment of critical supplies that minimizes social costs, designs suitable heuristic solution approaches, and assesses the performance of the heuristics using numerical experiments. The results, coupled with the insight gained from the models developed, are used to establish conclusions and policy recommendations.

Keywords: humanitarian logistics; social costs; deprivation cost; inventory allocation
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1. Introduction

The vulnerability of modern societies to natural and human-made catastrophes is a serious problem. On a worldwide basis, about 500 disasters occur every year, producing about 75,000 fatalities and affecting some 200 million people worldwide (Van Wassenhove 2006). In 2008 alone, the human and economic losses of the 354 reported natural disasters—including Cyclone Nargis in Myanmar and the Sichuan earthquake in China—caused more than 235,000 deaths, 214 million people to be affected, and over $190 billion in economic costs (Rodriguez et al. 2009). These and other recent disasters, such as the 2010 Haiti earthquake (160,000 deaths) and the 2011 Japan tsunami (19,000 deaths), illustrate the challenges that such events pose to postdisaster humanitarian logistics (PD-HL). In these situations, critical resources such as water and food are usually either made unavailable, destroyed by the event itself, or quickly depleted during the response, thereby requiring the rapid and efficient deployment of external assistance to reduce further property damage and loss of human life.

The benefits of enhancing preparedness as a way to improve disaster response are well known. Timely identification of threats as well as initiatives to reduce vulnerability and increase resilience are worthy undertakings. Even the best-laid plans, however, cannot anticipate all of the needs that may emerge after a large disaster. Consequently, the success of the humanitarian response largely relies on the ability of disaster responders to adapt to the changing conditions in the aftermath of a large disaster. PD-HL is central to this effort because it deals with the procurement, transportation, and distribution of the critical supplies needed by the surviving population and the response itself. PD-HL is also one of the most expensive components of any relief operation. The stakes could not be higher because the successful execution of PD-HL is not just a question of economics; it could mean the difference between life and death for scores of individuals.

Given its critical importance, it is reasonable to expect that PD-HL planning and research would be a top priority for scholars and practitioners. After all, commercial supply chains have experienced significant advances in recent years, making efficient logistics a highly coveted competitive advantage. However, this has not always been the case in PD-HL operations because, quite frequently, they are an afterthought (Holguín-Veras et al. 2012b). Part of the problem is that the primary focus of many humanitarian organizations has been on funding relief rather than on the processes that support the delivery of that relief (Pettit and Beresford 2005). Quite often, relief organizations have relegated humanitarian logistics to a secondary role, locking it in a self-reinforcing cycle where failure to recognize its importance leads managers to exclude it from budgetary and planning processes, thus letting humanitarian logistics function with a fire-fighting mentality (Van Wassenhove 2006). This reactive approach may introduce large inefficiencies and lead to poor levels of service, delayed or incorrect deliveries, redundant activities, and unmet needs. The situation is aggravated by a lack of scientific methods to analyze and coordinate the flow of goods under extreme conditions (Holguín-Veras et al. 2007). The resulting lack of knowledge about the nature and challenges of real-life operations further hinders the development of mathematical formulations that properly capture the inherent complexity of PD-HL.

While humanitarian and commercial logistics have the same general purpose—delivering goods and services to points of demand—they differ significantly in several aspects: objectives pursued, characteristics of the commodity flows, decision-making structures, and the state of the supporting systems (Holguín-Veras et al. 2012b). Based on fieldwork research after large disasters, together with the comprehensive study of real-life operations, Holguín-Veras et al. (2012b) established the importance of making a distinction between humanitarian assistance that takes place in chronic crisis regions (regular humanitarian logistics, or R-HL), and those after large disasters or catastrophic events (PD-HL). While R-HL is relatively similar to the commercial logistic paradigm (only the objective is different), PD-HL is profoundly different as a result of emergent behaviors such as material convergence (Holguín-Veras et al. 2012d) and extreme human suffering (Holguín-Veras et al. 2012b). However, many analytical formulations in the PD-HL literature do not consider these differences because they are extensions of models originally developed for commercial logistics where the focus is on the minimization of logistical costs or a similar objective function. However, in the aftermath of large disasters, minimizing logistical costs is not necessarily the most important consideration because in most cases the emerging needs cannot be met, so the goal is to deliver the scarce resources in a way that maximizes the effectiveness of the aid. Other publications acknowledge the need for analytical models specifically designed for humanitarian operations and have developed techniques to prioritize levels of service to impacted sites over operational costs. For the most part, these methods use proxy measures (e.g., equity constraints, priority factors, and penalties for unmet demands) to consider the beneficiaries' well-being. Examples include Barbarosoglu and Arda (2004), Yi and Kumar (2007), and Yi and Özdamar (2007). However, recent research based on welfare economics has called into question both the conceptual validity of these methods and their ability to provide sound delivery strategies (Holguín-Veras et al. 2013). This is further discussed later in this paper.

The root of the problem is the need to balance two seemingly disparate objectives: maximization of operational efficiency and minimization of human suffering. Delivering a high level of service (LOS) to beneficiaries could require a large expenditure of resources, which may not always be financially feasible. Aggressive measures to reduce operational costs could negatively affect the beneficiaries' welfare, even to the extent of loss of life. The fundamental tenet of this paper, and that of Holguín-Veras et al. (2013), is that these considerations could be seamlessly reconciled using welfare economics. Central to this proposition is the concept of social costs, which are incurred by all segments of society that are involved in, and impacted by, the relief distribution strategy. The costs to the relief group are assessed as logistical costs, whereas the impacts to the beneficiaries are measured as the effects that the relief distribution has on their deprivation costs, which are the economic value of human suffering resulting from the lack of access to a good or service (Holguín-Veras et al. 2013). The impact on beneficiaries, which become economic externalities in the event of the destruction of the markets, takes into account two components: the reduction in deprivation costs for the recipients of the aid and the increases in deprivation costs for those individuals who do not receive the aid at a delivery epoch (the opportunity costs of the delivery strategy) (Holguín-Veras et al. 2013). The consideration of social costs is the most important difference between the methods proposed here and the current literature. In addition, the models in this paper incorporate other unique characteristics of the phenomenon, including the nonadditive nature of demands and multiple commodities with varying degrees of priority.

The main goal of this paper is to contribute to the literature by developing formal mathematical formulations that build on the work of Holguín-Veras et al. (2013). To this effect, it uses the concepts outlined in Holguín-Veras et al. (2013) to create fully specified mathematical models of relief operations and suitable solution heuristics for both the single and multiple commodity cases. This is an important development because these are the first formal models, based on welfare economic concepts, capable of modeling relief operations with the level of detail required to support operational decision making in real-life conditions. This represents a major step forward with respect to the rest of the literature because, as established in Holguín-Veras et al. (2013), the social cost objective function used here provides a fuller description of the phenomenon being modeled.

The paper is organized as follows: §2 succinctly discusses the relevant literature, §3 discusses the economic foundations of the paper and describes the problem of interest, §4 proposes formulations for the integrated inventory allocation and vehicle routing problem that incorporate the deprivation costs associated with human suffering, and §§5 and 6 present mathematical models and numerical results for the allocation of supplies to demand nodes (also referred to as points of distribution, or PODs) in a disaster area. The single commodity case is discussed in §5, and the multiple commodity case is discussed in §6. Section 7 presents and discusses the results of the numerical experiments. The paper ends with a general discussion of conclusions and contributions in §8.

2. Basic Literature on Postdisaster Humanitarian Logistics Modeling

The amount of research on PD-HL is not commensurate with the importance of the subject, as made evident by the comprehensive reviews by Pérez (2011) and Holguín-Veras et al. (2013). Most publications in the literature have typically focused on the planning dimension of humanitarian relief, using approaches based on commercial logistics modeling techniques and methodologies. Few articles consider the unique features of PD-HL (Pérez 2011; Holguín-Veras et al. 2012b). The likely root of this situation is the extremely small number of publications describing and analyzing real-life PD-HL operations. Among the handful of publications on the subject, one could list Benini et al. (2006); Holguín-Veras et al. (2007, 2012a), and Holguín-Veras et al. (2014). Holguín-Veras et al. (2013) identified intertemporal effects, material convergence, and the impact of the distribution strategy on human suffering as three essential characteristics that must be considered for proper modeling of PD-HL.

The consideration of intertemporal effects is important because of the need to consider the opportunity costs of the delivery strategy—i.e., the increase in human suffering produced by not receiving critical supplies at a delivery epoch (Holguín-Veras et al. 2013). Because considering these intertemporal effects requires the use of multiperiod formulations, (static) single-period models (about half of the literature) cannot consider the impacts of shortages of supplies, which are the norm in the initial stages of the response. It is important to consider the terminal costs that accrue from the last delivery to the nodes and the end of the planning horizon to ensure proper modeling of the distribution effort. A second important, and frequently overlooked, phenomenon is the material convergence that takes place in the immediate aftermath of a large disaster. The main issue is that the bulk of the material convergence, about 60%, is comprised of nonpriority supplies that should not have been sent to the disaster area because, among other reasons, they are not needed, cannot be efficiently distributed, are not consistent with local needs and culture, or could do harm to the population (Fritz and Mathewson 1957; Holguín-Veras et al. 2012d). The complications associated with the handling of this huge inflow of unnecessary supplies are such that disaster responders refer to it as a second tier disaster (Newsweek 2002). Only recently, researchers have tackled this understudied subject (Holguín-Veras et al. 2012d). The third critical aspect is the impact of the relief distribution on human suffering.

The need to develop models that account for human suffering has been acknowledged by a number of researchers who have attempted to consider human suffering in the mathematical models they have developed. In terms of the mathematical structure, these formulations can be classified as (1) penalty based models (i.e., constant penalty, variable penalties, and hard constraint models) and (2) unmet demand models (Holguín-Veras et al. 2013). In terms of the objective function used, these formulations can be classified into those primarily based on (1) logistic costs (Haghani and Oh 1996; Barbarosoglu and Arda 2004; Han et al. 2007; Rawls and Turnquist 2010), (2) proxy measures of human suffering (Fiedrich, Gehbauer, and Rickers 2000; Özdamar, Ekinci, and Küçükyazici 2004; Chang, Tseng, and Chen 2007; De Angelis et al. 2007; Sheu 2007; Yi and Kumar 2007; Yi and Özdamar 2007; Campbell, Vandenbussche, and Hermann 2008; Ukkusuri and Yushimito 2008), and (3) logistic costs and a proxy measure of human suffering (Barbarosoglu, Özdamar, and Çevik 2002; Tzeng, Cheng, and Huang 2007; Balcik, Beamon, and Smilowitz 2008; Salmerón and Apte 2010; Yushimito, Jaller, and Ukkusuri 2012). The formulations in the first group minimize logistical costs such that demand constraints are met; they may be appropriate for planning purposes but certainly not for situations where supplies are insufficient to meet the needs on the ground, which is the focus of this paper. The limitations of these approaches, from the perspective of social costs, are summarized next. For a full discussion of the limitations of alternative approaches, see Holguín-Veras et al. (2013).

Penalty-based models, which are the majority of the available formulations, incorporate priority or penalty factors to encourage satisfaction of the most urgent needs (Chang, Tseng, and Chen 2007; Yi and Kumar 2007; Yi and Özdamar 2007; Rawls and Turnquist 2010). The penalty could be in the form of hard constraints that must be met, as in equity constraints, a constant penalty that is assessed whenever an equity constraint is not met, or a variable penalty that depends on the extent that the equity constraint is violated (as in soft-time windows routing models). Examples include introducing constraints requiring a minimum number of deliveries to the demand nodes, e.g., De Angelis et al. (2007), or limiting the headway between successive deliveries, e.g., Sheu (2007).

Holguín-Veras et al. (2013) concluded that the main problem of using (hard) equity constraints to mandate a minimum LOS is that there are no guarantees that the constraints could be met, for the simple reason that in situations of scarcity, by definition, the available resources cannot satisfy all needs. This, unavoidably, leads to unfeasible solutions that force modelers to alter the constraints until a solution is found. In essence, the modeling process becomes arbitrary. The constant penalty model has similar limitations. If the penalty is low, the model would accept the penalties and avoid making deliveries with high logistical costs—e.g., deliveries to far away nodes. However, if the penalty is set too high and the supplies are not available, there could be no feasible solutions. To avoid these outcomes, equity constraints are frequently used, leading to the feasibility problems discussed previously. Variable penalty models would assess a charge that depends on a measure of tardiness. These models are not structurally impeded to obtain optimal solutions in terms of social costs, if (1) the penalties match the deprivation cost function and (2) intertemporal effects and terminal costs are considered.

Other models focus exclusively on minimizing unmet demands, e.g., Özdamar, Ekinci, and Küçükyazici (2004). The inherent limitation of these approaches is that they do not account for differences in the degree of urgency at the various demand locations. A demand node that has experienced a long deprivation time is likely to be more urgent than a larger node that has been deprived of supplies for a shorter period of time, even if the unmet demands at both nodes are exactly the same. Moreover, these models implicitly assume that demands are additive, and in real life, they are not. For instance, a person deprived of food or water for three days cannot consume three days' worth of food and water when the supplies finally arrive (Holguín-Veras et al. 2013). In addition to imposing levels of service that may be unfeasible given the scarcity of critical supplies, these models generally do not account for the operational costs incurred in the response. A handful of publications consider both operational costs and proxy measures of the social impact of the delivery strategy (Barbarosoglu and Ozgur 1999; Tzeng, Cheng, and Huang 2007; Balcik, Beamon, and Smilowitz 2008), but they have difficulties reconciling these objectives. Unable to express operational and social welfare on a common scale, these methods rely on subjective parameters, thus biasing the optimization process toward the modelers' preference structure. The multiplicity of objective functions and the fact that they are likely to lead to different solutions led Holguín-Veras et al. (2013) to use welfare economics to analyze the appropriateness of the various formulations. Because of the importance of this research, the main findings of Holguín-Veras et al. (2013) are further discussed in §3.

3. Methodological Foundations

This section conceptually describes the analytical and conceptual foundations of the methodology as well as the most salient aspects of the relief distribution problem studied in the paper. The first part of the section provides an extended summary of Holguín-Veras et al. (2013), the foundation for the work reported in this paper. The second part focuses on the impact of alternative delivery strategies on the social costs and, particularly, on the opportunity costs that would be accrued. Holguín-Veras et al. (2013) analyzed the impact of the relief distribution of critical supplies from biological, philosophical, and economic points of view. Their chief conclusion is that PD-HL models must be based on welfare economics—the branch of economics that studies the impact of the allocation of resources—to ensure that all relevant impacts are properly accounted for. In doing so, the impact of the distribution strategy on the relief group itself and the beneficiaries must be considered. Although quantifying the impact on the former could be readily accomplished using logistical costs, assessing the impact on the former requires analyzing the impact of the distribution of relief aid in conditions where the supplies available do not fully satisfy the population needs. At the time of delivery (the delivery epoch), the arrival of supplies splits the population of beneficiaries into two groups: those that receive aid, and those that have to wait for future deliveries. Whereas the former group experiences a reduction of suffering due to the arrival of the needed supplies, the latter group experiences the opposite because they have to wait longer for the supplies. This leads to a path-dependent optimization problem in which the history of previous decisions determines the optimal solution at a given delivery epoch. The latter introduces a thick layer of complexity to an analytical problem that is already NP-hard. Considering the increases in suffering, which are the opportunity costs of the strategy, is an important component of the models developed here. Another important factor is that, because opportunity costs accrue over time, only multiperiod formulations could correctly compute them, implying that single-period formulations are not appropriate.

Quantifying these impacts is not straightforward because large disasters destroy the markets where, in normal times, the supplies are traded. Thus, these impacts become externalities that must be valued using economic valuation techniques (Bateman et al. 2002). Moreover, the need to consider private (relief group) and external (beneficiaries) costs necessitates the use of social costs (Varian 1992), which are equal to the summation of the private and external costs. Central to estimating the externalities produced by the delivery strategy is the idea that human suffering could be represented by a suitable deprivation cost function, which captures how human suffering increases as a function of the deprivation time (the length of time an individual has been without access to a good or service). This deprivation cost function is denoted by $\gamma(\theta, \delta_{it}, Z)$, where θ is a parameter vector, δ_{it} is the deprivation time of node i at time t, and Z is a vector of socioeconomic characteristics. However, the models in this paper use a generic deprivation cost function that only depends on deprivation time. This assumption is justified because, in the aftermath of a large disaster, it is unlikely that relief groups have detailed data about the individuals that need help. As a result, the individuals in need become observationally identical to outsiders, such as relief groups (Holguín-Veras et al. 2013).

Holguín-Veras et al. (2013) established that there are two key cases that differ to the extent and importance of the residual effects produced by the deprivation. In cases where the deprivation cycle leads to permanent damage (cost) that cannot be eliminated during the fulfillment cycle, the deprivation costs are said to be hysteretic (Cross 1993; Hughes-Hallett and Piscitelli 2002). In contrast, if the fulfillment cycle leads to the complete elimination of the damage inflicted by the deprivation cycle, the deprivation costs are nonhysteretic. Idealized deprivation cost functions, both hysteretic and nonhysteretic, are shown in Figure 1.

In the nonhysteretic case, shown in solid lines, the increase in deprivation time during the deprivation cycle leads to higher deprivation costs (path AB) that are completely eliminated in the fulfillment cycle (path BA). The dashed lines show the hysteretic case, where the costs produced by the deprivation cycle (path AB) are not eliminated when the

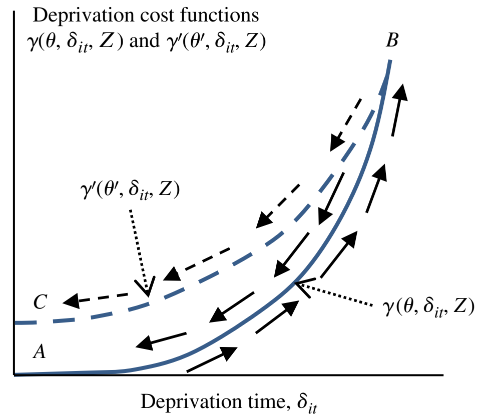

Figure 1 (Color online) Nonhysteretic and Hysteretic Deprivation Costs

fulfillment cycle takes place (path BC), which shows that the deprivation cost function has changed into $\gamma'(\theta', \delta_{it}, Z)$. It should be acknowledged that although the hysteretic case is the most general, it is also the most challenging from the computational point of view. Taking into account that this is the first investigation using deprivation costs, we decided to focus on the simpler nonhysteretic version.

To facilitate the understanding of the problem studied in this paper, it is useful to summarize the stylized models of Holguín-Veras et al. (2013) that consider the case of a relief group that delivers critical supplies to a set of demand nodes $i \in N$. The relevant distribution decisions are in what order, how much, and when to deliver to the demand nodes, during a planning horizon T. These decisions could be represented by an ordered sequence $X = \{(i_1, d_1, t_1), (i_2, d_2, t_2), \ldots, (i_j, d_j, t_j), \ldots, (i_J, d_J, t_J)\}$, which includes the return trips to the distribution center (DC). Subscript j is the position the node visited in X, J is the cardinality of X, i_j is the node visited during delivery j, d_j is the amount of supplies delivered at i_j, and t_j is the time at which delivery j is made. The order sequence X is assumed to be feasible and obtainable through the use of heuristics or algorithms. Two mathematical operators $\Omega_T(X, T)$ and $\Gamma_T^*(X, T)$ calculate the total logistical and deprivation costs, respectively, associated with X during the planning period T. Graphically, the distribution problem can be illustrated with the assistance of Figure 2, which shows two delivery routes (one in solid and another in dashed lines) undertaken by a single truck from a DC. In this case, the positions i_5 and i_9 would correspond to the DC (not shown in the figure to avoid confusion).

The deprivation cost at node i at time t is equal to the aggregation of individual costs

$$\Gamma_i(X, t) = \gamma_g(\theta_g, \delta_{it})\pi_{it}, \quad \forall\, i \in N, \tag{1}$$

where $\Gamma_i(X, t)$ are deprivation costs incurred at node i at time t as a result of implementing the sequence

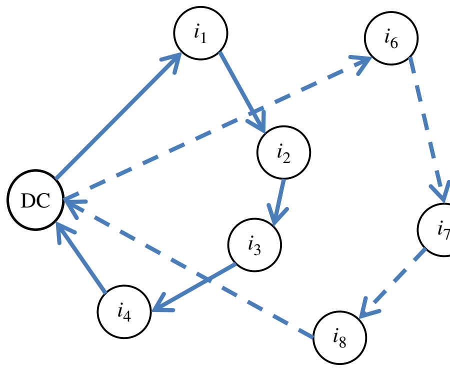

Figure 2 (Color online) Schematic of Distribution Problem of Interest

of delivery actions X; $\gamma_g(\theta_g, \delta_{it})$ is a deprivation cost function with parameter vector θ_g, δ_{it} is the deprivation time for node i at time t, π_{it} is the population in need at node i at time t, and t is a continuous time index, $0 \le t \le T$.

In mathematical terms (Holguín-Veras et al. 2013), the total social costs during the planning horizon, T, and the entire sequence, X, are equal to

$$\Psi_T^{SC}(X,T)=\Omega_T(X,T)+\sum_{j=1}^{J}\Gamma_{i_j}(X,t_j)+\sum_{i=1}^{N}\Gamma_i(X,T). \quad (2)$$

The first term in the right-hand side of Equation (2) represents the logistical costs associated with executing X, the second term represents the delivery costs at the moment of making the deliveries at the various nodes, while the third term captures the outstanding deprivation costs that accrue from the last delivery to a node and the end of the planning horizon T (including those not served by X). The second and third terms are the opportunity costs of the delivery strategy (Holguín-Veras et al. 2013), or Γ_T^*. It is worth noting that this model does not need the analyst to specify the desired LOS because the model itself will determine the highest LOS that could be provided based on the distribution capacity of the relief group.

The problem of interest bears some resemblance to both the inventory-routing problems (IRP) and vendor managed inventory (VMI) systems, where a logistician decides the timing and size of deliveries, e.g., Zachariadis, Tarantilis, and Kiranoudis (2009), although in this case the decision maker allocates the resources to minimize social costs. However, in terms of the mathematical structure of the objective function, there are profound differences. Most notably, the objective function in this paper is nonlinear, nonconvex, and path-dependent. It is nonlinear because of the nature of deprivation costs, which increase more rapidly as the deprivation times increase. It is nonconvex because of the impact of deliveries that introduce sharp discontinuities in the social costs. It is path-dependent because the payoffs to be obtained from delivering to a node depend on the history of deliveries.

In contrast, the IRP and VMI models generally assume that costs are a linear function of variables such as traveling distance and inventory holding costs; see Andersson et al. (2010) and Coelho et al. (2012) for comprehensive reviews. It is also common to add constraints to avoid stock-outs at the demand points, e.g., Archetti et al. (2007), although other applications relax this assumption by allowing for backorders or lost sales, such as Reiman, Rubio, and Wein (1999) and Kleywegt, Nori, and Savelsbergh (2004). Backorders or lost sales are usually discouraged by introducing fixed-penalty factors in the objective function, which are computationally easier to assess than the nonlinear deprivation costs defined in this paper. A literature review found no other IRP application with similar mathematical structure in its objective function, although there is a small number of distribution problems that consider some form of nonlinear cost function, e.g., Bolduc et al. (2010). These models generally assume that any losses resulting from backorders, lost sales, or any other relevant performance metric affect only the particular vendor or receiver bearing the cost of the occurrence, with no further implications for additional players in the system. In the humanitarian problem defined in this paper, all beneficiaries are impacted by the allocation decisions made by the relief group (either if they receive or do not receive supplies at a delivery epoch). These opportunity costs are not relevant to IRP and VMI problems. The assumption of identical individuals also complicates the problem because it may lead to multiple optima. In contrast, in commercial logistic applications, the vendor has a good idea about the relative importance of their customers, particularly which ones should receive the highest level of service. Such insight makes it easier to determine an order of priority, reducing the search space.

It is important to highlight a major difference between the stylized models in Holguín-Veras et al. (2013) and the ones developed in this paper. In the case of Holguín-Veras et al. (2013), the sole intent of the models was to highlight the differences between the social cost model of Equation (2) and the alternative objective functions used in the literature. These analyses clearly demonstrated the need to use welfare economic principles to define the objective function of humanitarian logistical models. It should be highlighted that, although these models facilitated the process of comparing and contrasting alternative objective functions, they are not suitable for modeling purposes because they do not have the level of mathematical formalism required by standard computational algorithms, which is why they assume that an external computational process defines the delivery strategy X. In contrast, the models developed in this paper use the concepts defined in Holguín-Veras et al. (2013) to develop fully specified mathematical models amenable for computational use in real-life settings. The sections that follow introduce formal mathematical formulations that minimize Equation (2).

4. Integrated Inventory Allocation and Routing Formulation

This section presents an inventory allocation and routing (IA-R) formulation that minimizes the social costs associated with relief distribution of aid for the problem described in §3. This section describes the assumptions and notation used as well as the analytical formulations we developed.

4.1. Assumptions

The formulation assumes that the relief manager has basic information about the distribution system, including (1) the set of demand nodes (or PODs), (2) the vehicles used to transport the supplies, and (3) the transportation network. In addition, the formulation assumes the following:

1. Deprivation costs are nonhysteretic.
2. Beneficiaries are observationally identical in terms of their needs and socioeconomic characteristics.
3. The time-dependent deprivation cost function $\gamma_c(\delta)$ captures deprivation costs for commodity $c \in C$ and deprivation time $\delta = t - t'_{cit}$ and is convex in the range $[0, t_c^{\max}]$.
4. The consumption rates u_c for commodity c are known and uniform for all beneficiaries.
5. The demands at the PODs are a function of the number of beneficiaries and the consumption rates for each commodity.
6. Backordering for unmet demands is not allowed (it is not reasonable to assume that a person who has not eaten in three days will consume three times the daily portions when supplies finally arrive).
7. Travel times, including loading and unloading of vehicles, are known for the arcs in the network.
8. Vehicles are assumed to operate continuously, which implies multiple drivers per vehicle, a frequent practice in humanitarian operations.
9. Vehicles are allowed to wait in nodes and to pick-up commodities from any DC in the system.
10. Demand nodes can be used as transshipment points to transfer commodities among vehicles.
11. Multiple and split deliveries are allowed, although a minimum shipment size q_c is required for each commodity c. This minimum is assumed to be large enough to serve all beneficiaries in a node for at least one period of time and small enough not to exceed vehicle capacity. This could be achieved by breaking up nodes with a large number of beneficiaries into smaller nodes of equal size such that the demand for commodities at each node is at most equal to the minimum shipment size.
12. The relief group has reasonable estimates for the number of beneficiaries, the amount and type of goods needed, and the resources initially available, if any.

4.2. Notation

Input parameters:

T: length of the planning horizon (discretized);
D: set of demand and transshipment nodes, $D \subset N$;
S: set of supply nodes, e.g., warehouses, depots, and DCs, $S \subset N$;
N: set of all nodes in the network, $N = D \cup S$;
A: set of arcs linking nodes in the transportation network;
C: set of commodities to be distributed;
K: set of vehicle types;
u_c: consumption rate, i.e., the amount of commodity c required to sustain a person per unit time;
τ_{ij}: travel time required to traverse arc $(i, j) \in A$;
$t_c^{\max}$: maximum deprivation time a person could survive without commodity c;
P_{it}: population (number of beneficiaries) in node $i \in D$ at time t;
$\gamma(t - t'_{ci,t-1})$: deprivation cost function evaluated at time t (the deprivation cost is a function of the deprivation time $t - t'_{c,i,t-1}$ outstanding at any time $t \in 1, \ldots, T$, for commodity $c \in C$);
q_c: minimum delivery quantity for commodity $c \in C$;
Q^k: load capacity for vehicle type $k \in K$;
c^k: travel cost per unit of time for vehicle type $k \in K$;
w_c: unit weight of commodity $c \in C$;
a_{cit}: amount of commodity $c \in C$ from external sources, e.g., donations, to node $i \in N$ at time t;
B: a big number;
b: a small number;
i: index of origin nodes $(i, j) \in A$;
j: index of destination nodes $(i, j) \in A$;
c: index of commodity types;
t'': index of time t.

Decision variables:

SC: social costs accrued during the planning horizon;
x_{cijt}^k: amount of commodity type $c \in C$ shipped from node i to node j, $(i, j) \in A$, at time t, using vehicle $k \in K$;
I_{cit}: inventory level, i.e., amount of commodity type $c \in C$ carried over from time t to time $t+1$ at node $i \in N$;

t'_{cit}: last time at which people in node $i \in D$ consumed commodity $c \in C$ prior to current time t;

α_{cit}: 1 if $I_{cit} \geq u_C P_{it}$ (node $i \in D$ has enough inventory of $c \in C$ for at least one period of time), Otherwise, 0 (there is a shortage of commodity c);

ϕ_{cit}: 1 if commodity $c \in C$ arrives to node $i \in D$ at time t; otherwise, 0;

ϑ^k_{ijt}: 1 if vehicle $k \in K$ leaves from node i to node j at time t, $(i,j) \in A$; otherwise, 0;

v^k_i: 1 if vehicle $k \in K$ is located at node $i \in N$ at time $t = 0$; otherwise, 0;

z^k_{it}: 1 if vehicle $k \in K$ is added to the fleet at node $i \in N$ at time t; otherwise, 0;

$\delta_{it} = t - t'_{ci,t-1}$: Deprivation time at node $i \in N$, time t.

4.3. Integrated Inventory Allocation and Routing Problem

The objective of the IA-R problem is to find the distribution strategy that minimizes social costs. The optimization problem can be formulated as follows:

Optimization Problem IA-R:

$$\min SC = \Bigg\{ \sum_{k\in K}\sum_{(i,j)\in A}\sum_{t=1}^{T} c^k \tau_{ij}\vartheta^k_{ijt} + \sum_{c\in C}\sum_{i\in D}\sum_{t=1}^{T}(\phi_{cit}P_{i,t-1}(1-\alpha_{ci,t-1})\gamma(t-t'_{ci,t-1})) + \sum_{c\in C}\sum_{i\in D} P_{iT}(1-\alpha_{ciT})\gamma(T-t'_{ciT})\Bigg\} \quad (3)$$

subject to:

$$I_{ci,t-1} + a_{cit} = I_{cit} + \sum_{m\in M}\sum_{k\in K}\sum_{(i,j)\in A_m} x^k_{cijt}, \quad \forall i \in S, c \in C, t \in 1,\ldots,T, \quad (4)$$

$$I_{ci,t-1} + a_{cit} + \sum_{k\in K}\sum_{(j,i)\in A} x^k_{cji,t-\tau_{ji}} - u_c P_{i,t-1}\alpha_{ci,t-1}, = I_{cit} + \sum_{k\in K}\sum_{(i,j)\in A} x^k_{cijt} \quad \forall i \in D, c\in C, t \in 1,\ldots,T, \quad (5)$$

$$\phi_{cj,t+t_{ij}} q_c \leq x^k_{cijt}, \quad \forall c \in C, (i,j)\in A : j \in D, k \in K, t \in 1,\ldots,T, \quad (6)$$

$$I_{cit} - u_c(P_{it} - b) \leq \alpha_{cit} B, \quad \forall i \in D, c \in C, t \in 1,\ldots,T, \quad (7)$$

$$(u_c P_{it})\alpha_{cit} \leq I_{cit}, \quad \forall i \in D, c\in C, t\in 1,\ldots,T, \quad (8)$$

$$\sum_{k\in K}\sum_{(j,i)\in A} x^k_{cji,t-\tau_{ji}} \leq \phi_{cit} B, \quad \forall c \in C, i \in D, t \in 1,\ldots,T, \quad (9)$$

$$(u_c P_{it})\phi_{cit} \leq \sum_{k\in K}\sum_{(j,i)\in A} x^k_{cji,t-\tau_{ji}}, \quad \forall c \in C, i \in D, t\in 1,\ldots,T, \quad (10)$$

$$t'_{cit} = t'_{ci,t-1} + \alpha_{cit}(t - t'_{ci,t-1}) \quad \forall c \in C, i \in D, t \in 1,\ldots,T, \quad (11)$$

$$t\phi_{cit} - t'_{ci,t-1} \leq t^{\max}_c, \quad \forall c \in C, i \in D, t \in 1,\ldots,T, \quad (12)$$

$$\sum_{(j,i)\in A} \vartheta^k_{jit} \leq 1, \quad \forall i \in N, t \in 1,\ldots,T, k \in K, \quad (13)$$

$$\sum_{(i,j)\in A} \vartheta^k_{ijt} \leq 1, \quad \forall i \in N, t \in 1,\ldots,T, k \in K, \quad (14)$$

$$v^k_i + \sum_{t''=1}^{t''=t}\Bigg[z^k_{it''} + \sum_{(j,i)\in A}\vartheta^k_{ji,t''-\tau_{ji}}\Bigg] - \sum_{t''=1}^{t''=t-1}\sum_{(i,j)\in A}\vartheta^k_{ijt''} \geq \sum_{(i,j)\in A}\vartheta^k_{ijt}, \quad \forall i \in N, t \in 1,\ldots,T, k\in K, \quad (15)$$

$$\sum_c w_c x^k_{cijt} - Q^k \vartheta^k_{ijt} \leq 0, \quad \forall (i,j) \in A, c \in C, k \in K, t \in 1,\ldots,T, \quad (16)$$

$$x^k_{cijt} \geq 0, \quad I_{cit} \geq 0, \quad t'_{cit} \geq 0, \quad \alpha_{cit} = 0,1, \quad \phi_{cit} = 0,1, \quad \vartheta^k_{ijt} = 0,1. \quad (17)$$

The objective function in Equation (3) captures the total social costs incurred during a planning period of length T. Its first term represents the total travel and handling costs for delivering commodities to the demand nodes (inventory costs were disregarded as a simplification). The other terms correspond to the deprivation costs at the time of deliveries, plus the ones experienced by beneficiaries at all nodes at the end of the planning period. Equation (3) collapses to the minimum travel cost if the relief group is able to meet all demands on time.

Constraints (4) balance commodity flows at the supply nodes. Likewise, constraints (5) enforce the conservation of commodity flows at demand nodes, allowing for commodity transfers among vehicles, if any. The parameters a_{cit} in these constraints represent the flow of commodities from external sources, such as donations. The term $u_c P_{it}\alpha_{cit}$ in constraints (5) defines the total consumption of commodities at the nodes per unit of time and restricts consumption to instances where the commodity is available, i.e., when inventory at a node exceeds at least the requirements for one period of time. This expression computes the demand based on the number of beneficiaries in a node, P_{it}, and the commodity consumption rate, u_c. The node populations, which could change over time as people arrive and leave the area, are an exogenous input to the model.

Constraints (6) restrict commodity flows to be at least as large as the minimum shipment size defined for the particular commodity type. Constraints (7) and (8) combine to define the binary variable α_{cit} indicating the existence of inventory ($\alpha_{cit} = 1$), or shortage ($\alpha_{cit} = 0$), at the demand nodes. It is assumed that a shortage of magnitude $u_c P_{it}$ occurs when there is no inventory at the node, or when the supplies available are insufficient to meet the needs of all of its population, for at least one period of time. This conservative stance penalizes a distribution plan for shortages when the inventory level at a demand node falls below the minimum required to sustain all beneficiaries because some individuals will be underserved as the available supplies are used.

Constraints (9) and (10) jointly define the binary variables ϕ_{cit} that capture the arrival of commodities to demand nodes. Deprivation costs increase continuously with deprivation times, but they are only charged when ϕ_{cit} signals that a delivery was received, which in turn resets the deprivation costs at that node to zero (nonhysteretic costs). Constraints (11) track t'_{cit}, the variables that record the last time prior to current time t at which the population at node i consumed commodity c. Constraints (12) prohibit deliveries to nodes that have been short of a critical good c for a period of time exceeding $t_c^{\max}$, the maximum time a person could survive without the commodity. It is assumed that people would die after this happens and that commodities would no longer be needed at the node. Such an occurrence is already penalized by the deprivation cost function.

Inequalities (13) and (14) allow for the construction of vehicle routes by requiring that every vehicle enters a node or leaves it, respectively, at most once at any point in time. Constraints (15) enforce the conservation of vehicle flows by allowing a vehicle to depart from a node at time t only if its cumulative number of entries up to that time exceeds the number of departures from the node. The initial location and the addition of a vehicle to a fleet at a node are also considered as entries to the node in these constraints. Constraints (16) link commodity and vehicle flows by allowing commodities to traverse arc (i, j) in vehicle k only if that vehicle is traveling along the arc simultaneously, without exceeding its cargo capacity. Finally, constraints (17) define the mathematical characteristics of the variables for the model.

Regrettably, the attempts to solve IA-R using Lagrangian relaxation, subgradient optimization, and tabu search proved unsuccessful. Details are available in Pérez (2011). The analyses of the performance of these solution attempts revealed that the main challenge is the nature of the objective function that is path-dependent, nonlinear, and nonconvex. The objective function is nonlinear because of the need to consider the deprivation costs, which are nonlinear. (Linearizing the objective function did not have a major effect on computational performance because the resulting function is still nonconvex and path-dependent.) The nonconvexity is introduced by the effects of deliveries on deprivation costs, which create sharp and discontinuous changes in the value of the objective function whenever deliveries are made. The path-dependent nature of the model—that arises because of the need to consider the opportunity cost of the delivery strategy—is, without a doubt, its most challenging feature.

The main issue is that, in situations of scarcity, delivering supplies to a node leads to changes in the deprivation costs at all other nodes. The deprivation cost at the node decreases, whereas the deprivation costs at the other nodes increase. The decision to deliver to a node changes the payoffs in terms of social costs to all subsequent nodes. As a result, the underlying network problem that is being solved is extremely difficult to solve because the arc costs constantly change. This differs from other notoriously hard problems, such as the traveling salesman problem (TSP), where the network does not change with the iterations. The model also requires two large sets of binary variables (α and ϕ) and the nonlinear constraint set (11) to capture the deprivation costs (i.e., deprivation times, the existence of inventory at demand nodes, and the arrival of deliveries to impacted areas). A third set of binary variables ϑ is also necessary to track the movement of every vehicle in the system. Another issue is that the two Big M constraint sets (7) and (9) require the careful definition of tight bounds to reduce the solution space for the mixed integer problem. These characteristics significantly increase the complexity of the already difficult computational challenge because even traditional vehicle routing problem (VRP) models are notoriously difficult NP-hard problems. The resulting formulation is a complex nonlinear combinatorial problem that cannot be solved by exact methods. For that reason, we developed the heuristic solution approaches that are explained in §5.

5. Inventory Allocation with Point-to-Point Distribution

This section introduces a heuristic approach to solve the inventory allocation subproblem discussed in the previous section as a standalone problem, independent of the routing. This particular problem is important because the allocation of the supplies to the demand nodes is not only the most computationally expensive task, but it is also arguably the most difficult decision the relief group has to make.

The heuristic developed exploits a key feature of the problem that allows for a meaningful decomposition of the inventory allocation and the routing problems. If the availability of vehicles is not a limiting constraint and delivery tours are not used, the inventory allocation and the VRP, in the general model of §4, could be treated as separate problems. In this case, the inventory allocation problem could be solved first; then the VRP could be solved once the inventory allocations are produced. Such an assumption is reasonable for humanitarian distribution problems because, unlike commercial supply chains, responders must rely on volunteers and local vehicles to transport supplies. As indicated by the fieldwork conducted in Haiti and Japan (Holguín-Veras et al. 2012a, c; 2014), in both cases there were enough trucks in or around the impacted areas to do local deliveries. The case of the United Nations (UN) effort following the Port-au-Prince earthquake in Haiti offers a revealing example. In the initial days of the response effort, the UN tried to undertake the response on its own and failed to do so because it did not have the number of vehicles needed for the job (New York Times 2010). However, as soon as they found a way to get in touch with local truckers, the so-called "truck crisis" disappeared (Holguín-Veras et al. 2012a, pp. 1629–1630). This is particularly true in large metropolitan areas where an over-supply of trucks is the norm. However, the situation could be different in disasters in remote areas because the local supply of vehicles is likely to be much smaller. There are other reasons that support the use of point-to-point (PTP) deliveries, which were made crystal clear in the case of Haiti. In the aftermath of the Port-au-Prince response, the Servicio Social de Iglesias Dominicanas (SSID, the social arm of the Evangelical Churches in the Dominican Republic) implemented a two echelon distribution network based on staging areas and distribution centers in Santo Domingo, Dominican Republic, Port-au-Prince, and the border between the two countries. This structure was later replaced with PTP to the internally displaced-people (IDP) camps, simply because PTP allowed them to use small trucks that could more easily navigate the streets clogged by debris. The fact that the smaller trucks did not attract much attention allowed the SSID to send the shipments without protection. (In fact, there were no truck robberies.) For details see Holguín-Veras et al. (2012a).

Consider the case of a DC with an initial inventory, and a single demand node i, located at a travel time τ_i from the DC. Assume that the node has no supplies in stock and depends on deliveries from the DC to satisfy the needs of its population P. Alternate distribution frequencies would lead to cases like the ones in Figure 3.

The objective is to determine the optimal number of deliveries, d_i, of size q to minimize total social costs (SCs) over a planning horizon T, which are the summation of the total delivery cost, $\omega = (2\tau_i c^T)d_i$ and $\gamma(T)$, the deprivation costs at the node at time $t = T$

$$\mathrm{SC} = \omega + \gamma(T). \tag{18}$$

If no deliveries are made to the demand node, then $\mathrm{SC} = \gamma(T)$, as shown in Figure 3(a). Therefore,

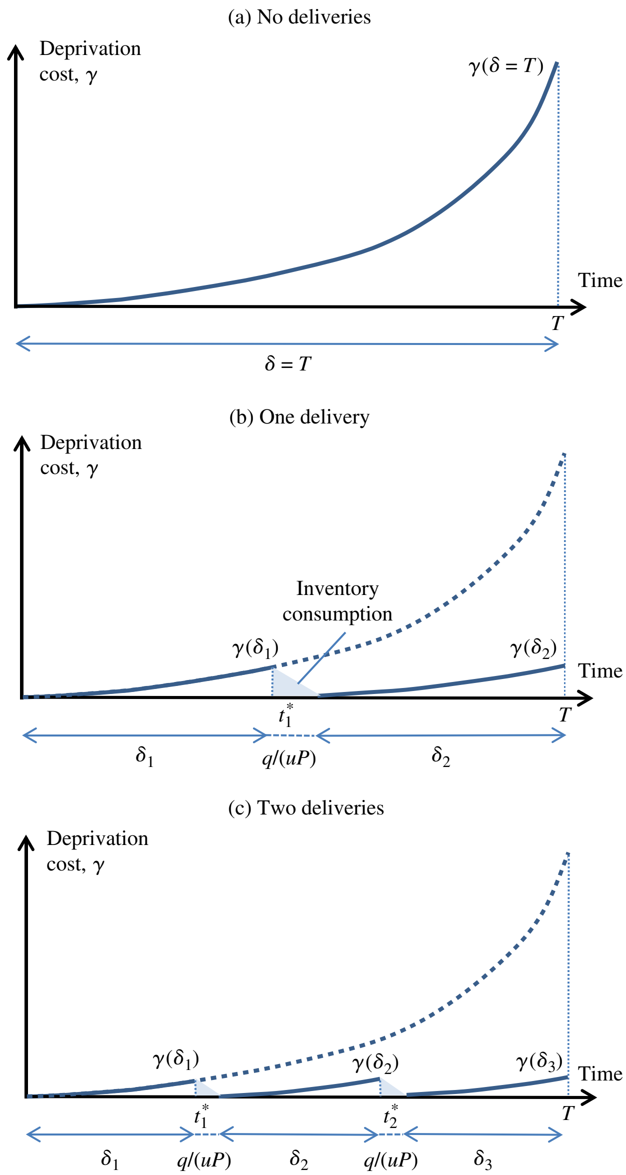

Figure 3 **(Color online) Impact of Deliveries on Deprivation Costs at a Demand Node**

if $\gamma(T) < 2\tau_i c^T$, the optimal decision would be to do nothing because the delivery costs of making a single delivery would exceed any potential reduction in the beneficiaries' deprivation costs at the demand node.

On the other hand, if $\gamma(T) > 2\tau_i c^T$, it would be optimal to make at least one delivery to the node. It is assumed that the arrival of supplies immediately resets the deprivation costs at the demand node (the time spent distributing and consuming the supplies is disregarded). The deprivation costs will start increasing again when the supplies are completely consumed by the population at the node. Thus, making a delivery to the node would reduce social costs if the reduction in deprivation costs is larger than the delivery cost. Because the travel costs of making a single delivery are fixed, the optimal SC occurs when γ is at a minimum. As shown in Figure 3(b), the timing of the delivery determines its impact on deprivation costs. If the supplies arrive early in the planning horizon, deprivation time δ_1 would be small, although δ_2 would be large. Conversely, a delivery close to the end of the planning period would lead to a small δ_2 and a large δ_1. Because the deprivation cost function is convex and nonlinear, deprivation costs in Figure 3(b) are minimized when commodities arrive at a time that makes δ_1 and δ_2 equal. This is because reducing the duration of one of the deprivation periods would automatically increase the duration of the other, and the decrease in deprivation costs during the former will be smaller than the increase of deprivation costs in the latter.

All of this implies that a single delivery of q units would satisfy the needs of the beneficiaries at the node for a period of $q/(u_cP)$ units of time, the total deprivation time experienced at this node would be $(T-q/(u_cP))$. Deprivation costs are thus minimized when this time is distributed evenly between δ_1 and δ_2

$$\delta^* = \frac{1}{2}\left(T - \frac{q}{u_cP}\right). \tag{19}$$

Therefore, if only a single delivery is made, social costs are minimized when supplies arrive at time $t=\delta^*$. Similarly, making a second delivery could decrease social costs if the additional delivery reduces deprivation costs in excess of the delivery costs between the DC and the node. The supplies delivered in these two shipments would satisfy the needs of the population P for a period of $2q/(u_cP)$ units of time, thus reducing total deprivation time to $(T-2q/(u_cP))$. Total deprivation costs would then be minimized in Figure 3(c) when the deprivation times δ_1, δ_2, and δ_3 are equal

$$\delta^* = \frac{1}{3}\left(T - \frac{2q}{u_cP}\right). \tag{20}$$

Therefore, deprivation costs are minimized when deliveries arrive at times $t_1=\delta^*$ and $t_2=2\delta^*+q/(u_cP)$. This delivery schedule would create three deprivation periods of equal duration at the demand node. The key insight is that the optimal strategy to deliver d_i shipments of size q to demand node i is to do so in such a way that all intranodal deprivation times are equal. This observation can be generalized to situations with a larger number of deliveries using Equation (19)

$$\delta_i^* = \frac{1}{d_i+1}\left(T - \frac{d_iq}{u_cP_i}\right). \tag{21}$$

This relationship can also be extended to situations in which some or all of the demand nodes have initial inventory, I_i^0. In this case, any initial inventory available at a demand node i would satisfy the needs of the beneficiaries for a period of $I_i^0/(u_cP_i)$ units of time, thus reducing the total deprivation time to be distributed among any resulting deprivation periods

$$\delta_i^* = \frac{1}{d_i+1}\left(T - \frac{I_i^0 + d_iq}{u_cP_i}\right). \tag{22}$$

Equation (22) implies that there is a unique optimal deprivation time (and consequently an optimal delivery time) that minimizes the social costs. This key observation is the starting point for the inventory allocation model proposed in this section. However, before the formulation is formally introduced, it is necessary to take a close look at the initial deprivation time, δ_i^*. At the start of the delivery process, the beneficiaries at a demand node i with no initial inventory will always experience a deprivation of at least τ_i units of time prior to the first delivery there because it would take τ_i to travel from the DC to node i. Similarly, if a node i has already experienced a deprivation time δ_i^0 by the start of the optimization, no one can completely eliminate the welfare effects of the deprivation costs that accrued. These effects can be captured by defining δ_i^1 as the deprivation time outstanding at i at the time of the first delivery

$$\delta_i^1 \geq \delta_i^0 + \tau_i - \frac{I_i^0}{u_cP_i}. \tag{23}$$

As implied by the last term of Equation (23), the initial inventory at a demand node may sustain beneficiaries long enough to allow a vehicle to reach the node before the supplies run out, and thus δ_i^1 could be as low as zero. However, by defining δ_i^1 in that way the formulation accounts for the fact that some suffering may be unavoidable. Another aspect to consider is that nodes with initial deprivation times require further consideration to minimize the deprivation costs of subsequent delivery actions. Figure 4 shows the deprivation costs at a demand node with a deprivation cost $\gamma(\delta^0)$, resulting from an initial deprivation period δ^0.

Figure 4 shows two distinct types of deprivation times. The initial deprivation time, δ_i^1, measures the amount of time until the first delivery. The regular deprivation time, δ_i^*, represents the length of time from the moment the supplies run out until the next delivery arrives. By construction, the regular deprivation time is the same during the planning horizon.

As shown, the relief group has at least two alternatives. One is to arrange deliveries in such a way that all deprivation periods are equal (Figure 4(a)). As discussed earlier, for a given number of deliveries, social costs are minimized when deprivation times are equal. In this case, $\delta_i^* \geq \delta_i^0 + \tau_i$ imposes a lower bound on the optimal deprivation time. However, if

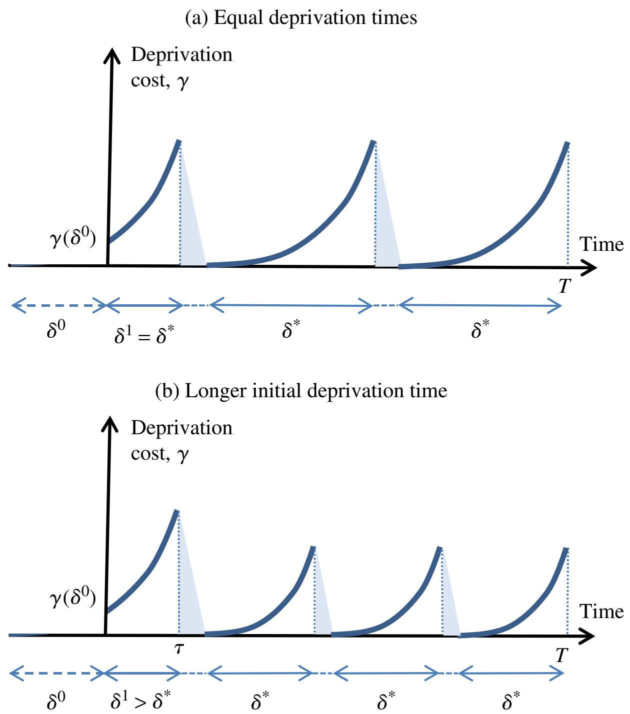

Figure 4 (Color online) Effect of Initial Deprivation Times

the relief group has the resources to provide higher levels of service than those corresponding to $(\delta_i^0 + \tau_i)$, the social costs would decrease by allowing δ_i^1 to differ from the deprivation times at subsequent deliveries (Figure 4(b)). In this strategy, the relief group would expedite a special delivery to nodes with initial deprivation times to eliminate the deprivation cost there. Because the earliest that those supplies could reach the demand node i would be at time τ_i, the system would accrue a cost at least equal to $\gamma(\delta_i^0 + \tau_i)$, the minimum deprivation cost possible given the starting conditions at i. If the relief group makes any additional $(d_i - 1)$ deliveries to this node, social costs would be minimized by splitting any remaining deprivation time in d_i equal deprivation periods, although their duration δ_i^* could be shorter than δ_i^1, as captured in Equation (24)

$$\delta_i^* = \frac{1}{d_i}\left(T + \delta_i^0 - \delta_i^1 - \frac{q d_i + I_i^0}{u_c P_i}\right). \tag{24}$$

Factors such as travel time and the existence of nodes with initial deprivation times may affect the finding that social costs are minimized when the deprivation times over the planning horizon at the node are equal, although the influence of these factors is limited to the first deprivation period only. Once the timing of the first delivery is set, the convexity and nonlinearity of the deprivation cost function guarantees that social costs are minimized when additional shipments are scheduled so that the deprivation times at the node are equal. Thus, the inventory allocation problem simplifies to finding the optimal delivery strategy to each demand node constrained by the responder's ability to meet all requests and the operational considerations.

The following inventory allocation model builds on the insight gained from the previous discussion for the case of PTP distribution. This formulation relaxes the assumption of a constant shipment size used up to now because both the number of deliveries and shipment sizes are decision variables. The parameters and variables could be redefined as follows:

Parameters:

τ_i: travel time between demand node $i \in D$ and the DC;
δ_i^0: initial deprivation time at node $i \in D$;
I_i^0: initial inventory available at node i;
h: number of time periods satisfied by a full serving of supplies;
P_i: population at node $i \in D$.

Variables:

d_i: number of deliveries to node $i \in D$;
q_i: shipment size (number of commodities) to demand node $i \in D$;
δ_i^1: initial deprivation time, experienced at node $i \in D$ during the first deprivation period;
δ_i^*: regular deprivation times, experienced at node $i \in D$ during deprivation periods $2, \ldots, d_i + 1$;
$\Gamma_i = P_i[\gamma_c(\delta_i^1) + d_i \gamma_c(\delta_i^*)]$: total deprivation cost at node $i \in D$;
$\gamma_c(\cdot)$: deprivation cost function for commodity c.

Optimization Model IA-PTP:

$$\min \ \mathrm{SC} = \sum_{i \in D} \Gamma_i + \sum_{i \in D} (2\tau_i c^T) d_i \tag{25}$$

subject to

$$\delta_i^1 \geq \delta_i^0 + \tau_i - \frac{I_i^0}{u_c P_i}, \quad \forall\, i \in D, \tag{26}$$

$$\delta_1^* \geq \frac{1}{d_i}\left(T + \delta_i^0 - \delta_i^1 - \frac{q_i d_i + I_i^0}{u_c P_i}\right), \quad \forall\, i \in D, \tag{27}$$

$$\delta_i^1 \geq \frac{1}{d_i + 1}\left(T + \delta_i^0 - \frac{q_i d_i + I_i^0}{u_c P_i}\right), \quad \forall\, i \in D, \tag{28}$$

$$\Gamma_i \geq P_i\big(\gamma_c(\delta_i^1) + d_i \gamma_c(\delta_i^*)\big), \quad \forall\, i \in D, \tag{29}$$

$$q_i \geq h u_c P_i, \quad \forall\, i \in D, \tag{30}$$

$$q_i \leq Q, \quad \forall\, i \in D, \tag{31}$$

$$\sum_{i \in \mathrm{DN}} d_i \tau_i \leq \mathrm{SC}, \quad \forall\, i \in D, \tag{32}$$

$$\sum_{i \in \mathrm{DN}} q_i d_i \leq I_j^0, \tag{33}$$

$$d_i \geq 1, \quad \forall\, i \in D, \tag{34}$$

$$\delta_i^* \geq 0, \quad \gamma_i \geq 0, \quad q_i \geq 0, \quad d_i \geq 0, \quad \text{integer}. \tag{35}$$

The objective function (25) minimizes total social costs along the planning horizon T. Constraint sets (26) and (27) capture, for each demand node, the deprivation times at the first and subsequent deprivation periods, respectively. Constraint set (28) requires δ_i^1 to be at least equal to δ_i^*. Constraints (29) capture the total deprivation costs at the demand nodes; this constraint is relaxed to an inequality for computational efficiency (i.e., at the optimum, deprivation costs are minimized when Γ_i equals the right side of the equation). As discussed before, allocating d_i shipments to a demand node creates $d_i + 1$ deprivation periods. The cost for the first deprivation period is equal to $\gamma_c(\delta_i^1)$. Because the optimal delivery schedule would make all subsequent deprivation times equal, their deprivation costs are equal to $\gamma_c(\delta_i^*)$ times d_i. Constraint set (30) prevents partial deliveries from creating a time-expanded network by requiring that any delivery to a demand node be large enough to meet the needs of all of its population for at least h periods of time. Constraints (31) define an upper bound for q_i based on the maximum vehicle capacity Q. This model incorporates total vehicle travel time (constraint set 32) and supply inventory restrictions (constraints 33) to ensure feasibility of the optimal solution. The last set of constraints (34) requires at least one delivery to each demand node. Although it may lead to suboptimal allocations by requiring a delivery to nodes that otherwise may not receive any supplies, this lower bound on d_i is necessary to avoid dividing by zero in constraints (27). Constraints (34) are not likely to have any impact on the optimal solution because they are expected to be binding only in very specific instances, such as (a) cases of extreme scarcity where it may not be possible to provide a single delivery to all nodes with the supplies at the DC; (b) cases where some demand nodes are able to sustain themselves for most of the planning horizon and thus require assistance only for brief periods of time whereas other nodes have much more urgent needs; or (c) when the population at certain demand nodes is so small that even letting deprivation costs increase for long periods of time does not justify the delivery and opportunity costs to supply them. Moreover, the modeler may evaluate the possible loss in efficiency caused by constraints (34) by removing any node receiving a single delivery in the optimal solution from the set D and then performing a new optimization on the reduced problem. If social costs for the new solution are lower (after accounting for the increased deprivation costs at the nodes receiving a single delivery in the initial solution to the full problem), then it is reasonable to implement the solution obtained during this postprocessing step.

The IA-PTP model is significantly more compact and efficient than IA-R. However, this mixed integer formulation has several nonlinear constraints, which are generally difficult to solve. However, the model found exact solutions to the real-life size networks considered in this research. These examples, solved using specialized commercial software packages for nonlinear mathematical optimization, are discussed in §7.

6. Inventory Allocation with Point-to-Point Distribution and Multiple Commodities

The model described in this section is the multiple commodity version of the one in §5. Transporting multiple commodities in the same vehicle could increase operational efficiency by allowing a more efficient use of the vehicles. This would be the case if small shipments of different commodities could be transported in the same vehicle. Assuming that commodities are allocated to demand nodes in fixed shipment sizes, the inventory allocation model in the previous section can be modified for this multiple commodity scenario (IA-PTP-MC). However, it is necessary to redefine several parameters and decision variables as follows:

- δ_{ic}^0: initial deprivation time at node $i \in D$ for commodity type $c \in C$;
- I_{ic}^0: initial inventory of commodity type $c \in C$ available;
- u_c: consumption rate for commodity type $c \in C$;
- h_c: number of time periods satisfied by a full serving of supplies of commodity $c \in C$;
- $\gamma_c(\delta)$: deprivation cost function for commodity type $c \in C$;
- q_{ic}: amount of commodity $c \in C$ per delivery allocated to demand node $i \in D$;
- δ_{ic}^*: deprivation times for commodity type $c \in C$ at node $i \in D$ during deprivation periods $2, \ldots, (d_i + 1)$.

The multiple commodity inventory allocation model can then be formulated as follows:

Optimization Model IA-PTP-MC:

minimize

$$\mathrm{SC} = \sum_{i\in D}\Gamma_i + \sum_{i\in D}(2\tau_i c^T)d_i, \tag{36}$$

$$\delta_i^1 \ge \delta_{ic}^0 + \tau_i - \frac{I_{ic}^0}{u_c P_i} \quad \forall\, i \in D,\, c \in C, \tag{37}$$

$$\delta_{ic}^* \ge \frac{1}{d_i}\left(T + \delta_{ic}^0 - \delta_i^1 - \frac{q_{ic} d_i + I_{ic}^0}{u_c P_i}\right), \quad \forall\, i \in D,\, c \in C, \tag{38}$$

$$\delta_i^1 \ge \frac{1}{d_i + 1}\left(T + \delta_{ic}^0 - \frac{q_{ic} d_i + I_{ic}^0}{u_c P_i}\right), \quad \forall\, i \in D,\, c \in C, \tag{39}$$

$$\Gamma_i \ge P_i \sum_{c\in C}(\gamma_c(\delta_i^1) + d_i \lambda_c(\delta_{ic}^*)), \quad \forall\, i \in D, \tag{40}$$

$$q_{ic} \geq h_c u_c P_i, \quad \forall i \in D, c \in C, \tag{41}$$

$$\sum_{c\in C} q_{ic} \leq Q, \quad \forall i \in D, \tag{42}$$

$$\sum_{i\in DN} d_i \tau_i \leq SC, \quad \forall i \in D, \tag{43}$$

$$\sum_{i\in DN} q_{ic} d_i \leq I^0_{ic}, \quad \forall c \in C, \tag{44}$$

$$d_i \geq 1, \quad \forall i \in D, \tag{45}$$

$$\delta^*_i \geq 0, \quad \Gamma_i \geq 0, \quad q_i \geq 0, \quad d_i \geq 0, \quad \text{integer}. \tag{46}$$

In this model, the number of deliveries d_i is independent of the commodity type because it is assumed that any shipments to a particular demand node will always have the same commodity mix. The initial deprivation time, δ^1_i, is also independent of the commodity type because supplies will be consolidated in the same vehicle and thus will reach demand nodes at the same time. The shipment sizes q_{ic} are allowed to vary across nodes and commodity types, so optimal deprivation times δ^*_{ic} can also differ among commodities.

7. Numerical Experiments

The models described in §§5 and 6 were tested with the assistance of numerical experiments. To assess the role of scarcity in the delivery policies, the experiment considers four levels of inventory that correspond to 25%, 50%, 75%, or 100% of the total demand. For ease of reference, the cases of 25% and 50% are referred to as the "scarcity scenarios," the case of 75% is referred to as a "shortage," and the one with 100% as a situation of "plenty." The experiments also consider two commodities with different levels of priority and, consequently, different deprivation cost functions. The parameter values used in the experiments are shown in Table 1.

To gain insight into optimal policies, the first set of numerical experiments focus on the relatively small

Table 1 Parameters Used in Numerical Experiments

Deprivation costs functions	
High priority	$\Gamma^H(\delta) = e^{1.50+0.12*\delta} - e^{1.50}$
Low priority	$\Gamma^L(\delta) = e^{0.065*\delta} - 1$
Initial inventory at the DC	
25%	30,000 supply units
50%	60,000 supply units
75%	90,000 supply units
100%	120,000 supply units
Population at demand nodes	100 individuals
Consumption rate (u)	1 unit/person-hour
Number of periods per serving of supply unit (h)	1
Planning horizon	10 days (240 hours)
Transportation cost	$50/hour
Vehicle capacity (Q)	1,000 supply units
Number of vehicles	Not limited

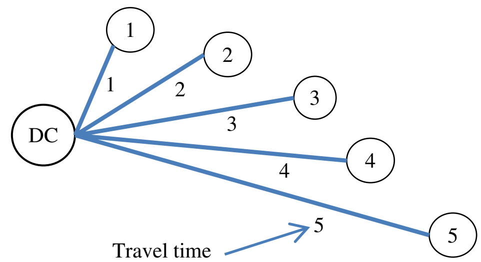

Figure 5 (Color online) Network for Numerical Examples

network of Figure 5, where PTP deliveries are made from a DC to five PODs (demand nodes). These results are discussed in §7.1–7.3. It is assumed that the relief group considers larger networks because the intent is to test the computational limits of the model.

7.1. Single Commodity Case

This section discusses the results for the IA-PTP model for both high- and low-priority supplies. These experiments consider two different supply allocation scenarios. The first is based on the use of a fixed shipment size; the second treats the shipment size as a decision variable computed by the model.

7.1.1. Fixed Shipment Size. The experiments here are based on the use of a standard shipment size, q^*, that is delivered at various frequencies to the demand nodes. The use of a fixed shipment size could be made operational, among other possibilities, by defining the demand nodes so that they are equal in population and of such size that their needs could be satisfied by q^*. In this case, the only decision variable is d_i, the number of deliveries to each node. This strategy simplifies operations by fixing one of the key parameters in the planning process, thus allowing for faster updates to current plans when new information becomes available and increasing operational efficiency through standardization. The scenarios considered in this section assume that the relief group delivers supplies to the PODs using a shipment size equal to full truck loads, $q^* = Q = 1{,}000$ units, and that there are no initial inventories or deprivation times at the demand nodes.

7.1.1.1. *Case* 1: *High-priority commodity.* This case considers a situation in which only high-priority supplies are delivered to the PODs. Table 2 shows the results for the four experimental scenarios with fixed shipment size. As shown, when resources are scarce ($\rho = 25\%$), the optimal policy is to distribute the same amount of supplies to all nodes. This leads to long deprivation periods (25.71 hours) but minimizes social costs by spreading the suffering among all nodes and avoiding high deprivation costs at some PODs. A similar pattern is observed for $\rho = 50\%$.

However, as the availability of supplies increases ($\rho = 75\%$), social costs are minimized by delivering

Table 2 Results for Fixed Shipment Size Policy (High-Priority Commodity)

Node	Number of deliveries (d)	Initial dep. time (δ^1) (hours)	Regular dep. time (δ^*) (hours)	Total supplies delivered	Deprivation costs ($)	Delivery costs ($)
			$\rho = 25\%$ SC = $313,668			
1	6	25.71	25.71	6,000.0	60,933.5	600
2	6	25.71	25.71	6,000.0	60,933.5	1,200
3	6	25.71	25.71	6,000.0	60,933.5	1,800
4	6	25.71	25.71	6,000.0	60,933.5	2,400
5	6	25.71	25.71	6,000.0	60,933.5	3,000
Total	30	—	—	30,000.0	304,667.5	9,000
			$\rho = 50\%$ SC = $74,985.3			
1	12	9.23	9.23	12,000.0	11,397.1	1,200
2	12	9.23	9.23	12,000.0	11,397.1	2,400
3	12	9.23	9.23	12,000.0	11,397.1	3,600
4	12	9.23	9.23	12,000.0	11,397.1	4,800
5	12	9.23	9.23	12,000.0	11,397.1	6,000
Total	60	—	—	60,000.0	56,985.5	18,000
			$\rho = 75\%$ SC = $45,544.4			
1	21	1.36	1.36	21,000.0	1,714.0	2,100
2	19	2.50	2.50	19,000.0	3,061.0	3,800
3	17	3.89	3.89	17,000.0	4,672.4	5,100
4	17	3.89	3.89	17,000.0	4,672.4	6,800
5	16	4.71	4.71	16,000.0	5,624.2	8,000
Total	90	—	—	90,000.0	19,743.9	25,800
			$\rho = 100\%$ SC = $36,679			
1	24	1.00	0.00	24,000.0	55.9	2,200
2	24	2.00	0.00	24,000.0	118.8	4,800
3	24	3.00	0.00	24,000.0	189.4	7,200
4	23	4.00	0.26	23,000.0	589.9	9,200
5	23	5.00	0.22	23,000.0	625.0	11,500
Total	118	—	—	118,000.0	1,579.0	35,100

more often to nodes closer to the DC. Because it is not possible to meet all demands and shipment sizes are uniform, the lower delivery costs makes it attractive to allocate more supplies to these nodes, letting deprivation costs increase at remote nodes. However, the differences in deprivation times δ_i^* are relatively small (1.36 versus 4.71 hours at the farthest nodes), suggesting that distributing equal amounts of supplies would be a fairly good solution.

In situations of plenty ($\rho = 100\%$), however, the optimal solution is to provide high levels of service that eliminate the deprivation costs after the first delivery. Therefore, the inventory allocation problem for this particular scenario could also be solved using simpler objective functions, such as the minimization of unsatisfied demands or the maximization of delivery volumes to demand nodes.

General Policy. The results suggest that the relief group should allocate supplies uniformly among all beneficiaries, delivering shipments of equal size q^* in such a way that the deprivation times at a node, along the planning horizon T, are equal to its optimal value. This strategy may be controversial because it rations supplies that are available at the DC and causes short-term deprivation at some nodes that some could view as unnecessary. However, it prevents beneficiaries from experiencing large deprivation costs later on.

7.1.1.2. *Case* 2: *Low-priority commodity.* In this case, a low-priority commodity is delivered to the demand nodes. Table 3 shows the results. Unlike the high-priority scenario, the optimal solutions generally call for relatively low levels of service. The initial stock at the DC was allocated in its entirety only in the case of severe scarcity ($\rho = 25\%$). In this scenario, the optimal allocation policy is to distribute the supplies uniformly among all beneficiaries, just as in the high-priority case.

As before, social costs are minimized by rationing, even though there could be supplies available at the DC (e.g., the DC had 18.3% of the initial inventory in stock at the end of the planning horizon for $\rho = 50\%$). Thus, having larger initial inventories did not change the optimal solution. As shown in Table 3, the trade-off between delivery and deprivation costs leads to optimal solutions in which nodes closer to the depot received significantly more deliveries than remote nodes. Node 5, the farthest from the DC, experienced long deprivation periods that are almost four times those at node 1 (21.23 hours versus 5.63 hours).

Table 3 Results for Fixed Shipment Size (Low-Priority Commodity)

Node	Number of deliveries (d)	Initial dep. time (δ^1) (hours)	Regular dep. time (δ^*) (hours)	Total supplies delivered	Deprivation costs ($)	Delivery costs ($)
			$\rho = 25\%$ SC = $24,119.2			
1	6	25.71	25.71	6,000.0	3,023.8	600
2	6	25.71	25.71	6,000.0	3,023.8	1,200
3	6	25.71	25.71	6,000.0	3,023.8	1,800
4	6	25.71	25.71	6,000.0	3,023.8	2,400
5	6	25.71	25.71	6,000.0	3,023.8	3,000
Total	30	—	—	30,000.0	15,119.2	9,000
			$\rho = 50\%$ SC = $21,015.3			
1	15	5.63	5.63	15,000.0	706.3	1,500
2	10	12.73	12.73	10,000.0	1,415.8	2,000
3	9	15.00	15.00	9,000.0	1,651.2	2,700
4	8	17.78	17.78	8,000.0	1,958.2	3,200
5	7	21.25	21.25	7,000.0	2,383.9	3,500
Total	49	—	—	49,000.0	8,115.3	12,900
			$\rho = 75\%$ SC = $21,015.3			
1	15	5.63	5.63	15,000.0	706.3	1,500
2	10	12.73	12.73	10,000.0	1,415.8	2,000
3	9	15.00	15.00	9,000.0	1,651.2	2,700
4	8	17.78	17.78	8,000.0	1,958.2	3,200
5	7	21.25	21.25	7,000.0	2,383.9	3,500
Total	49	—	—	49,000.0	8,115.3	12,900
			$\rho = 100\%$ SC = $21,015.3			
1	15	5.63	5.63	15,000.0	706.3	1,500
2	10	12.73	12.73	10,000.0	1,415.8	2,000
3	9	15.00	15.00	9,000.0	1,651.2	2,700
4	8	17.78	17.78	8,000.0	1,958.2	3,200
5	7	21.25	21.25	7,000.0	2,383.9	3,500
Total	49	—	—	49,000.0	8,115.3	12,900

An additional trip to node 5 would reduce deprivation costs there by $425.7 (making them equal to the deprivation costs at node 4), but it would increase transportation costs to the relief group by $500. Therefore, the system minimizes total social costs by accepting larger deprivation costs at the remote areas.

7.1.2. Variable Shipment Size. The assumption of a uniform shipment size q^* used in the previous section can be relaxed to allow for a more flexible allocation of supplies to the PODs. This is accomplished by treating q as a variable in the model. In this set of numerical experiments, the variable q_i represents the optimal shipment size to node $i \in D$. Separate analyses are presented for both low- and high-priority supplies.

7.1.2.1. *Case* 1: *High-priority commodity.* Table 4 presents the results for the high-priority commodity with variable shipment sizes. As shown, when resources are scarce ($\rho = 25\%$, 50%), the optimal policy is to deliver small shipments frequently to the nodes closer to the DC and deliver larger, less frequent shipments to remote nodes. The results show that there is a significant difference in the amount of supplies allocated to the demand nodes, depending on their location relative to the DC as remote nodes received a larger amount of supplies. However, even with this increased volume of supplies, beneficiaries at remote nodes experienced longer periods of deprivation.

The optimal solutions to the scarcity scenarios minimize social costs by preventing deprivation times from reaching high levels at any node. Because deprivation costs are a nonlinear function of deprivation times, even small deliveries to nodes close to the DC significantly reduce the deprivation costs at these nodes (which is not efficient for far away nodes because of the higher delivery costs). Thus, although beneficiaries at node 1 experienced total deprivation times almost twice as high as those at node 5 in both scenarios, their total deprivation costs were only about 50% higher because the relatively shorter deprivation periods (5.34 hours versus 8.12 hours) do not lead to high deprivation costs.

These results indicate that, in situations of scarcity ($\rho = 25\%$, 50%), it is possible to reduce social costs by delivering lower amounts to closer nodes and allocating the resources along the planning horizon so that the regular deprivation times are low. The key objective should be to reduce the maximum deprivation times at any node, which requires larger shipments to remote nodes to reduce transportation costs.

Delivery costs play a more obvious role in the scenarios with more abundant resources ($\rho = 75\%$, 100%)

Table 4 **Results for Variable Shipment Size (High-Priority Commodity)**

Node	Number of deliveries (d)	Shipment size (q)	Initial dep. time (δ^1) (hours)	Regular dep. time (δ^*) (hours)	Total supplies delivered	Total deprivation (hrs/person)	Deprivation costs ($)
				$\rho = 25\%$ SC = $105,776			
1	37	100.0	5.34	5.34	3,700.0	203.0	14,867.5
2	31	100.0	6.53	6.53	3,100.0	209.0	16,543.7
3	28	100.0	7.31	7.31	2,800.0	212.0	17,672.6
4	19	408.7	8.12	8.12	7,764.7	162.4	14,289.7
5	13	971.9	8.12	8.12	12,635.3	113.6	10,002.8
Total	128	—	—	—	30,000.0	900.0	73,376.3
				$\rho = 50\%$ SC = $69,716.2			
1	40	100.0	4.88	4.88	4,000.0	200.0	14,216.4
2	15	1,000.0	5.63	5.63	15,000.0	90.0	6,713.5
3	14	1,000.0	6.67	6.67	14,000.0	100.0	7,986.8
4	14	1,000.0	6.67	6.67	14,000.0	100.0	7,986.8
5	13	1,000.0	7.86	7.86	13,000.0	110.0	9,512.8
Total	96	—	—	—	60,000.0	600.0	46,416.2
				$\rho = 75\%$ SC = $45,544.4			
1	21	1,000.0	1.36	1.36	21,000.0	30.0	1,714.0
2	19	1,000.0	2.50	2.50	19,000.0	50.0	3,061.0
3	17	1,000.0	3.89	3.89	17,000.0	70.0	4,672.4
4	17	1,000.0	4.00	3.88	17,000.0	70.0	4,672.4
5	16	1,000.0	5.00	3.88	16,000.0	67.1	5,624.7
Total	90	—	—	—	90,000.0	287.1	19,744.4
				$\rho = 100\%$ SC = $36,679			
1	24	995.8	1.00	0.00	23,900.0	1.0	55.9
2	24	991.7	2.00	0.00	23,800.0	2.0	118.8
3	24	987.5	3.00	0.00	23,700.0	3.0	189.4
4	23	1,000.0	4.00	0.26	23,000.0	10.0	589.9
5	23	1,000.0	5.00	0.22	23,000.0	10.0	625.0
Total	118	—	—	—	117,400.0	26.0	1,579.0

because the optimal solution is to distribute supplies in large shipments in order to minimize transportation costs per unit. Thus, these solutions are similar to the example with fixed order quantities: the number of deliveries to each node is the same in both examples, and the optimal order sizes are essentially equal to the vehicle capacity.

General Policy: The variable shipment size policy, in the case of high-priority supplies, only reduced social costs for scenarios with high levels of scarcity. In these cases, the optimal strategies minimized deprivation costs by making frequent small deliveries to nodes closer to the DC and fewer but larger deliveries to remote nodes. As the availability of supplies increases, the solution approaches that of constant shipment sizes. Thus, in situations of small shortages the relief group should allocate supplies uniformly among beneficiaries, delivering to demand nodes in shipments as large as possible in such a way that the deprivation times are equal to the optimal value of the regular deprivation time. As the available supplies increase, and at some point match total demand, the problem increasingly resembles the minimization of unsatisfied demand.

7.1.2.2. *Case* 2: *Low-priority commodity.* As shown in Table 5, the flexibility to adjust shipment sizes offered relatively small improvements in terms of total social costs for the low-priority case. In fact, using variable shipment sizes reduced social costs in only one instance ($\rho = 25\%$). In the other three cases, the optimal solutions were the same as the ones for the fixed shipment size. These results suggest that once the most pressing needs are satisfied, delivery costs dominate deprivation costs, even at a low level of service ($\rho = 50\%$). Thus, the optimal solution is to hold supplies in stock at the DC even if doing so leads to unsatisfied demands and increased deprivation times. Because delivery costs are a larger portion of the social costs in this case, the optimal allocation strategy is to maximize the benefits produced by each delivery by delivering shipments as large as possible.

Even in the scenario with severe scarcity ($\rho = 25\%$), the optimal shipment sizes to all but node 1 were full truck loads. Compared with the solution with shipment sizes, social costs in this scenario with variable order sizes were minimized by significantly reducing the amount of supplies allocated to node 1 (from 6,000 to 2,000) and by increasing the total supplies allocated

Table 5 Results for Variable Shipment Size (Low-Priority Commodity)

Node	Number of deliveries (d)	Shipment size (q)	Initial dep. time (δ^1) (hours)	Regular dep. time (δ^*) (hours)	Total supplies delivered	Total deprivation (hrs/person)	Deprivation costs ($)
				$\rho = 25\%$ SC = $23,037.9			
1	13	153.8	15.71	15.71	2,000.0	220.0	2,488.0
2	8	1,000.0	17.78	17.78	8,000.0	160.0	1,958.2
3	7	1,000.0	21.25	21.25	7,000.0	170.0	2,383.9
4	7	1,000.0	21.25	21.25	7,000.0	170.0	2,383.9
5	6	1,000.0	25.71	25.71	6,000.0	180.0	3,023.8
Total	41	—	—	—	30,000.0	900.0	12,237.9
				$\rho = 50\%$ SC = $21,015.3			
1	15	1,000.0	5.63	5.63	15,000.0	90.0	706.3
2	10	1,000.0	12.73	12.73	10,000.0	140.0	1,415.8
3	9	1,000.0	15.00	15.00	9,000.0	150.0	1,651.2
4	8	1,000.0	17.78	17.78	8,000.0	160.0	1,958.2
5	7	1,000.0	21.25	21.25	7,000.0	170.0	2,383.9
Total	49	—	—	—	49,000.0	710.0	8,115.3
				$\rho = 75\%$ SC = $21,015.3			
1	15	1,000.0	5.63	5.63	15,000.0	90.0	706.3
2	10	1,000.0	12.73	12.73	10,000.0	140.0	1,415.8
3	9	1,000.0	15.00	15.00	9,000.0	150.0	1,651.2
4	8	1,000.0	17.78	17.78	8,000.0	160.0	1,958.2
5	7	1,000.0	21.25	21.25	7,000.0	170.0	2,383.9
Total	49	—	—	—	49,000.0	710.0	8,115.3
				$\rho = 100\%$ SC = $21,015.3			
1	15	1,000.0	5.63	5.63	15,000.0	90.0	706.3
2	10	1,000.0	12.73	12.73	10,000.0	140.0	1,415.8
3	9	1,000.0	15.00	15.00	9,000.0	150.0	1,651.2
4	8	1,000.0	17.78	17.78	8,000.0	160.0	1,958.2
5	7	1,000.0	21.25	21.25	7,000.0	170.0	2,383.9
Total	49	—	—	—	49,000.0	710.0	8,115.3

to nodes 2, 3, and 4. Similar to the optimal strategy for the high-priority commodity, deprivation costs at node 1 were kept in check by delivering smaller but more frequent shipments.

7.2. Multiple Commodity Case

The multiple commodity model was used to determine the optimal allocation of a mix of high- and low-priority commodities for the scenarios discussed in §7.1. The results, shown in Table 6, have features in common with the ones from the single commodity problems. For example, other than in the scenario with extreme scarcity ($\rho = 25\%$), the optimal solutions call for full truck deliveries to maximize the utilization of vehicular capacity and minimize delivery costs. This resemblance is strongest with the high-priority case. Table 6 also shows that social costs are minimized by performing frequent deliveries to nodes close to the DC and making less frequent but larger shipments to remote nodes. In situations of scarcity, this strategy allocates a larger share of the available supplies to remote nodes.

In contrast, the delivery patterns produced by the multiple commodity model for the low-priority supply are very different from the optimal allocation produced for the low-priority commodity by the single commodity model. Other than in the scenario with $\rho = 25\%$, the multiple commodity model allocates a significantly higher amount of low-priority supplies to nodes located close to the DC. This leads to differences in the levels of service among demand nodes. For example, when $\rho = 50\%$, the optimal allocation strategy satisfies all demands for the low-priority commodity at node 1 (except the stock-out prior to the arrival of the first delivery), whereas beneficiaries at node 5 experienced 16 deprivation periods of 12.44 hours each.

The results show that, even in situations of plenty, remote nodes receive only a fraction of the amount of low-priority commodities allocated in the single commodity problem. For example, the total amount of the low-priority commodity delivered to node 5 decreased by 64.3% (from 7,000 to 2,500 units) when using the multiple commodity model for the scenario with $\rho = 100\%$. Interestingly, deprivation costs at this node did not increase for the multiple commodity solution. In fact, these costs decreased by 22.4% (from $2,384 to $1,850), notwithstanding the significant reduction in the amount of supplies delivered. The determining factor in this apparently counterintuitive finding is the timing of the smaller shipments to the remote nodes. Because the low-priority

Table 6 Results for the Multiple Commodity Allocation Problem

		Shipment size			Deprivation times (hrs)			Total delivered		Deprivation costs ($)		
Node	d	q^H	q^L	Total	δ^1	δ^H	δ^L	Σq^H	Σq^L	$\Sigma\Gamma^H$	$\Sigma\Gamma^L$	Total
						$\rho = 25\%$ SC = $113,714						
1	38	100	100	200	5.18	5.18	5.18	3,800	3,800	14,640	1,561	16,201
2	32	100	176.3	276.3	6.30	6.30	5.54	3,200	5,642	16,219	1,438	17,656
3	28	100	277.1	377.1	7.31	7.31	5.54	2,800	7,758	17,673	1,274	18,947
4	17	583.4	416.6	1,000	9.40	7.73	9.40	9,918	7,082	12,172	1,516	13,687
5	16	642.6	357.4	1,000	10.75	7.90	10.75	10,282	5,718	12,102	1,720	13,822
Total	131	—	—	—	—	—	—	30,000	30,000	72,805	7,509	80,314
						$\rho = 50\%$ SC = $79,816.2						
1	33	286	714	1,000	4.29	4.29	0.00	9,429	23,571	9,973	32	10,006
2	24	488	511.7	1,000	4.91	4.91	4.68	11,720	12,280	8,748	891	9,638
3	20	635	365.1	1,000	7.95	5.25	7.95	12,699	7,301	8,342	1,421	9,764
4	18	722.6	277.4	1,000	10.00	5.55	10.00	13,007	4,993	8,420	1,740	10,161
5	16	821.6	178.4	1,000	12.44	6.01	12.44	13,145	2,855	8,832	2,116	10,948
Total	111	—	—	—	—	—	—	60,000	51,000	44,316	6,200	50,516
						$\rho = 75\%$ SC = $59,612						
1	32	621	379	1,000	3.60	1.17	3.60	19,885	12,115	2,357	870	3,228
2	23	818	181.9	1,000	8.26	1.90	8.26	18,815	4,185	3,305	1,705	5,010
3	20	900	100.0	1,000	10.48	2.48	10.48	18,000	2,000	4,113	2,049	6,162
4	19	900.0	100.0	1,000	11.05	3.05	11.05	17,100	1,900	4,862	2,102	6,964
5	18	900.0	100.0	1,000	11.68	3.68	11.68	16,200	1,800	5,688	2,161	7,849
Total	112	—	—	—	—	—	—	90,000	22,000	20,326	8,886	29,212
						$\rho = 100\%$ SC = $50,923.8						
1	37	641	359	1,000	2.82	0.00	2.82	23,718	13,282	176	765	941
2	26	893	107.1	1,000	7.86	0.00	7.86	23,214	2,786	679	1,800	2,479
3	26	893	107.1	1,000	7.86	0.00	7.86	23,214	2,786	679	1,800	2,479
4	26	892.9	107.1	1,000	7.86	0.00	7.86	23,214	2,786	679	1,800	2,479
5	25	900.0	100.0	1,000	8.27	0.27	8.27	22,500	2,500	1,096	1,850	2,946
Total	140	—	—	—	—	—	—	115,861	24,139	3,310	8,014	11,324

commodity is bundled with the high-priority supplies, beneficiaries at all nodes receive more frequent deliveries of the low-priority commodity than they do in the optimal solution for the single commodity problem. Therefore, these relatively small deliveries are large enough to reset the accumulation of deprivation costs for the low-priority commodity, and relatively low costs are maintained by the frequent deliveries.

Table 7 shows that the multiple commodity model generally leads to lower social costs than the ones associated with solving independent allocation problems of single commodities. For the scenarios considered in this experiment, the multiple commodity model reduced overall social costs by 11.56%. The bulk of the savings were obtained in the form of lower delivery costs across all scenarios; 20.11% lower for the multiple commodity model. Thus, consolidating the shipments of different commodities to the demand nodes could reduce the relief agency transportation costs. Beneficiaries, however, are relatively unaffected by the decision to consolidate the deliveries. Total deprivation costs were just 3.56% lower using the multiple commodity model. Moreover, the solutions to the multiple commodity problem increased deprivation costs in two scenarios ($\rho = 25\%$ and $\rho = 50\%$), although these increases were offset by larger savings in delivery costs.

General Policy: The results suggest that multiple commodity deliveries are likely to lead to a more efficient use of assets. Both the relief group and the beneficiaries are generally better off with the allocations obtained from this model, although most of the savings are realized by the relief group in the form of lower delivery costs. The numerical experiments suggest that the relief group should plan for full truck deliveries in most instances; with situations of severe scarcity being the only exception in which social costs are minimized by delivering less than full truck loads to nodes relatively close to the DC. As observed before, social costs are minimized by spacing out consecutive deliveries to a particular node such that the regular deprivation time is equal to the optimal value.

However, it is important to note that the optimal allocation strategies for multiple commodity problems depend on the relative importance of the commodities, which is determined by how critical they are to the beneficiaries. In the cases considered in these experiments, the allocation strategies were dominated by the high-priority supplies. Thus, the multiple commodity model minimizes social costs by

Table 7 Optimal Social Costs for Single and Multiple Commodity Models

	Single commodity			Multiple commodity			Difference		
ρ	Deprivation cost	Logistic cost	Social cost	Deprivation cost	Logistic cost	Social cost	Deprivation cost (%)	Logistic cost (%)	Social cost (%)
25%	85,614	43,200	128,814	80,314	33,400	113,714	−6.19	−22.69	−11.72
50%	54,532	36,200	90,732	50,516	29,300	79,816	−7.36	−19.06	−12.03
75%	27,860	38,700	66,560	29,212	30,400	59,612	4.85	−21.45	−10.44
100%	9,694	48,000	57,694	11,324	39,600	50,924	16.81	−17.50	−11.74
Total	177,700	166,100	343,799	171,366	132,700	304,066	−3.56	−20.11	−11.56

allocating supplies in delivery patterns that resemble the optimal solutions for the high-priority commodity with variable shipment sizes. These policies depend on the level of resources available at the DC: (a) in situations of plenty, the focus should be on minimizing unmet demand for the high-priority supplies; (b) whereas in situations of scarcity, the relief group could minimize the duration of the deprivation times at any node by making small and frequent deliveries to nodes close to the DC and making fewer but larger deliveries to remote nodes. The multiple commodity model seems to favor a complementary allocation policy for the low-priority commodity, i.e., whereby small amounts of this commodity are allocated to remote nodes and higher volumes to nodes close to the DC. This strategy minimizes social costs in two ways: (a) by maximizing the utilization of the vehicles already scheduled to visit nodes relatively close to the DC and (b) by reducing the deprivation costs at remote nodes without concomitant increasing in delivery costs.

The experiments conducted so far have provided insight into general policies of relief distribution. The experiments in §7.3 focus on assessing the performance of the models in real-life size networks.

7.3. Large Networks

To test the capabilities of the IA-PTP model to solve large problems, we created a test case inspired by a hypothetical disaster in New Orleans. To this effect, New Orleans was subdivided into districts, as exemplified in Figure 6, assuming that each district would house a POD as its center. Five different configurations, with different numbers of PODs (10, 30, 50, 80, and 100) and four different levels of scarcity ($\rho = 25\%$, 50%, 75%, and 100%) were tested. The populations are based on the U.S. Census Bureau's 2005 statistics.

The test cases assume that 2.5% of the population needs assistance, there are no initial inventories or deprivation times at the demand nodes, and a relief group with a DC located in Metairie, Louisiana, needs to decide the optimal allocation of a single commodity (drinking water) to the beneficiaries in the disaster area. Each beneficiary requires three liters of water per day (thus, $u = 3/24 = 0.125$ l/hr $= 0.125$ kg/hr), which are consumed in six servings per day (hence, $h = 24/6 = 4$). Therefore, any delivery to a node i should provide at least $huP_i = 4 \times 0.125P_i = 0.5P_i$, or half a liter of water per beneficiary. The planning horizon is set as 10 days, during which the 11,731 beneficiaries in New Orleans would require a total of 351,930 kg of water. However, it is assumed that the relief group has enough supplies at the DC to meet at most ρ% of the total need. The deprivation cost function was assumed to be equal to the one used in §§7.1 and 7.2 for water.

Transportation costs are assumed at \$50/hr. Because only last-mile distribution is considered, it is assumed that the relief group uses small trucks with a maximum capacity of four metric tons to deliver supplies to the beneficiaries. The travel speed was assumed to be 2.0 miles/hour, which is consistent with the ones observed in postdisaster operations. The data for each scenario, such as the number of beneficiaries and travel times, were updated accordingly using GIS software. The mathematical problems were solved with MINLP, a commercial solver using branch and bound and other techniques for nonlinear integer programs. The data for the case of the 10 PODs are shown in Table 8.

Figure 7 shows the computational performance of the model for the various scenarios considered (i.e.,

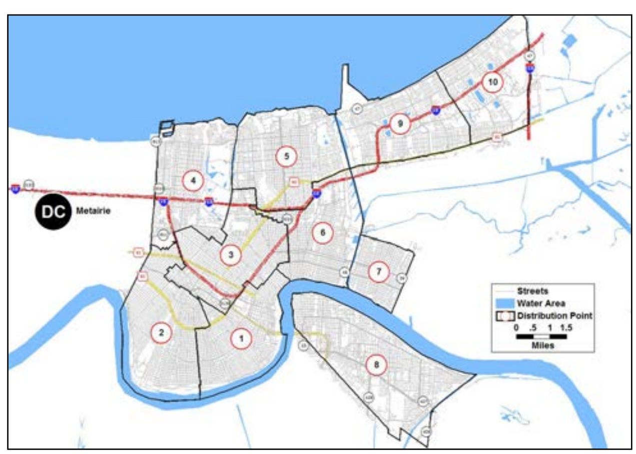

Figure 6 (Color online) Disaster Districts and Distribution Center

Table 8 **Data for Demand Nodes** ($n = 10$)

Node	Area description	Population (2005)	Beneficiaries (2.5% of total)	Total daily demand (kg)	Travel time to DC (hrs)
1	FQ Central Garden	54,297	1,357	162,891.0	3.90
2	Uptown	67,083	1,677	201,249.0	2.99
3	Mid-City	79,441	1,986	238,323.0	3.15
4	Lakeview	25,897	647	77,691.0	2.43
5	Gentmy	44,133	1,103	132,399.0	4.09
6	Bywater	41,163	1,029	123,489.0	4.29
7	Lower 9th Ward	21,773	544	65,319.0	5.38
8	Aigiers	55,635	1,391	166,905.0	5.94
9	New Orleans East	48,941	1,224	146,823.0	5.94
10	Little Woods Read Blvd	30,867	772	92,601.0	7.52
	Totals	469,230	11,731	1,407,690.0	—

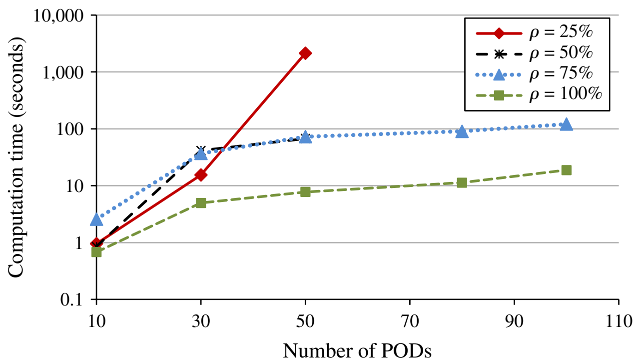

Figure 7 **(Color online) Computation Time vs. Number of PODs**

combinations of networks of different sizes and levels of scarcity). The results show that, if there is no scarcity of the critical supply ($\rho = 100\%$), the model is able to compute the optimal solution quickly for all networks. Figure 7 shows that the relationship between computational time and the number of PODs is fairly linear for the range of more than 30 PODs. However, as the level of scarcity increases, the model starts to struggle to find optimal solutions. As shown, for networks with more than 50 PODs no solutions were found for $\rho = 25\%$ and 50%, which exhibit a highly nonlinear relation between the computation time and number of PODs, and is most evident in the case of $\rho = 25\%$. However, these results are not likely to impede the use of the model in real-life networks because in most cases relief groups deliver to 30–40 PODs, at most.

Tables 9 through 12 show the detailed results for the different test runs. When interpreting the results, it is important to keep in mind that the social costs for networks of different sizes are not comparable for the simple reason that IA-PTP does not estimate the cost of walking to the PODs. In postdisaster operations, where infrastructure is typically severely impacted, the costs of walking could be significant. Obviously, longer walking distances lead to higher social costs for the beneficiaries. This is not an issue for the simple reason that the main focus of IA-PTP is on inventory allocation, not on the design of the POD network where consideration of walking costs is crucial. IA-PTP assumes that the structure of the POD network is provided as an exogenous input. For an example of a POD network design model, see Jaller and Holguín-Veras (2013).

The results shown in Tables 9 through 12 are conceptually valid. As shown, in conditions of plenty the

Table 9 **Experimental Results** ($\rho = 100\%$)

	Experimental scenario				
Characteristic	$n = 10$	$n = 30$	$n = 50$	$n = 80$	$n = 100$
Total social cost ($)	$0.076 M	$0.083 M	$0.084 M	$0.072 M	$0.097 M
Deprivation cost ($)	$0.037 M	$0.037 M	$0.038 M	$0.027 M	$0.038 M
Travel cost ($)	$0.039 M	$0.046 M	$0.046 M	$0.044 M	$0.059 M
Total supplies delivered (kg)	345,436	345,451	345,300	287,122	345,322
Total number of deliveries	90	103	104	110	132
Avg. deliveries per node	9.0	3.4	2.1	1.4	1.3
Avg depriv. time (hrs/day/person)	0.44	0.45	0.45	0.41	0.45
Minimum δ^* (hrs)	0.00	0.00	0.00	0.00	0.00
Maximum δ^* (hrs)	0.03	0.24	6.56	3.02	3.02
Standard deviation for δ^* (hrs)	0.01	0.04	0.92	0.34	0.31
Average delivery quantity (kg/delivery)	3,838.2	3,353.9	3,320.2	2,610.2	2,616.1
Average vehicle utilization (%)	95.95	83.85	83.00	65.25	65.40
Computation time (seconds)	0.68	4.96	7.75	11.32	18.77

Table 10 **Experimental Results** ($\rho = 75\%$)

Characteristic	$n = 10$	$n = 30$	$n = 50$	$n = 80$	$n = 100$
	Experimental scenario				
Total social cost ($)	$0.506 M	$0.585 M	$0.635 M	$0.603 M	$0.744 M
Deprivation cost ($)	$0.445 M	$0.482 M	$0.503 M	$0.449 M	$0.548 M
Travel cost ($)	$0.061 M	$0.102 M	$0.131 M	$0.154 M	$0.196 M
Total supplies delivered (kg)	263,925	263,993	263,948	219,038	263,925
Total number of deliveries	155	280	373	469	570
Avg. deliveries per node	15.5	9.3	7.5	5.9	5.7
Avg depriv. time (hrs/day/person)	6.00	6.00	6.00	6.00	6.00
Minimum δ^* (hrs)	2.77	3.87	4.41	5.38	5.76
Maximum δ^* (hrs)	3.11	8.89	6.56	6.52	6.72
Standard deviation for δ^* (hrs)	0.11	0.82	0.29	0.24	0.25
Average delivery quantity (kg/delivery)	1,702.7	942.8	707.6	467.0	463.0
Average vehicle utilization (%)	42.57	23.57	17.69	11.68	11.58
Computation time (seconds)	2.57	36.99	72.83	90.18	121.29

Table 11 **Experimental Results** ($\rho = 50\%$)

Characteristic	$n = 10$	$n = 30$	$n = 50$	$n = 80$	$n = 100$
	Experimental scenario				
Total social cost ($)	$1.075 M	$1.098 M	$1.389 M	—	—
Deprivation cost ($)	$0.946 M	$0.962 M	$1.079 M	—	—
Travel cost ($)	$0.129 M	$0.136 M	$0.310 M	—	—
Total supplies delivered (kg)	175,950	175,995	175,965	—	—
Total number of deliveries	287	778	773	—	—
Avg. deliveries per node	28.7	25.9	15.5	—	—
Avg depriv. time (hrs/day/person)	12.00	12.00	12.00	—	—
Minimum δ^* (hrs)	3.63	3.87	5.38	—	—
Maximum δ^* (hrs)	4.65	5.06	6.79	—	—
Standard deviation for δ^* (hrs)	0.34	0.36	0.48	—	—
Average delivery quantity (kg/delivery)	613.1	226.2	227.6	—	—
Average vehicle utilization (%)	15.33	5.66	5.69	—	—
Computation time (seconds)	0.85	41.60	67.55	—	—

Table 12 **Experimental Results** ($\rho = 25\%$)

Characteristic	$n = 10$	$n = 30$	$n = 50$	$n = 80$	$n = 100$
	Experimental scenario				
Total social cost ($)	$2.378 M	$2.518 M	$2.652 M	—	—
Deprivation cost ($)	$2.310 M	$2.311 M	$2.314 M	—	—
Travel cost ($)	$0.068 M	$0.208 M	$0.338 M	—	—
Total supplies delivered (kg)	87,975	87,998	87,983	—	—
Total number of deliveries	150	450	742	—	—
Avg. deliveries per node	15.0	15.0	14.8	—	—
Avg depriv. time (hrs/day/person)	18.00	18.00	18.00	—	—
Minimum δ^* (hrs)	11.25	11.25	10.35	—	—
Maximum δ^* (hrs)	11.25	11.25	12.27	—	—
Standard deviation for δ^* (hrs)	0.00	0.00	0.46	—	—
Average delivery quantity (kg/delivery)	586.5	195.6	118.6	—	—
Average vehicle utilization (%)	14.66	4.89	2.96	—	—
Computation time (seconds)	0.95	15.38	2,133.82	—	—

social costs are minimal because the distribution effort could be organized in a way that leads to relatively high load factors (65.40% to 95.95%). As the scarcity increases, the social costs drastically increase (the ones for $\rho = 25\%$ are about 30 times the ones for conditions of plenty). Moreover, in response to the need to expedite deliveries to the PODs in need, the load factors and delivery quantities plummet, and there is a significant increase in the number of deliveries.

Taken together, the results of the test cases discussed in this section are consistent with the ones discussed in §7.1. Equally important is the finding that IA-PTP is capable of solving problems of a size that exceeds the typical size of the distribution problems encountered in real life. This suggests that the model could be used to organize distribution strategies in real-life disasters.

8. Conclusions

The paper uses welfare economic principles to formulate inventory allocation and distribution models for postdisaster humanitarian logistics (PD-HL). The use of welfare economics leads to an objective function based on social costs, which account for the impact on both the relief group and the beneficiaries. The latter effects include both the reduction on deprivation costs experienced by the individuals that receive the aid and the increase in deprivation costs for those that have to wait for future deliveries (the opportunity costs of the delivery strategy). The impact on the relief group are assumed to be captured by the corresponding logistical costs. The paper develops mathematical formulations that significantly contribute to the humanitarian logistic literature because they are based on a conceptually solid objective function and are capable of modeling real-life size operations in the level of detail needed for practical applications. This is accomplished by building on the concepts outlined in Holguín-Veras et al. (2013) to develop a fully specified optimization program that can be efficiently solved. Additional details about the chief findings and the process that we used are provided next.

The paper introduces three different formulations. The first one is an inventory allocation and routing model (IA-R) that intends to compute the optimal allocation of supplies to the demand nodes—together with the sequence of deliveries and the routing—that minimizes social costs during the planning horizon. Unfortunately, the complexity of IA-R is such that it could not be solved using standard computational algorithms such as Lagrangian relaxation, subgradient optimization methods, or heuristics like tabu search. Such complexity arises because the objective function is path-dependent, nonlinear, and nonconvex. The path-dependent nature of the objective function, which adds tremendous complexity to the computational process, is a consequence of the need to consider the opportunity costs. The chief challenge is that the payoffs to be obtained from delivering to a node change as a function of the history of the previous deliveries. In essence, the underlying network problem constantly evolves as deliveries are made. This feature, combined with the nonlinear and nonconvex objective function, severely reduces the effectiveness of standard solution approaches and metaheuristics.

As an alternative to IA-R, we developed a formulation (IA-PTP) that assumes point-to-point deliveries, which are often used in actual postdisaster operations. Based on insight gained from the analyses of the impact produced by deliveries on the deprivation costs at a demand node, we created a nonlinear program to compute the optimal size and frequency of the shipments to PODs. The objective in this model is to minimize the social costs, subject to vehicle capacity and inventory constraints. A third formulation, a variation of the second, considers multiple commodities. The performance of these models was assessed using numerical experiments.

The experimental results stress the important role played by deprivation times. In general, social costs for a given node are minimized when the deprivation times between consecutive deliveries are equal, which leads to the equalization of deprivation costs along the planning horizon. The experiments suggest that, in situations of scarcity, responders should deliver small shipment sizes frequently to nodes close to the DC, while delivering larger but less frequent shipments to remote nodes. However, the experimental results also suggest that a different strategy should be followed in cases of plenty. As the amount of supplies approaches the total demand, the inventory allocation problem reduces to the minimization of unmet demands, or the maximization of delivery volumes; both objective functions are easier to solve than social costs. It should be highlighted that, from the perspective of social costs, these alternative objective functions are only valid in situations in which the relief group has enough resources to meet all of the demands at the disaster site.

Of great significance is that IA-PTP was able to efficiently solve problems that are larger than the ones typically observed in real-life operations. As discussed in the paper, depending on the level of scarcity, IA-PTP is able to solve problems to optimality of problems between 50 ($\rho = 25\%$) and 100 PODs ($\rho = 100\%$). The numerical experiments also indicated that the larger the level of scarcity, the longer the computation time.

The models and findings from this investigation form a basis for future developments that could lead to better methods of relief aid distribution. For example, metaheuristics able to efficiently solve large problems could be developed to find the allocation strategies that would minimize deprivation times among disaster areas, as opposed to methods focused on nonlinear deprivation costs that may take longer to converge. The relaxation of the assumptions regarding equal shipment sizes per node, and the implications of limited vehicle capacity, also present opportunities for future research. Another important extension of this research would be to consider the dynamic nature of disaster relief operations, in terms of both availability of critical supplies and changes in demand, and the inherent uncertainties that impact the optimization process.

The research reported in this paper has clearly established the practicality and potential of using social costs as the objective function of PD-HL models. The explicit consideration of deprivation costs into the social costs lead to insight into the optimal relief strategies that is not possible to obtain from alternative objective functions. However, significant challenges remain to be overcome. The computational and empirical complexity of computing optimal solutions and estimating suitable deprivation cost functions are such that a significant amount of research is needed for the research community to be able to transition these promising models to practice.

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