Socio-Economic Planning Sciences 47 (2013) 191–204

Contents lists available at SciVerse ScienceDirect

Socio-Economic Planning Sciences

journal homepage: www.elsevier.com/locate/seps

Weekly scheduling models for traveling therapists

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ARTICLE INFO

Article history:
Available online 14 July 2012

Keywords:
Home healthcare
Therapist scheduling
Mixed-integer programming
Weekly planning

ABSTRACT

This paper presents a series of models that can be used to find weekly schedules for therapists who provide ongoing treatment to patients throughout a geographical region. In all cases, patient-appointment times and visit days are known prior to the beginning of the planning horizon. Variations in the models include single vs. multiple home bases, homogeneous vs. heterogeneous therapists, lunch break requirements, and a nonlinear cost structure for mileage reimbursement and overtime. The single home base and homogeneous therapist cases proved to be easy to solve and so were not thoroughly investigated. This left two cases of interest: the first included only lunch breaks while the second added nonlinear overtime and mileage reimbursement costs. For the first case, 40 data sets were solved, each consisting of either 15 or 20 therapists and between roughly 300 and 540 patient visits over five days. For each instance, we were able to obtain the minimum cost of providing residential healthcare services using a commercial solver. The results showed that CPU time increases more rapidly than total cost as the total number of visits grows. For the second case, which was much more difficult, it was necessary to develop heuristics to find good solutions quickly. Results for 5- through 20-therapist instances are presented and compared to the linear programming relaxation lower bounds. In the first of two parametric analyses, the tradeoff between the number of therapists on staff and the cost of providing service was examined. In the second, a similar tradeoff was explored between cost can the number of home bases used by the therapists.

1. Introduction

Home healthcare is the business of providing professional healthcare services in the patient's place of residence. These healthcare services include nursing, physical therapy, speech therapy, occupational therapy, medical treatments, and general assistance to the infirm [11]. As the population ages, the need for healthcare professionals grows. The Census Bureau projects that by 2030 there will be more than 70 million Americans aged 65 and older, more than twice their number in 1995. By 2040, one out of every five Americans will be over 65. The number of "old old," aged 85 and older, is projected to triple or quadruple [16]. There are now more than 10,000 home healthcare organizations in the U.S. alone. This number will continue to increase at an accelerated rate leading to greater competition between provider organizations. To remain profitable, these organizations must find new ways to better manage their human resources. A critical aspect of this effort is the derivation of cost-efficient schedules for up to a week at a time for their therapists. This involves constructing travel routes and assigning patients to therapists while taking into account a number of operational and logical constraints [3]. In fact, effective staff planning and scheduling must aim at cost minimization.

In this paper, we present a range of models that can be used to solve the weekly therapist scheduling problem and investigate the two most detailed. The objective of the models is to minimize the total cost of providing residential healthcare services for up to a week at a time without violating availability constraints, the requirement for lunch breaks, and patient-appointment times. We use the term "residential healthcare" as a generalization of "home healthcare" to reflect the fact that patients can be treated at locations other than their home. The type of organization that we have in mind is an independent agency that contracts with therapists to work on an hourly basis. Costs include wages for treatment and drive time, mileage reimbursement, and a premium for overtime. Because of different training levels and experience, therapists get paid at different rates. Travel costs arise from the fact that therapists must often travel for several hours between various locations to see their patients on any day of the week. Indeed, it is shown in a report of The National Association for Home Care and Hospice

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http://dx.doi.org/10.1016/j.seps.2012.07.001

that healthcare providers drive nearly 5 billion miles each year [16]. It's important to minimize the cost of the travel time.

In the next section, we review the literature related to residential healthcare scheduling. In Section 3, a description of the operational procedures of our collaborating organization is provided that includes scheduling issues and cost structure. The problem statement and models are given in Section 4. In Section 5, we present two simple heuristics that proved to be extremely effective. This is followed in Section 6 with an example and the presentation of our computational results for 40 randomly generated data sets that include either 15 or 20 therapists and upwards of 540 visits per week, and 100 instances with 5 to 20 therapists and up to 650 visits per week. We also present two independent parametric analyses which demonstrate that as the number of available therapists increases and as the number of their home bases increases, total costs decrease linearly. The data sets reflect the actual demand faced by Key Rehab, a Midwestern rehabilitation services company that provides physical, occupational, and speech therapy. We close with a summary of the work and suggestions for future research.

2. Literature review

The problem discussed in the paper is posed in the context of residential healthcare delivery but it is not unique to that industry. More generally, we are solving a vehicle scheduling problem with time windows and multiple depots. There is a vast quantity of literature on algorithms for the multiple depot vehicle scheduling problem (MDVSP), which was first introduced by Carpaneto and Dell'Amico [7]. They present a branch and bound algorithm for the MDVSP which is based on an additive lower bound and the use of dominance criteria. Ribeiro and Soumis [13] improved Carpaneto's results by introducing a new formulation of the MDVSP as a set partitioning (SP) problem with side constraints whose continuous relaxation could be solved by column generation. They showed that the continuous relaxation of the SP formulation provides a much tighter lower bound than the additive bound procedure. Binaco et al. [5] studied the same problem using a similar approach. However, in addition to column generation, they describe a procedure for computing a lower bound to the MDVSP that is based on heuristically solving the dual of the continuous relaxation of SP without using the SP matrix. The dual solution obtained is used to reduce the number of variables in the SP in such a way that the resulting problem can be solved by standard branch and bound techniques. Dell'Amico et al. [9] aimed to minimize a weighed combination of the number of vehicles used and overall operational costs for the MDVSP. They designed a new polynomial-time heuristic that always guarantees the use of the minimum number of vehicles. Extensive computational results on test problems involving up to 1000 trips and 10 depots are reported.

The multiple depot crew scheduling problem (MDCSP) which is quite similar to our residential healthcare problem is an extension of the MDVSP. In that case, each crew is required to return to their starting depot within imposed limits on both the elapsed time and the working time of any duty. Boschetti et al. [6] developed a bounding procedure based on Lagrangian relaxation and column generation. Their procedure is an effective alternative to classical column generation as it is not affected by the typical degeneration drawbacks in solving the linear relaxation of the master problem with a standard LP code.

With respect to residential healthcare routing and scheduling, the literature is much sparser. Cheng and Rich [8] considered healthcare staff planning with full-time and part-time workers. They developed two mixed-integer linear programming formulations which, like us, included lunch break constraints. A two-phase heuristic was implemented and numerical results were presented for small data sets with up to 2 full-timers, 2 part-timers and 10 patients. Bertels and Fahle [4] present the core optimization components of PARPAP which is an optimization and planning tool for highly constrained staff routing problems, again for residential healthcare. A combination of linear programming, constraint programming and heuristics is used. Their model is flexible enough to accommodate many of the components of real-world problems.

Different than others who aim to minimize total cost, Steeg and Schröder [15] choose the objective of minimizing the number of nurses that visit patients during the day. Their model combines the nurse rostering problem with a periodic vehicle routing problem. Their algorithm first applies constraint programming to compute an initial feasible solution and then calls an adaptive large neighborhood search heuristic to improve the results.

3. Description of operations

Therapists that provide rehabilitation services can be divided into three categories: physical, occupational and speech. Physical therapists are healthcare professionals who diagnose and treat individuals of all ages. Their patients range from newborns to the very old who have medical problems or other health-related conditions, illnesses, or injuries that limit their ability to move and perform functional activities to the degree that they would like in their daily lives. Physical therapy is performed by either a physical therapist (PT) or sometimes by a physical therapist assistant (PTA) acting under their direction. Occupational therapists work with individuals who suffer from a mentally, physically, developmentally, or emotionally disabling condition. They provide treatments that develop, recover, or maintain a person's daily living functions. Speech and language pathologists assess, diagnose, treat, and help to prevent disorders related to speech, language, cognitive-communication, voice, swallowing, and fluency. Because the problem decomposes by therapist type, we only consider PTs and PTAs in this paper.

Therapists who contract to work 40 h per week are classified as "full-time"; otherwise, they are "part-time" and work on a fixed pre-defined schedule. For example, a specific part-timer may work Tuesday morning and all day Thursday. Full-timers are typically, but not always, available five days a week between 8:00 am and 5:30 pm; however, they are not necessarily given schedules that fully cover this interval each day. On a low demand day, for example, they might be assigned a set of patients in the morning only. PTs and PTAs could be either full-timers or part-timers, but the wages of the former are usually higher so if the aim is cost minimization, it is better to schedule part-timers first whenever possible.

In this paper, we focus on patients who have fixed appointment times which means that the number of treatments each week for a patient and the start and end time of the treatment are precisely defined. Patients can be treated at hospitals, nursing centers, medical lodges, clinics or their home. Most of the patients need one or two treatments weekly with a handful requiring daily visits (in some cases by PTs only). Each treatment lasts from 15 min to an hour.

The above factors imply that the problem has both a routing and scheduling aspect that must be considered. Given the information on therapist availability, wages and classification, and patient-appointment times, the objective is to develop a weekly schedule for each therapist that minimizes the total cost of providing residential healthcare services while satisfying a collection of hard constraints that are discussed in the remainder of this section. These are standard throughout the industry. An overview of the issues relevant to providing rehabilitative therapy is shown in Fig. 1 (adapted from Ref. [4]).

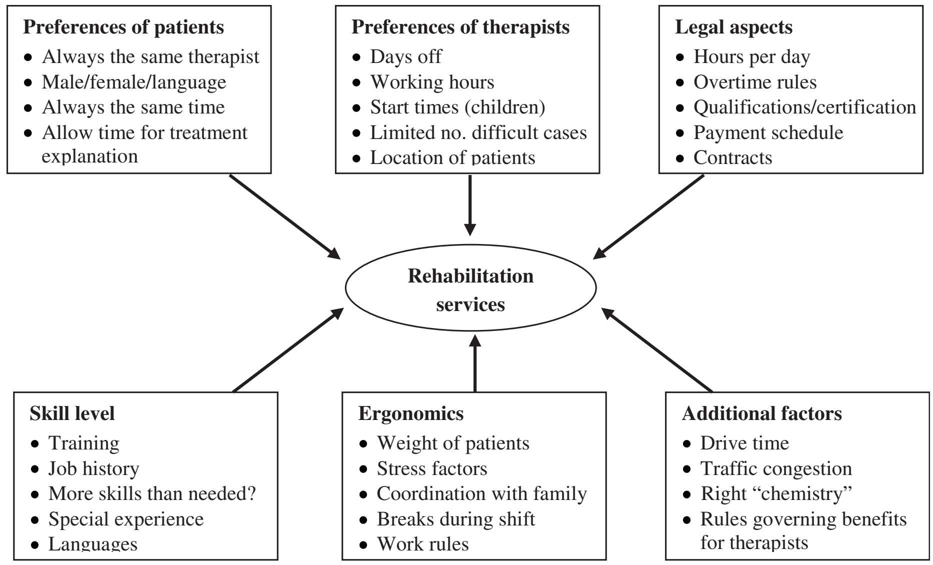

Fig. 1. Major issues in providing rehabilitation services.

3.1. Scheduling issues

Typically, therapists start their day at home, visit a series of patients at different sites and then return home after seeing their last patient. As in most service organizations, when a therapist is assigned 6 or more hours of work including driving time, a lunch break must be provided. Because most patients not residing in their homes receive lunch at a set time (e.g., 12:00 pm), this is often the best time for the therapists to have a break. Here we assume that the lunch break is 30 minutes and should be assigned between 11:00 am and 1:00 pm. In addition to treating patients, a therapist's work also includes performing some administrative tasks. Such tasks do not need to be performed until the end of the day but in the case of treatment it is best to do them while memory is fresh. Therefore, administrative time is added to treatment time in the models.

3.2. Cost issues

Therapists get paid for time spent seeing patients and for driving between patient locations. In most cases, the treatment rate and driving rate is therapist dependent.Therapists are also paid for the time required to perform administrative functions that typically include document treatment and notifying relatives. To determine administrative time, an important parameter— *productivity* – is introduced. Productivity is defined as the ratio of treatment time to treatment plus administrative time. It is included as input along with the administration rate.

For a given week, the total working time for a therapist is derived by summing all treatment, driving, and administrative time. When a therapist works more than 40 h in a week, overtime begins to accumulate at a rate equal to one and a half times the regular wage rate for each overtime hour. However, when a therapist is idle and not driving to a patient, there is no compensation. Similarly, time spent waiting is not compensated and accumulated idle time is not counted toward overtime.

Another component of cost is the mileage reimbursement. On a given day, therapists are paid a supplement proportional to the number of miles drives in excess of 25.

3.3. Travel time estimation

In order to determine travel time, we need to determine the distance between each location as well as the driving speed. In this paper, we use the Euclidean metric to calculate the distance between any two points. Given two coordinates (X_i,Y_i) and (X_j,Y_j) expressed in degrees of longitude and latitude, the distance (in miles) between i and j is calculated by the following equation.

$$D(i,j) = \sqrt{(53(X_i - X_j))^2 + (69.1(Y_i - Y_j))^2}$$

The numbers in this equation are the factors for converting degrees of longitude and degrees of latitude, respectively, into miles. In determining the driving speed for therapists, we use the algorithm below for computing vehicle velocity (MPH).

$$\text{If } (D(i,j) < 1) \quad \text{then} \quad D(i,j) = 1$$

$$\text{If } (D(i,j) \le 20 \quad \text{and} \quad \text{Metro} = {''}\text{Y}{''}) \quad \text{then}$$

$$\text{MPH } (i,j) = 18.285 + 0.45159 \times D(i,j)$$

$$\text{ElseIf } (D(i,j) \le 20 \quad \text{and} \quad \text{Metro} = {''}\text{N}{''}) \quad \text{then}$$

$$\text{MPH } (i,j) = 5.447 + \log_e D(i,j) + 11.11$$

$$\text{ElseIf } (D(i,j) > 20) \quad \text{then}$$

$$\text{MPH } (i,j) = 17.326 + 14.4356 \times \log_e D(i,j) + 11.11$$

$$\text{End If}$$

$$\text{MPH } (i,j) = \min\{\text{MPH } (i,j), 50\}$$

The "Metro" flag is set to "N" for selected depots located in sparsely populated regions [2].

4. Model formulations

The problems described below are referenced by their generic names which come from the vehicle routing literature but will be defined in the context of therapist scheduling. In particular, a therapist is equivalent to a vehicle, a patient is equivalent to a customer, and treatment is equivalent to service. The full problem spans five days and has the objective of minimizing treatment, driving, administrative, mileage and overtime costs while observing flow balance, time window, and resource constraints. The first model we present is the single depot vehicle scheduling problem (SDVSP). The second model we have is the MDVSP with lunch break while the last one is MDVSP with overtime, mileage reimbursement and lunch breaks. The following basic notation is used in the formulations. Additional notation will be introduced as needed.

Indices and sets

- i,j: index for patients
- k: index for therapists
- d: index for days
- $o(k)$: origin (and destination) of therapist k
- K: set of all therapists
- I: set of patients to be seen over the planning horizon
- $IF(i,d)$: set of patients that can follow patient i in a schedule on day d including $o(k)$
- $IP(i,d)$: set of patients that can precede patient i in a schedule on day d including $o(k)$
- $IL(d)$: set of patients that can precede a lunch break on day d
- $IL(i,d)$: set of patients that can follow patient $i \in IL(d)$ in a schedule on day d

Data and parameters

- c_{ij}^k: hourly traveling rate from patient i to patient j by therapist k
- c_{id}^k: hourly treatment rate by therapist k to see patient i on day d
- τ_{ij}: travel time between patient i and j
- s_{id}: time required to provide treatment to patient i on day d
- a_{id}: appointment time for patient i on day d

Decision variables

- x_{ijd}^k: 1 if therapist k visits patients i and j in succession on day d, 0 otherwise

4.1. Single depot vehicle scheduling problem (SDVSP)

In this version of the problem, all therapists in the set K are identical, which means that they have the same working windows, treatment rates and travel rate, and are based at the same clinic where they start and end their day. The SDVSP decomposes by day and can be formulated as a min-cost flow problem (MCFP) [10,12], which implies that it is polynomial solvable. The homogeneity assumption allows us to drop the index k. This leads to the following model for day d.

$$\phi_d^{\text{SDVSP}} = \text{Minimize} \sum_{i \in I \cup \{o\}} \sum_{j \in IF(i,d)} \bar{c}_{ijd} x_{ijd} \tag{1a}$$

$$\text{subject to} \sum_{j \in IF(i,d)} x_{ijd} = 1, \quad \forall\ i \in I \tag{1b}$$

$$\sum_{i \in IP(j,d)} x_{ijd} - \sum_{i \in IF(j,d)} x_{jid} = 0, \quad \forall\ j \in I \cup \{o\} \tag{1c}$$

$$\sum_{j \in I} x_{ojd} \le |K| \tag{1d}$$

$$x_{ijd} \in \{0,1\}, \quad \forall\ i \in I \cup \{o\}, \quad j \in IF(i,d) \tag{1e}$$

The objective in (1a) is to minimize the cost of treating all patients on a specific day. The coefficients $\bar{c}_{ijd}$ include the travel cost c_{ij} between location i and j and the cost of providing treatment c_{id} for patient i on day d. Constraints (1b) ensure that each patient is visited exactly once while identifying her immediate successor. Constraints (1c) guarantee flow balance at each location visited while (1d) limits the number of schedules each day to at most $|K|$ – one for each therapist. The variables are defined in (1e). Model (1) is quite simple and is used as a foundation.

Those familiar with routing problems such as this one will note that there are no constraints in the formulation that prevent subtours that do not include the clinic; that is, there are no subtour elimination constraints that prevent a therapist from being assigned a route that cycles through a subset of patients and never returns to the clinic. While such a solution is logically infeasible, it is often necessary to explicitly exclude it from occurring. For the problems that we are addressing, however, subtours can never arise because all patients have fixed appointment times. This implies that there is no patient j on day d who is simultaneously a member of the sets $IF(i,d)$ and $IP(i,d)$; either j can precede i or j can follow i or the two are incompatible. These are the only options. If we constructed a graph with the nodes being the patients and the arcs indicating whether or not it is feasible for the same therapist to treat patient j after treating patient i, we would see that the graph is directed (all arcs are one way only) and acyclic (no cycles), and hence no need for subtour elimination constraints. This is true for all the models in the paper.

4.2. Multiple depot vehicle scheduling problem (MDVSP) with lunch break

In this problem, each therapist k has a unique location $o(k)$ which is referred to as the home base where therapist k starts and ends the day. Therapists in the set K are no longer assumed to be identical, which means that they may have different working windows, treatment rates and mileage reimbursement rates. Therefore, it is necessary to reintroduce the index k in the notation.

In most organizations, when a worker's shift extends beyond a certain number of hours, an uncompensated lunch break must be provided. In our case, when a therapist is assigned 6 or more hours of work, including driving and administrative time, a ½-h break between 11:00 am and 1:00 pm is required. To identify when the lunch break can be assigned, we define pairs of patients (i,j) whose appointment times are far enough apart to allow the break to be inserted between their treatments. For therapist k on day d, let $IL(i,k,d)$ be the set of immediate successors of patient i that admit a lunch break, let $IF(i,k,d)$ be set of patients including $o(k)$ that can immediately follow patient i, and let $IP(i,k,d)$ be the set of patients including $o(k)$ that can immediately precede patient i. The conditions for $j \in IL(i,k,d)$ are as follows: (i) $a_{jd} - (a_{id} + s_{id} + \tau_{ij}) \ge 0.5$, that is, at least 30 min of idle time exist between the end of patient i's treatment and the beginning of patient j's treatment, and (ii) $|[a_{id} + s_{id}, a_{jd}] \cap [11,13]| \ge 0.5$, that is, the ½-h break can fit between 11:00 am and 1:00 pm.

Let $IK(k,d)$ be the set of patients that therapist k can be assigned on day d. By updating model (1) to accommodate lunch breaks and to take into account the fact that therapists must be viewed individually, we get the following model for day d.

$$\phi_d^{\text{MD-L}} = \text{Minimize} \sum_{k \in K} \sum_{i \in I \cup \{o(k)\}} \sum_{j \in IF(i,d)} \bar{c}_{ijd}^k x_{ijd}^k \tag{2a}$$

$$\text{subject to} \sum_{k \in K} \sum_{j \in IF(i,d)} x_{ijd}^k = 1, \quad \forall\ i \in I \tag{2b}$$

$$\sum_{i \in IP(j,k,d)} x_{ijd}^k - \sum_{i \in IF(j,k,d)} x_{jid}^k = 0, \ \ \forall\ k \in K, \ \ j \in IK(k,d) \cup \{o(k)\} \tag{2c}$$

$$\sum_{j\in I} x^k_{o(k),j,d} \le 1, \quad \forall\ k\in K \tag{2d}$$

$$\sum_{i\in IL(d)} \sum_{j\in IL(i,k,d)} x^k_{ijd} \ge 1, \quad \forall\ k\in K \tag{2e}$$

$$x^k_{ijd}\in\{0,1\}, \quad \forall\ k\in K, \quad i\in I\cup\{o(k)\}, \quad j\in IF(i,k,d) \tag{2f}$$

Model (2) is the same as model (1) except for the reintroduction of the index k and the addition of (2e). These constraints enforce the lunch break requirement for all therapists by insisting that each schedule include a lunch break arc (i,j), where the set $IL(d)$ is limited to those patients whose appointment times are early enough in the day to satisfy the above two conditions on day d. Note that the depot or home base $o(k)$ is an element in $IL(i,k,d)$ if only to ensure that (2) is feasible. In practice, the earliest appointment time is at 8:00 am so it is not possible for the lunch break to occur after the last patient assigned to therapist k is treated, given that the break must conclude no later than 1:00 pm. Constraints (2e) do not permit the break to precede the first appointment.

In general, the binary requirements for the x-variables in (2f) cannot be relaxed. A special case exists, however, that preserves the MCFP characteristics of (2) when there is a single depot, that is, $o(k) = o$ for all k.

Proposition 1. Assume that there are m identical therapists and that all routes from o back to o in $G = (V,A)$ include at most one lunch break arc, where $V = \{o,1,2,\ldots,n\}$ and $A = \{(i,j)$: $i, j \in V$ and it is feasible to travel from i to $j\}$. Then model (2) is polynomial solvable.

The proof is given in Shao [14]. It is an open question whether the more general case with identical therapists is polynomial solvable.

4.3. MDVSP with overtime, mileage and lunch break

The above models decompose by day. When a therapist works more than 40 h in a week, time and a half is paid for the overtime hours. This factor ties the days of the week together and prevents a daily decomposition. To incorporate overtime in the model, it is necessary to keep track of the billable, travel, treatment and administrative time that each therapist accumulates each day. Idle time, however, is not compensated. To calculate the administration time, we use the following formula

$$\mathrm{adm}_{ikd} = (1/p_k - 1)\times c^k_{id}$$

where p_k is the productivity for therapist k.

A second factor not yet considered is the nonlinear nature of mileage reimbursement. For the first 25 miles, therapist k is not reimbursed but for all miles above that, he is paid at the rate of c^{miles}_k. In the formulation, we make use of these parameters and the following additional notation.

Indices and sets

$DU(i)$: set of days in the planning horizon that patient i is to be treated

$DK(k)$: set of days in the planning horizon that therapist k works

Data and parameters

c^{reg}_k: hourly treatment rate for therapist k

c^{miles}_k: hourly travel cost for every mile that therapist k drives in a day beyond 25 miles (assumed to be constant at \$0.55/mi)

adm_{ikd}: time required by therapist k to perform administrative functions after treating patient i on day d

e_{kd} earliest start time of therapist k on day d

f_{kd}: latest end time of therapist k on day d

$\tau^{\max}_{dk}$: upper bound on the number of working hours permitted for therapist k on day d

$\tau^{\mathrm{max\_over}}_k$: upper limit on the amount of overtime permitted for therapist k

dist_{ij}: distance between locations i and j

$\overline{T}$: number of hours in a week above which overtime is paid; $\overline{T} = 40$

$\overline{D}$: number of miles driven in a day for which a therapist is not reimbursed; $\overline{D} = 25$

Decision variables

T^1_{dk}: time (hours) for therapist k to go from his home base to his first patient on day d

T^2_{dk}: time (hours) for therapist k to return to his home base after treating his last patient on day d

T_{dk}: total time for which therapist k is paid on day d

T^{over}_k: number of hours that therapist k works in a week beyond 40

D_{dk} number of miles that therapist k drives on day d

ΔD_{dk} number of miles above 25 that therapist k drives on day d

Based on model (2), we add constraints for overtime and mileage reimbursement. This leads to model (3).

$$\begin{aligned}\phi^{\mathrm{MD-L-O-M}} = \text{Minimize} &\sum_{k\in K}\sum_{d\in DK(k)}\sum_{i\in IK(k,d)\cup\{o(k)\}}\sum_{j\in IF(i,k,d)} \\ &\times\left(c^k_{ij}+c^k_{id}\right)x^k_{ijd} + \sum_{d\in D}\sum_{k\in K} c^{\mathrm{miles}}_k \Delta D_{dk} \\ &+\sum_{k\in K} 0.5 c^{\mathrm{reg}}_k T^{\mathrm{over}}_k\end{aligned} \tag{3a}$$

$$\text{subject to} \sum_{k\in K(i,d)}\sum_{j\in IF(i,k,d)} x^k_{ijd} = 1, \quad \forall\ i\in I, \quad d\in DU(i) \tag{3b}$$

$$\sum_{j\in IK(k,d)} x^k_{o(k),j,d} \le 1, \quad \forall\ k\in K, \quad d\in DK(k) \tag{3c}$$

$$\begin{aligned}\sum_{i\in IP(j,k,d)} x^k_{ijd} - \sum_{i\in IF(j,k,d)} x^k_{jid} = 0, \quad \forall\ k\in K, \quad d\in DK(k), \\ j\in IK(k,d)\cup\{o(k)\}\end{aligned} \tag{3d}$$

$$\sum_{j\in IK(k,d)} \tau_{o(k),j} x^k_{o(k),j,d} = T^1_{dk}, \quad \forall\ k\in K, \quad d\in DK(k) \tag{3e}$$

$$\sum_{i\in IK(k,d)} \left(\tau_{i,o(k)} + s_{id} + \mathrm{adm}_{ikd}\right) x^k_{i,o(k),d} = T^2_{dk}, \quad \forall\ k\in K, \quad d\in DK(k) \tag{3f}$$

$$\begin{aligned}T^1_{dk} + T^2_{dk} + \sum_{i\in IK(k,d)}\sum_{j\in IF(i,k,d)/\{o(k)\}} (\tau_{ij} + s_{id} + \mathrm{adm}_{ikd}) x^k_{ijd} \\ = T_{dk}, \quad \forall\ k\in K, \quad d\in DK(k)\end{aligned} \tag{3g}$$

$$\sum_{d\in DK(k)} T_{dk} - \overline{T} \le T^{\mathrm{over}}_k, \quad \forall\ k\in K \tag{3h}$$

$$\sum_{i\in I\cup\{o\}}\sum_{j\in IF(i,d)} \mathrm{dist}_{ij} x^k_{ijd} = D_{dk}, \quad \forall\ d\in D, \quad k\in K \tag{3i}$$

$$D_{dk} - \overline{D} \leq \Delta D_{dk}, \quad \forall\ d \in D, \quad k \in K \tag{3j}$$

$$\sum_{i \in IK(k,d)} \sum_{j \in IL(i,k,d)} x^k_{ijd} \geq 1, \quad \forall\ k \in K, \quad d \in DK(k) \tag{3k}$$

$$\begin{aligned} & x^k_{ijd} \in \{0,1\}, \quad T^1_{dk} \geq 0, \quad T^2_{dk} \geq 0, \quad T_{dk} \in \left[0, \tau^{\max}_{dk}\right], \\ & T^{\text{over}}_k \in \left[0, \tau^{\text{max\_over}}_k\right], \quad D_{dk} \geq 0, \quad \Delta D_{dk} \geq 0, \\ & \forall\, i \in I \cup \{o(k)\}, \quad d \in DU(i), \quad j \in IF(i,k,d), \quad k \in K(i,d) \end{aligned} \tag{3l}$$

The objective function (3a) has three terms. The first minimizes the cost of traveling between all pairs of patients plus the cost of providing treatment. For those patients to be seen on day d, the set $IK(k,d)$ must be defined accordingly. The second and third terms minimize the mileage reimbursement and overtime payments, respectively.

Constraints (3b) ensure that each patient has exactly one successor on a route on day d. The first summation is over the set of therapists who are qualified to provide treatment to patient i and the second is over all patients who are compatible successors of i and hence can be on the same route. An ordered pair of patients (i,j) is said to be *compatible* on day d for therapist k if it satisfies the following conditions: $a_{id} + s_{id} + \tau_{ij} \leq a_{jd}$, $e_{kd} + \tau_{o(k),i} \leq a_{id}$, and $a_{jd} + s_{jd} + \text{adm}_{jkd} + \tau_{j,o(k)} \leq f_{kd}$. As such, if $j \notin IF(i,k,d)$, then either i and j are separated by too great a distance, therapist k cannot reach i at her appointment time, patient j's appointment time does not allow sufficient time for therapist k to return to his home base prior to the end of his day, or therapist k is not suited to treat patient j. These situations are sorted out in a preprocessing step where all the sets are defined. The requirement that patient i be seen by a particular therapist k on day d, can be accommodated by appropriately defining the set $K(i,d)$. We treat this as a hard constraint but it is an easy matter to treat patient preferences as a soft constraint, penalizing non-preferred assignments.

Constraints (3c) limit each therapist $k \in K$ to at most one schedule per day. By implication, when $x^k_{o(k),j,d} = 0$ for all $j \in IK(k,d)$, therapist k is not given a schedule on day d. If a subset of therapists, say, n_k of them, have the same profile and can be viewed as interchangeable (that is, homogeneous), then the right-hand side of (3c) can be replaced with n_k and K redefined to be the set of therapist types rather than individuals. Constraints (3d) impose route continuity for each therapist k and each compatible patient $j \in IF(i,k,d)$ by requiring that tours (loops) are constructed rather than open paths. The start and end of a tour for therapist k is assumed to be the same location, $o(k)$. Given positive flow on network G^k, when (3d) is combined with (3b), we see that each patient j who is treated by k has a unique predecessor and a unique successor.

Constraints (3e) and (3f) keep track of the amount of paid time accrued daily by therapist k. Therapists may work for more than 8 h a day but overtime does not apply until their billable plus administrative plus travel time exceeds $\overline{T}$ $(= 40)$ hours in a week. The summation on the left-hand side of (3e) determines the time to go from the home base of therapist k to his first patient. The left-hand side of (3f) determines the time required by therapist k to treat the last patient in his schedule on day d, perform the associated administrative functions, and then return to his home base.

The total number of working hours that therapist k accrues on day d is calculated in (3g). An upper bound is placed on this value in (3*l*). Constraint (3h) sums the working hours over the week and subtracts $\overline{T}$ to provide a lower bound on the number of overtime hours for therapist k. An upper bound is likewise imposed in (3*l*). Because we are trying to minimize overtime compensation in (3a), we are able to write the constraints in (3h) as inequalities rather than equalities; T^{over}_k will always assume its smallest value in any solution and hence will be binding in either (3h) or (3*l*). Constraints (3i) are similar to those in (3g) and are used to determine the total distance that therapist k drives each day. Whether mileage reimbursement is due is determined by (3j) in a manner similar to (3h).

Constraints (3k) are written for those therapists whose time window on day d spans 6 or more hours and ensures that they traverse at least one arc (i,j) that admits sufficient time for the lunch break. Because the demand for therapists generally exceeds the supply, we have assumed for modeling purposes that all those whose time windows are at least 6 h will be assigned at least 6 h of work and hence will require a break. The equations needed to determine whether or not these conditions hold and hence whether a therapist is indeed entitled to a break on day d are given by Shao [14]. Finally, variable definitions are stated in (3l).

Compared with models (1) and (2), model (3) is much more difficult both theoretically and computationally and remains so even when all therapists are identical and only overtime is included as a side constraint to the SDVSP.

5. Heuristics

Although model (2) was relatively easy to solve, model (3) proved much more burdensome. Consequently, we developed three heuristics for finding feasible solutions. The first two are described below; the third is based on a rolling horizon procedure and is detailed in Bard et al. [1] who also developed a branch-and-price-and-cut (B&P&C) algorithm for (3). The results provided by the latter two algorithms were in general not as good as those presented here. For example, the B&P&C algorithm could only solve instances with up to 5 therapists to optimality with any reliability and then only with unreasonably long runtimes.

5.1. Relaxed MIP I

Accounting for overtime and mileage reimbursement makes the computations difficult. For real instances, though, the corresponding costs are likely to be marginal in comparison to the other costs. Therefore, solving a relaxed version of model (3) in which overtime and reimbursement costs are either ignored or simplified should speed up the computations without significantly compromising solution quality.

In formulating the relaxed problem, the first point to note is that most full-time therapists are available during the week for no more than 45 or 50 h, while part-timers always work less than 40 h. As a consequence, feasible rosters will contain little overtime, which would not be realized until Friday if at all. From a computational point of view, this means that the overtime constraints have minimal effect on the optimization process, at least for the first four days. Thus in relaxed MIP I, we initially ignore the overtime costs by removing constraints (3h) and the cost term in T^{over}_k in (3a) from the model. When a solution is found, any cost that would have been incurred due to overtime are added to the objective function value, call it $\phi^{\text{IP}}_{\text{relaxed}}$, in a post-processing step.

The impact of the mileage reimbursement constraints is slightly different since most therapists drive more than 25 miles per day. If we knew for sure that therapist k's schedule on day d required him to drive more than 25 miles, his cost could be taken as linear and an adjustment made for the first 25 miles. Moreover, even when some therapists drive less than 25 miles on some days, the solution of the fully linearized problem would be near-optimal because the reimbursement rate of \$0.55/mi is relatively low. The linearized problem, however, can be solved much more quickly than the original problem because the mileage cost can be included in the arc cost. This allows us to remove constraints (3i) and (3j) and the objective function term in ΔD_{dk} to get a relaxed solution, call it $(\overline{x}_{ijd}^{k}, \overline{T}_{dk}, \overline{T}_{dk}^{1}, \overline{T}_{dk}^{2})$ for all i, j, d, k, with objective function value $\phi_{\text{relaxed}}^{\text{IP}}$. This solution is always feasible. Now, letting $\overline{T}_{k}^{\text{over}}$ be the overtime incurred in the relaxed solution and $\overline{D}_{dk}$ the distance traveled by therapist k on day d in the relaxed solution, the actual cost can be calculated as follows,

$$\phi_{\text{actual}}^{\text{IP}} = \phi_{\text{relaxed}}^{\text{IP}} + \sum_{k\in K} 0.5c_k^{\text{reg}} \cdot \max\{\overline{T}_k^{\text{over}} - 40, 0\} - \sum_{d\in D}\sum_{k\in K} c_k^{\text{miles}} \cdot \min\{\overline{D}_{dk}, 25\} \qquad (4)$$

The second term on the right-hand side of (4) is 0 when $\overline{T}_k^{\text{over}} \leq 40$ and positive otherwise, so when overtime does occur, we are adding back the cost $0.5c_k^{\text{reg}}\overline{T}_k^{\text{over}}$. The third term on the right-hand side removes the mileage reimbursement included in the relaxed solution for up to the first $\overline{D}_{dk}$ or 25 miles, whichever is less.

5.2. Relaxed MIP II

As mentioned, the overtime constraints rarely take effect due to the problem characteristics. Our experience also has shown that if we keep the original piecewise linear cost structure, runtimes do not significantly increase for most instances with up to 20 therapists. Thus in relaxed MIP II, we keep the overtime cost while relaxing the mileage reimbursement as in relaxed MIP I. After the relaxed solution is found, the actual cost can be calculated as follows.

$$\phi_{\text{actual}}^{\text{IP}} = \phi_{\text{relaxed}}^{\text{IP}} - \sum_{d\in D}\sum_{k\in K} c_k^{\text{miles}} \cdot \min\{\overline{D}_{dk}, 25\} \qquad (5)$$

In Eq. (5) on the right-hand side, we once again subtract off the mileage reimbursement from the relaxed solution for up to 25 miles.

6. Computational results

6.1. Data description

For the first set of experiments models (2) and (3) were tested using the same 40 randomly generated instances. For the first 20 instances, it is assumed that all therapists have the same location (single depot; $o(k) = o$ for all $k \in K$), but otherwise have different working windows, wage rates and reimbursement rates. For the second 20 instances, all therapists start at their home base and have different weekly profiles and cost characteristics. Of the 40 instances (20 with a single depot and 20 with multiple depots), five contain 15 therapists and 150 patients, five contain 15 therapists and 195 patients, five contain 20 therapists and 200 patients and the remaining five contain 20 therapists and 280 patients. Most patients require multiple treatments during the week.

The three tables in Appendix A contain the input data used in the study. Each instance was generated by sampling from Key Reb's data base which contains 109 fixed patients most of whom having multiple visit requirements each week. In the construction process, the number of therapists and patients were fixed but the total number of weekly visits was a function of the instance. For the single depot instances, one of the facilities in Table A1 in Appendix A was selected at random to be the home base for all therapists.

For the second set of experiments, 100 instances of model (3) were solved with both heuristics and CPLEX for the multiple depot case only. The data sets contain 5, 10, 15 and 20 therapists, each with a fixed number of patient visits (rather than patients); the number of patients varied with each random instance. Because we included roughly 20% more visits in these data sets, the 15- and 20-therapist instances turned out to be much more difficult to solve than the comparable instances in the first set of experiments.

Before presenting the results, an example is given to illustrate the input data and the type of results obtained from the analysis. A 15-therapist, 150-patient instance is used for this purpose. Table A1 identifies the location of each facility while Table A2 highlights the input data for each therapist for Monday (the data for the remaining four days is similar and is therefore omitted). Note that not all therapists are available every day; e.g., therapist 9 is not available on Monday. Table A3 provides the basic information for each patient.

6.2. Sample results

Models (2) and (3) were solved using a Java code and CPLEX Concert Technology for the first set of experiments. In the computations, the travel cost for every mile that therapist k drives in a day beyond 25 miles, $c_k^{\text{miles}} = \$0.55$, and the upper limit on the amount of overtime permitted for therapist k was 20 h, that is, $\tau_k^{\text{max\_over}} = 20$. The analysis was done for the more difficult scenario where each therapist k starts and ends at his home base, $o(k)$.

The solution to model (2) gives the minimum total cost by day without considering overtime and mileage reimbursement. Table 1 identifies the schedules derived for the 14 therapists who are available on Monday for the example data set (therapist 4 does not work on Monday). The first column is the therapist's ID. The second column is his corresponding route order given by patient ID and appointment time. For instance, on Monday morning, therapist 7 starts at his home base, visits patient 1064 from 9:00 am to 9:30 am (see Table A3 for treatment times), and then patient 1101 from 10:00 am to10:30 am. The third patient is 1088 whose appointment is from11:00 am to 11:30 am. After a lunch break, therapist 7 visits patient 1038 between 2:00 pm and 2:45 pm followed by 1057 from 3:30 pm to 4:15 pm. At that point his day is complete and he is free to return to his home base.

The solution to model (3) gives the minimal total cost for the week considering overtime and mileage reimbursement. Table 2 presents the schedule for each therapist who is available on

Table 1
Monday schedule for therapists from model (2).

Therapist ID	Route (patient-appointment time)
0	1098–8:00 am, 1105–1:00 pm, 1095–4:00 pm
1	1022–9:30 am, 1001–10:30 am, 1034–11:30 am, 1117–1:00 pm, 1136–1:30 pm, 1102–3:00 pm, 1074–4:00 pm
2	1075–1:30 pm, 1015–2:00 pm, 1073–4:00 pm
3	1013–8:00 am, 1042–10:30 am, 1007–1:00 pm, 1052–2:00 pm, 1090–2:30 pm, 1062–3:30 pm
5	1120–8:30 am, 1040–10:00 am, 1068–11:15 am, 1021–1:00 pm, 1059–4:00 pm
6	1139–8:30 am, 1091–9:00 am, 1036–10:00 am, 1033–10:30 am, 1100–11:00 am, 1030–1:00 pm, 1028–2:00 pm, 1106–3:00 pm, 1065–3:30 pm, 1116–4:30 pm
7	1064–9:00 am, 1101–10:00 am, 1088–11:00 am, 1038–2:00 pm, 1057–3:30 pm
8	1094–9:00 am, 1025–9:30 am, 1053–10:00 am, 1008–11:00 am, 1066–1:00 pm, 1147–2:00 pm, 1055–3:30 pm
10	1129–10:30 pm, 1110–11:00 am
11	1005–8:00 am, 1107–9:00 am, 1061–10:00 am, 1016–11:00 am, 1035–1:30 pm, 1138–2:00 pm, 1071–2:30 pm, 1115–3:30 pm, 1089–4:30 pm
12	1112–8:30 am, 1058–1:45 pm, 1039–3:30 pm
13	1081–7:30 am, 1097–8:30 am, 1029–10:30 am, 1027–11:00 am, 1054–1:00 pm, 1063–2:00 pm, 1087–2:45 pm, 1041–4:00 pm
14	1124–10:00 am, 1014–10:30 am, 1111–1:45 pm, 1032–4:00 pm

Table 2
Monday schedule for therapists from model (3).

Therapist ID	Route (patient-appointment time)
0	1112–8:00 am, 1066–1:00 pm, 1147–2:00 pm, 1095–4:00 pm
1	1022–9:30 am, 1001–10:30 am, 1034–11:30 am, 1117–1:00 pm, 1136–1:30 pm, 1102–3:00 pm, 1074–4:00 pm
2	1075–1:30 pm, 1015–2:00 pm
3	1013–8:00 am, 1042–10:30 am, 1007–1:00 pm, 1063–2:00 pm, 1087–2:45 pm, 1062–3:30 pm
5	1120–8:30 am, 1040–10:00 am, 1068–11:15 am, 1059–4:00 pm
6	1139–8:30 am, 1091–9:00 am, 1036–10:00 am, 1033–10:30 am, 1100–11:00 am, 1030–1:00 pm, 1028–2:00 pm, 1106–3:00 pm, 1065–3:30 pm, 1116–4:30 pm
7	1107–9:00 am, 1101–10:00 am, 1008–11:00 am, 1038–2:00 pm, 1055–3:30 pm
8	1094–9:00 am, 1025–9:30 am, 1053–10:00 am, 1088–11:00 am, 1105–1:00 pm, 1052–2:00 pm, 1090–2:30 pm, 1073–4:00 pm
10	1129–10:30 pm, 1110–11:00 am
11	1098–8:00 am, 1064–9:00 am, 1061–10:00 am, 1016–11:00 am, 1035–1:30 pm, 1138–2:00 pm, 1071–2:30 pm, 1115–3:30 pm, 1089–4:30 pm
12	1005–8:00 am, 1054–1:00 pm, 1041–4:00 pm
13	1081–7:30 am, 1097–8:30 am, 1029–10:30 am, 1027–11:00 am, 1021–1:00 pm, 1057–3:30 pm
14	1124–10:00 am, 1014–10:30 am, 1111–1:45 pm, 1032–4:00 pm

Monday. Comparing Tables 1 and 2, we see that the schedules for therapists 0, 2, 3, 5, 7, 8, 11, 12 and 13 change while the others remain the same. The reason is due to the mileage reimbursement since none of the schedules include overtime, that is, T_k^{over} for all $k \in K$. The objective of model (3) is to minimize the total cost of providing treatment over the week. The second term in the objective function (3a) is the total mileage reimbursement, that is, $\sum_{d \in D} \sum_{k \in K} c_k^{\text{miles}} \Delta D_{dk}$, and directly affects the solution. Looking at Table 3 we see that the total number of miles above 25 that all therapists drive on Monday is 212. 37 when model (2) was solved. When model (3) was solved, this number dropped to 176.43 miles, as might have been expected. The inclusion of mileage costs push down the number of miles driven.

The minimal total cost for model (2) for the week was $12,092.49 while the minimal total cost for model (3) was $12,565.20. Somewhat surprisingly, runtimes were dramatically different. It took 53.49 s to get the optimal solution for model (2) and 1705.30 s for model (3). The 30-fold increase was due directly to the vastly greater number of constraints in model (3).

Table 3
Comparison of driving distance for all therapists on Monday.

Therapist ID	Model (2) distance	Model (2) distance above 25 miles	Model (3) distance	Model (3) distance above 25 miles
0	3.00	0.00	21.53	0.00
1	15.39	0.00	15.39	0.00
2	5.04	0.00	4.92	0.00
3	21.67	0.00	20.43	0.00
4	0.00	0.00	40.86	15.86
5	54.76	29.76	54.76	29.76
6	23.83	0.00	23.83	0.00
7	90.20	65.20	90.20	65.20
8	94.39	69.39	58.46	33.46
9	0.00	0.00	0.00	0.00
10	2.00	0.00	2.00	0.00
11	31.96	6.96	29.88	4.88
12	14.55	0.00	2.00	0.00
13	66.06	41.06	52.31	27.31
14	15.14	0.00	15.14	0.00
Total	438.00	212.37	431.72	176.47

6.3. *Output summary for first set of experiments*

We tested 40 instances of models (2) and (3) on a workstation running Ubuntu linux with a dual core, hyperthreading 3.73 GHz Xeon processors and 24 GB of shared memory. Output summaries are presented in Appendix B, Tables B1–B4. In the majority of instances, the stopping criteria were an optimality gap of 0.5% or 10,000 nodes, whichever occurred first. The gaps reported are those realized at the final node in the search tree. When model (3) was solved for problems 16–20, the node limit was raised to 20,000 as seen in Table B4.

All runs terminated with a feasible solution. For instances with the same number of therapists, more patients means more total visits, a greater average number of working days, longer average working hours, and a greater average number of visits per therapist per week. The total cost and CPU time increase with number of therapists and patients in the data sets. Specifically, we found that when the total number of visits grows, the total cost grows linearly and the CPU time grows approximately exponentially. These relationships are illustrated in Figs. 2 and 3 where the horizontal axes represent the total number of visits per week. In Fig. 2 the vertical axis is the corresponding total cost in Table B1 and in Fig. 3 the vertical axis is the corresponding CPU time. The plots for the data in Tables B2–B4 were similar.

Tables B3 and B4 indicate when model (3) was tested with the same data sets, it took much more time to obtain a solution. Although the number of integer variables is the same in each model, the fact that the constraints span the full week in model (3) greatly adds to its complexity. A final observation from these results is that it is slightly more difficult to solve either model when the therapists are at different locations than when they all start and end their day at the same location.

6.4. *Output summary for second set of experiments*

Table 4 identifies the characteristics of the 100 random instances by data set used to test the model (3) heuristics. Each line represents the average of five instances. The first column gives the data set number, the next three columns list the parameter values associated with the patients, and the last six columns characterize the therapists. For example, for data set 1, 112 appointments averaging 0.59 h each need to be scheduled for 52.40 patients. Treatments are to be provided by 5 therapists whose average wage rate is $41.00/h, who are on duty an average of 4.64 days/wk and who

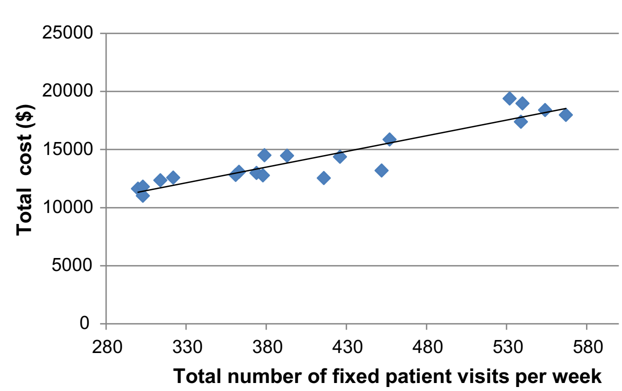

Fig. 2. Total visits and total cost from model (2) for different therapists at the same location.

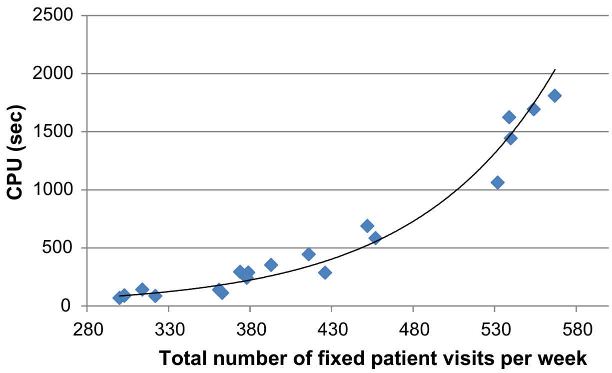

Fig. 3. CPU time and total cost from model (2) for different therapists at the same location.

are available to work an average of 37.75 h/wk. Their productivity rate is 75% (recall that this is an input parameter used to calculate administrative time), and each must treat an average of 5.32 patients in 8 h. Note that the data sets in Table 4 are sorted in increasing order of problem size as measured by the 'number of therapists' × 'number of visits.' As we go down the rows, runtimes were seen to generally increase.

Table 5 summaries the output statistics by data set. Similar to Table 4, each line represents the average of five instances. Columns 2–5 report the results for relaxed MIP I and columns 6–9 for relaxed MIP II. Columns 10–12 give the best results obtained by CPLEX, this time after 5000 B&B nodes, and the last column identifies the best lower bound provided by CPLEX at termination. A gap of 0% in the next to last column indicates that the optimal solution was found. As a point of reference, we have also included the relaxed solutions found for MIP I and MIP II that were used in Eqs. (4) and (5) to compute the actual solution. For MIP I, $\phi^{\mathrm{IP}}_{\mathrm{relaxed}}$ was 4.55% above $\phi^{\mathrm{IP}}_{\mathrm{actual}}$, and for MIP II, the difference was 4.94%. In all cases for both MIPs, the relaxed solutions were always greater than the actual solutions implying that overtime costs never dominated mileage reimbursement costs. When the relaxed costs and the actual costs were the same for both MIPs, as with data sets 1–5, for example, the implication is that the overtime costs were zero.

For relaxed MIP I, the computational effort appears to grow less than linearly (column 3) with problem size, while the runtimes for relaxed MIP II (column 6) and CPLEX (column 9) grow exponentially. Among the three approaches, relaxed MIP I is the most efficient for the obvious reason that the complicating constraints for overtime and mileage reimbursement are relaxed. CPLEX is the most expensive and is only able to optimally solve instances with 5 therapists within the allotted number of nodes. Relaxed MIP II is also very efficient, often providing better solutions than CPLEX, again for the reason that the overtime constraints are seldom binding and therefore have little effect on the computations. However, when overtime comes into play, this approach can be very time consuming as evidenced in the runtimes for data sets 19 and 20.

Regarding the solution quality, we always compare costs with the best lower bound in the last column of Table 5, which is the lowest objective function value among all unexplored nodes in CPLEX's search tree at termination. All three approaches provided very good solutions with gaps of at most 2% and often less than 1%. When the problem size is not too large, as in data sets 1–15, CPLEX yields the best solutions but is outperformed by relaxed MIP II for the largest data sets 16–20.

Based on these results, we can first conclude that relaxed MIP II is the better choice for instances that have ample therapist time and therefore require little overtime; otherwise, relaxed MIP I is preferred. For instances with fewer than 15 therapists, CPLEX can reliably find optimal or near-optimal solutions as we saw in the first set of experiments. We want to emphasize again, however, that the success of the relaxed MIPs is specifically related to the unique structure of the MDVSP under investigation. First, because overtime rarely takes effect and second, because the mileage reimbursement costs were linearized, thus removing the effects of the mileage threshold from the problem. In fact, we actually get the optimal solution for the MDVSP by solving the relaxed MIPs when each therapist is required to travel over 25 miles a day and no overtime is required. However, even if the distance traveled in a day does not exceed 25 miles, the extra cost is only \$13.75 = 0.55 × 25 which is negligible when compared to the total cost associated with a solution. Nevertheless, when overtime is present or when mileage

Table 4
Average problem characteristics for second set of experiments.

Data set	Patients			Therapists					
	Avg. no.	Total no. visits	Avg. time/visit	No.	Avg. days/wk	Avg. hrs/wk	Avg. wage rate (\$/h)	Avg. prod. (%)	Avg. visits/therapist in 8 h
1	52.60	112	0.59	5	4.64	37.75	41.00	75	5.32
2	56.80	125	0.59	5	4.56	38.15	38.74	76	5.86
3	58.80	137	0.57	5	4.92	41.19	39.84	76	5.92
4	66.20	150	0.59	5	4.96	42.44	39.62	75	6.32
5	68.60	162	0.58	5	4.96	44.24	40.04	79	6.59
6	90.00	225	0.59	10	4.42	35.05	39.17	76	5.70
7	97.60	250	0.60	10	4.62	38.52	41.04	76	5.79
8	108.20	275	0.59	10	4.68	37.89	37.94	75	6.44
9	118.20	300	0.58	10	4.70	39.01	39.08	76	6.86
10	127.00	325	0.58	10	4.82	40.99	37.98	77	7.06
11	128.40	337	0.59	15	4.41	36.38	39.14	78	5.55
12	141.00	375	0.59	15	4.40	35.70	37.47	76	6.23
13	155.60	412	0.59	15	4.48	36.11	38.25	77	6.75
14	168.20	450	0.58	15	4.63	38.18	38.01	77	6.99
15	178.40	487	0.58	15	4.81	41.33	39.47	77	7.02
16	167.60	450	0.60	20	4.44	36.08	39.00	78	5.55
17	184.60	500	0.59	20	4.44	35.92	38.11	78	6.18
18	201.60	550	0.59	20	4.60	37.87	38.14	75	6.46
19	218.40	600	0.58	20	4.54	37.76	39.65	77	7.07
20	236.00	650	0.58	20	4.71	39.96	39.45	77	7.25

Table 5
Algorithmic comparisons.

	Relaxed MIP I				Relaxed MIP II				CPLEX			LB[b] ($)
Data set	$\phi^{IP}_{relaxed}$	Cost ($)	Time (s)	Gap[a] (%)	$\phi^{IP}_{relaxed}$	Cost ($)	Time (s)	Gap[a] (%)	Cost ($)	Time (s)	Gap[a] (%)	
1	5459	5182	0.15	0.06	5459	5182	0.13	0.06	5180	0.52	0	5179
2	5391	5103	0.19	0.03	5391	5103	0.19	0.03	5102	0.52	0	5102
3	5964	5657	0.22	0.03	5964	5657	0.21	0.03	5655	0.42	0	5655
4	6723	6397	0.33	0.09	6723	6397	0.40	0.09	6391	0.47	0	6391
5	7020	6694	0.37	0.04	7020	6694	0.36	0.04	6691	0.64	0	6691
6	9106	8635	2.47	0.32	9106	8630	1.59	0.27	8616	51.64	0.10	8607
7	10,402	9912	3.66	0.37	10,402	9906	3.96	0.31	9888	90.65	0.12	9876
8	11,004	10,561	6.13	1.04	11,013	10,484	6.62	0.30	10,466	134.50	0.13	10,453
9	11,785	11,363	9.12	1.45	11,795	11,249	9.35	0.43	11,225	226.24	0.21	11,201
10	12,302	11,820	10.71	0.96	12,307	11,741	19.95	0.29	11,722	259.94	0.13	11,707
11	12,721	12,088	27.29	0.95	12,721	12,088	23.39	0.95	12,066	573.50	0.76	11,977
12	13,575	12,912	29.70	0.89	13,576	12,907	32.39	0.85	12,880	774.51	0.65	12,798
13	15,330	14,726	66.06	1.46	15,334	14,636	80.77	0.87	14,619	1,329.20	0.75	14,510
14	16,246	15,577	53.41	1.13	16,250	15,533	78.50	0.85	15,511	1,900.90	0.70	15,403
15	17,908	17,266	76.32	1.39	17,916	17,182	143.64	0.90	17,160	2,946.21	0.76	17,029
16	16,206	15,427	135.17	1.21	16,205	15,427	183.95	1.21	15,434	2,066.10	1.25	15,243
17	17,324	16,553	120.46	1.46	17,325	16,506	165.17	1.20	16,507	5,555.80	1.21	16,310
18	20,026	19,335	185.88	2.01	20,035	19,154	375.84	1.08	19,151	6,228.13	1.07	18,949
19	21,656	20,971	304.66	1.91	21,680	20,787	965.70	1.04	20,816	10,494.91	1.19	20,572
20	23,249	22,515	659.63	1.81	23,263	22,347	8,251.18	1.06	22,384	15,689.97	1.22	22,113

[a] 100 × (Cost − LB)/LB (%).
[b] Provided by CPLEX after 5000 B&B nodes.

reimbursement becomes significant, relaxed MIP II is likely to bog down and take much longer than the first relaxation to converge.

6.5. Parametric analysis for number of therapists

This experiment was aimed at determining the tradeoff between the number of therapists and total cost when the patient population remains fixed. Fig. 4 plots the total cost for problem number 1 in Table B4 as the number of therapists is increased from 15 to 20. This instance has 150 patients, 306 visits, and a nonhomogeneous set of 15 therapists all starting their day at their home base. Each additional therapist was randomly selected from our data base.

For the original case with 15 therapists, the total cost is $12,045. As therapists are added, the cost decreases linearly down to $10,783, a drop of 10.5%. The main explanation for this savings is the elimination of overtime and a reduction in total miles driven during the week. What is missing from the calculation, though, is the fixed cost of employment, which is primarily associated with benefits. Nevertheless, if the number of working hours in the week drops below 30 for an employee, then he or she becomes a part-timer and is not entitled to benefits. Adapting such a strategy can significantly reduce agency costs, but it could also lead to high turnover and additional costs for training, as well as a dissatisfied workforce often manifesting itself in poor service and an accompanying loss of patients.

6.6. Parametric analysis for number of home bases

The second parametric study we preformed examined the tradeoff between the number of different home bases from which the therapists could operate and the cost of a schedule. The working hypothesis was that the more home bases, the lower the cost. Because patients are widely distributed over the service area, it could be argued that it would be better for the therapists to be widely distributed as well. This would allow schedules to be constructed that more closely matched therapists with patients in their "neighborhood" and hence lower the travel, mileage reimbursement and even overtime costs.

Using data set 1 in Table 4 with 112 patient visits during the week and five therapists randomly drawn from our data base, we solved the scheduling problem with CPLEX for five different

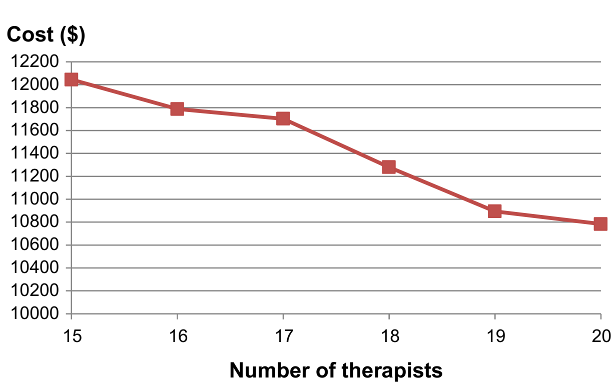

Fig. 4. Relationship between number of therapists and total cost.

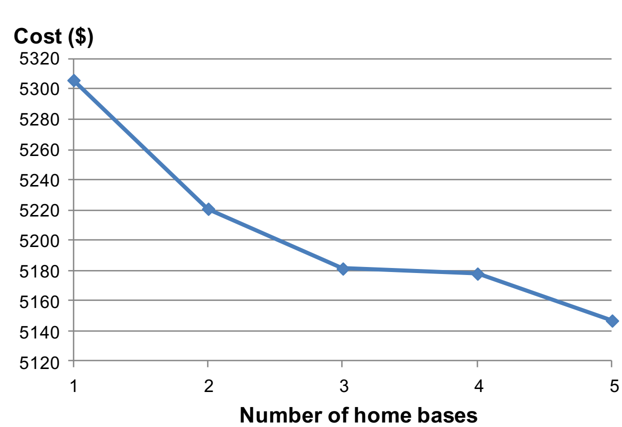

Fig. 5. Relationship between number of home bases and total cost.

scenarios, each for five instances, giving a total of 25 runs. In the first scenario, all five therapists were located at the same home base; the average cost was $5,306. In the second scenario, three therapists were located at the first home base and two at the second home base; the average cost was $5221 – slightly less than the first scenario. In all cases, the home bases were randomly selected from those of the five therapists.

Fig. 5 plots the results for the five scenarios. As can be seen, as the number of home bases increases, the costs decrease almost linearly but not significantly. The additional cost incurred when each therapist starts and ends his day at a central location rather than at his current home base was only $159. This 3% increase would appear to be small enough for management to insist on such a policy if it afforded better oversight. From a job satisfaction and cost point of view, though, maintaining the current policy has a small but measurable advantage.

7. Summary

In this paper, we began with 40 data sets to evaluate two different models for scheduling therapist visits over the week. The first model is referred to generically as a VSP with lunch breaks and either a single depot or multiple depots, and the second similarly, but with the addition of overtime and mileage reimbursement costs. For all of these instances, CPLEX was able to find feasible solutions well within 2.04% of optimality and often within 1%. For model (2) with 15 therapists, the gap was within 0.5%, averaging 0.17%; for the 20-therapist case, the gap averaged 0.21%. For model (3) with 15 therapists, the gap averaged 0.73%; for the 20-therapist case, the gap averaged 1.18%. Two parametric analyses were also conducted to get a better understanding of how costs might vary with changes in the number of therapists and the number of home bases. In both cases, the relationships were decreasing.

To get high-quality solutions for model (3), we proposed two relaxation heuristics. For all but the most difficult instances with 20 therapists, both proved to be quick and effective for the 100 instances solved. We believe that the heuristics are simple enough for someone skilled in programming to implement and maintain them in a decision support system. It would also be necessary to create an easy-to-update patient/therapist data base if the use of the system is to be sustained.

With respect to future research, we are now investigating the more general problem in which not all patients have fixed appointment times but can be seen at various times during the day. Little has been written on the “flexible” patient problem by those in the residential healthcare field although there is some related VRP literature. Nevertheless, it is an open question as to whether algorithms can be developed to optimally solve the dynamic patient problem when the additional constraints discussed in this paper are included in the model, and when patients have multiple treatment requirements per day. These features introduce a much greater level of complication than when the appointment times are fixed.

Appendix A. Input data for study.

Table A1 identifies the 17 facilities (depots) in the Key Rehab data base where patients are treated. The second column gives the ID, the third column identifies the type of facility, and the last two columns provide longitude and latitude data. The latter values are used to compute distances.

Table A2 highlights the input data for the therapists. The treatment rate (TR), driving rate (DR), administration rate (AR) and productivity (Prod) are listed in the cost columns. The ID numbers in the first column begin with 0. Longitude and latitude are given for each therapist’s home base as is the clinic he reports to.

Table A3 provides sample input data for the patients who have appointments on Monday. This includes his or her ID, the facility code associated with the treatment location, and the appointment time on Monday. Patient IDs begin with 1000.

Table A1
Facility data.

No.	ID	Facility type	Longitude	Latitude
1	KS109	Medical lodge	−97.0054215	37.5155992
2	KS130	Medical lodge	−97.4397647	37.7330952
3	KS245	Rest haven	−97.246866	37.560118
4	KS249	Rest haven	−97.3146551	37.6525878
5	KS532	Medical lodge	−97.572676	37.73481
6	KS593	Medical lodge	−97.645003	37.3863735
7	KS605	Medical lodge	−97.234295	37.5519869
8	KS636	Hospital	−97.1839959	37.657345
9	KS637	Hospital	−97.200431	37.7034049
10	KS763	Medical lodge	−97.2496716	37.6993938
11	KS803	Nursing center	−97.366046	37.648453
12	KS863	Nursing center	−97.447053	37.73173
13	KSH01	Home health	−97.267299	37.68347
14	KSH02	Home health	−97.2113605	37.7685329
15	KSH03	Home health	−97.58307	37.661173
16	KSH04	Home health	−97.466316	37.710647
17	KSH05	Home health	−97.657722	37.869227

Table A2
Input data for therapists.

ID	Type	Cost data ($)				Clinic ID	Longitude	Latitude	Availability (Monday)
		TR/h	DR/h	AR/h	Prod/h				
0	PT	45	45	45	0.65	KS863	−97.4456	37.7313	8:00 am–5:30 pm
1	PT	40	40	40	0.77	KS763	−97.2289	37.7035	7:00 am–6:00 pm
2	PT	50	50	50	0.75	KS763	−97.2289	37.7035	1:00 pm–5:00 pm
3	PT	42	37	37	0.8	KS249	−97.3123	37.6499	7:00 am–6:00 pm
4	PT	45	45	45	0.65	KS637	−97.2014	37.7025	1:00 pm–5:00 pm
5	PT	45	45	45	0.65	KS803	−97.366	37.6313	7:00 am–6:00 pm
6	PTA	27.5	27	27	0.9	KS763	−97.2289	37.7035	7:00 am–6:00 pm
7	PT	38.5	38.5	38.5	0.65	KS763	−97.2289	37.7035	7:00 am–6:00 pm
8	PT	30	30	30	0.65	KSH02	−97.2406	37.749	7:00 am–6:00 pm
10	PT	45	45	45	0.8	KS249	−97.3123	37.6499	8:00 am–12:00 pm
11	PT	40	40	40	0.77	KS130	−97.4432	37.7318	7:00 am–6:00 pm
12	PT	50	50	50	0.77	KSH04	−97.4665	37.7168	8:00 am–5:30 pm
13	PT	40	40	40	0.77	KS130	−97.4432	37.7318	7:00 am–6:00 pm
14	PT	50	50	50	0.77	KS863	−97.4456	37.7313	7:00 am–6:00 pm

Table A3
Input data for patients.

ID	Facility code	Appointment (Monday)	ID	Facility code	Appointment (Monday)
1001	KSH01	10:30 am–11:00 am	1065	KS605	3:30 pm–4:00 pm
1005	KSH04	8:00 am–8:30 am	1066	KS532	1:00 pm–1:30 pm
1007	KS605	1:00 pm–1:30 pm	1068	KS593	11:15 am–11:45 am
1008	KSH05	11:00 am–12:00 pm	1071	KS130	2:30 pm–3:15 pm
1013	KS605	8:00 am–8:30 am	1073	KS763	4:00 pm–4:45 pm
1014	KS803	10:30 am–11:00 am	1074	KSH02	4:00 pm–4:45 pm
1015	KSH01	2:00 pm–2:45 pm	1075	KSH01	1:30 pm–2:00 pm
1016	KSH02	11:00 am–11:30 am	1081	KS593	7:30 am–8:00 am
1021	KS593	1:00 pm–1:30 pm	1087	KS803	2:45 pm–3:30 pm
1022	KSH01	9:30 am–10:00 am	1088	KSH05	11:00 am–11:30 am
1025	KSH02	9:30 am–10:00 am	1089	KS130	4:30 pm–5:00 pm
1027	KS593	11:00 am–12:00 pm	1090	KS249	2:30 pm–3:00 pm
1028	KS245	2:00 pm–2:30 pm	1091	KS636	9:00 am–9:30 am
1029	KS593	10:30 am–11:00 am	1094	KSH02	9:00 am–9:30 am
1030	KS245	1:00 pm–1:30 pm	1095	KS863	4:00 pm–4:30 pm
1032	KS130	4:00 pm–4:30 pm	1097	KS593	8:00 am–8:30 am
1033	KS636	10:30 am–11:00 am	1098	KS130	8:00 am–8:30 am
1034	KS637	11:30 am–12:00 pm	1100	KS636	11:00 am–11:30 am
1035	KS130	1:30 pm–2:00 pm	1101	KS763	10:00 am–10:30 am
1036	KS636	10:00 am–10:30 am	1102	KSH02	3:00 pm–3:30 pm
1038	KSH05	2:00 pm–2:45 pm	1105	KS863	1:00 pm–1:30 pm
1039	KSH03	3:30 pm–4:15 pm	1106	KS605	3:00 pm–3:30 pm
1040	KS245	10:00 am–10:30 am	1107	KS763	9:00 am–9:30 am
1041	KSH04	4:00 pm–4:45 pm	1110	KS249	11:00 am–11:45 am
1042	KS605	10:30 am–11:00 am	1111	KS803	1:45 pm–2:30 pm
1052	KS249	2:00 pm–2:30 pm	1112	KSH03	8:30 am–9:00 am
1053	KSH02	10:00 am–10:30 am	1115	KS130	3:30 pm–4:15 pm
1054	KSH04	1:00 pm–1:30 pm	1116	KS763	4:30 pm–5:00 pm
1055	KS593	3:30 pm–4:15 pm	1117	KS637	1:00 pm–1:30 pm
1057	KS593	3:30 pm–4:15 pm	1120	KS245	8:30 am–9:00 am
1058	KSH03	1:45 pm–2:30 pm	1124	KS803	10:00 am–10:30 am
1059	KS593	4:00 pm–4:30 pm	1129	KS249	10:30 am–11:00 am
1061	KSH02	10:00 am–10:15 am	1136	KS637	1:30 pm–2:00 pm
1062	KS803	3:30 pm–4:15 pm	1138	KS130	2:00 pm–2:30 pm
1063	KS803	2:00 pm–2:30 pm	1139	KS636	8:30 am–9:00 am
1064	KS763	9:00 am–9:30 am	1147	KS532	2:00 pm–2:30 pm

Appendix B. Summary of results for first set of experiments for models (2) and (3)

Each table in this appendix has the same format. Column 1 gives the problem number. The next three columns indicate the number of patients, the total number of patient visits per week, and their average treatment time per visit in hours, respectively. Columns 5–10 are associated with the therapists. Column 5 gives the number of therapists, columns 6 and 7 are data input that indicate the average wage rate and productivity for each therapist, respectively. Output statistics are presented in columns 8–9. Column 8 gives the average number of days per week that each therapist is available, and column

Table B1
Results from model (2) for different therapists at the same location.

Data set	Fixed patients			Therapists						Results			
	No.	No. visits	Avg. treatment time/visit	Input			Output			Cost ($)	CPU (s)	Gap (%)	Nodes no.
				No.	Avg. wage/h	Avg. prod. (%)	Avg. no. days/wk	Avg. hrs/wk	Avg. no. visits/ therapist per day				
1	150	314	0.61	15	39.03	0.78	4.20	21.79	4.19	12,351.52	140.05	0.33%	0+
2	150	300	0.58	15	40.37	0.74	4.47	20.98	4.00	11,628.82	67.99	0.16%	0+
3	150	322	0.50	15	39.47	0.75	4.13	22.45	4.29	12,572.60	85.16	0.04%	0+
4	150	303	0.60	15	40.30	0.77	4.07	21.12	4.04	11,801.22	87.55	0.02%	0+
5	150	303	0.60	15	37.79	0.77	4.27	21.19	4.04	11,021.32	90.98	0.18%	0+
6	195	363	0.60	15	39.00	0.81	4.27	23.89	4.84	13,079.29	112.05	0.04%	0+
7	195	374	0.59	15	37.69	0.78	4.40	25.02	4.99	12,978.33	292.30	0.14%	0+
8	195	379	0.58	15	40.33	0.74	4.40	25.59	5.05	14,492.26	287.93	0.23%	0+
9	195	361	0.59	15	37.97	0.76	4.27	24.86	4.81	12,799.61	138.11	0.14%	0+
10	195	378	0.59	15	37.19	0.77	4.47	25.12	5.04	12,768.01	241.56	0.26%	0+
11	200	416	0.59	20	35.90	0.78	3.90	19.75	4.16	12,530.81	442.99	0.24%	0+
12	200	393	0.62	20	37.32	0.76	4.20	20.36	3.93	14,468.96	352.61	0.17%	0+
13	200	457	0.55	20	39.72	0.75	4.50	21.51	4.57	15,849.28	583.02	0.15%	0+
14	200	452	0.50	20	38.67	0.79	4.20	18.81	4.52	13,177.75	687.18	0.50%	336
15	200	426	0.59	20	37.73	0.78	4.05	20.55	4.26	14,363.94	286.14	0.01%	0+
16	280	554	0.59	20	38.57	0.75	4.70	26.64	5.54	18,387.26	1692.36	0.03%	0+
17	280	567	0.59	20	36.24	0.76	4.70	27.55	5.67	17,967.23	1808.24	0.40%	0+
18	280	540	0.59	20	38.95	0.76	4.45	26.64	5.40	18,962.48	1441.52	0.40%	0+
19	280	539	0.60	20	37.07	0.80	4.60	25.51	5.39	17,368.71	1622.66	0.03%	0+
20	280	532	0.60	20	39.85	0.77	4.65	26.15	5.32	19,379.01	1061.66	0.50%	0+

Table B2
Results from model (2) for different therapists at different locations.

Data set	Fixed patients			Therapists						Results			
	No.	No. visits	Avg. treatment time/visit	Input			Output			Cost ($)	CPU (s)	Gap (%)	No. nodes
				No.	Avg. wage/h	Avg. prod. (%)	Avg. no. days/wk	Avg. hrs/wk	Avg. no. visits/ therapist per day				
1	150	306	0.60	15	39.33	0.74	4.60	20.89	4.08	11,670	72.67	0.16%	0+
2	150	325	0.59	15	37.33	0.75	4.40	21.17	4.33	10,838	80.89	0.14%	0+
3	150	307	0.60	15	38.87	0.78	4.40	19.65	4.09	10,147	85.76	0.36%	0+
4	150	312	0.61	15	41.20	0.75	4.40	20.98	4.16	12,092	53.49	0.00%	0+
5	150	306	0.62	15	41.40	0.76	4.33	21.10	4.08	12,606	44.05	0.23%	0+
6	195	386	0.59	15	39.73	0.74	4.60	26.41	5.28	14,463	300.70	0.50%	0+
7	195	383	0.60	15	40.65	0.76	4.53	24.41	5.11	13,935	144.99	0.10%	0+
8	195	387	0.60	15	37.39	0.75	4.53	25.69	5.16	13,187	199.83	0.15%	0+
9	195	376	0.60	15	36.93	0.78	4.40	23.95	5.01	12,562	262.86	0.08%	0+
10	195	351	0.60	15	35.79	0.76	4.40	23.06	4.68	11,417	132.27	0.06%	0+
11	200	402	0.60	20	41.75	0.76	4.20	19.65	4.02	15,349	188.97	0.08%	0+
12	200	419	0.60	20	39.47	0.76	4.45	20.53	4.19	14,792	2105.04	0.28%	1702
13	200	421	0.59	20	38.84	0.77	4.25	19.84	4.21	13,771	370.12	0.15%	0+
14	200	432	0.60	20	40.05	0.76	4.50	20.71	4.32	15,161	545.55	0.24%	0+
15	200	416	0.61	20	43.55	0.77	4.45	19.94	4.16	16,090	223.30	0.16%	0+
16	280	504	0.60	20	36.54	0.79	4.65	23.16	5.04	15,251	4151.89	0.27%	1443
17	280	524	0.60	20	40.50	0.77	4.45	25.13	5.24	19,070	797.38	0.07%	0+
18	280	540	0.60	20	35.58	0.76	4.50	25.87	5.40	17,138	1654.47	0.24%	410
19	280	549	0.60	20	38.61	0.75	4.80	27.86	5.49	18,651	2302.81	0.14%	210
20	280	534	0.61	20	40.55	0.74	4.65	26.59	5.34	20,432	1688.89	0.21%	311

9 reports the average number of working hours per therapist per week. Column 10 gives the average number of visits per therapist per day. The last four columns highlight the results. The total cost of providing treatment for a week is given in column 11. Column 12 indicates the CPU time in seconds for solving the model. Column 13 shows the optimality gap provided by CPLEX while column 14 gives the number of final number of nodes in the search tree upon termination. The "+" symbol in the last column indicates that the solution was found at the root node by CPLEX's heuristic.

Table B1 gives the results for model (2) for the 20 instances associated with the case of different therapists who begin and end their day at the same location, while Table B2 gives the results for the case of different therapists at different locations, also for 20 instances and model (2). Tables B3 and B4 give parallel results for model (3). On average, it took 576.103 s to obtain the results for the 20 instances associated with different therapist at the same location using model (2), and an average of 12,531.05 s to obtain the results for the same instances using model (3). This corresponds to a nearly 22-fold increase in runtime. For the 20 instances associated with different therapist at different location, it took 770.30 s on average for model (2) and 18,181.82 s on average for model (3), a 23.6-fold increase.

Table B3
Results from model (3) for different therapist at the same location.

Prob. no.	Fixed patients			Therapists						Results			
	No.	No. visits	Avg. treatment time/visit	Input			Output			Cost ($)	CPU (s)	Gap (%)	Nodes no.
				No.	Avg. wage/h	Avg. prod. (%)	Avg. no. days/wk	Avg. hrs/wk	Avg. no. visits/ therapist per day				
1	150	314	0.61	15	39.03	0.78	4.20	21.66	4.19	12,849	1211.93	0.54%	10,000
2	150	300	0.58	15	40.37	0.74	4.47	20.75	4.00	12,221	1422.89	1.13%	10,000
3	150	322	0.50	15	39.47	0.75	4.13	22.40	4.29	13,164	2175.74	0.54%	9481
4	150	303	0.60	15	40.30	0.77	4.13	21.02	4.04	12,418	383.86	0.47%	603
5	150	303	0.60	15	37.79	0.77	4.33	21.00	4.04	11,573	5176.30	0.63%	10,000
6	195	363	0.60	15	39.00	0.81	4.33	23.81	4.84	13,751	2115.18	0.73%	10,000
7	195	374	0.59	15	37.69	0.78	4.40	24.88	4.99	13,680	2120.95	0.81%	10,000
8	195	379	0.58	15	40.33	0.74	4.53	25.53	5.05	15,102	1848.04	0.50%	3985
9	195	361	0.59	15	37.97	0.76	4.27	24.66	4.81	13,639	5328.43	0.68%	10,000
10	195	378	0.59	15	37.19	0.77	4.47	24.95	5.04	13,444	3242.68	0.57%	10,000
11	200	416	0.59	20	35.90	0.78	3.95	19.65	4.16	13,086	6946.45	0.77%	10,000
12	200	393	0.62	20	37.32	0.76	4.20	20.30	3.93	15,097	4775.64	1.38%	10,000
13	200	457	0.55	20	39.72	0.75	4.50	21.52	4.57	16,410	9830.18	1.07%	10,000
14	200	452	0.50	20	38.67	0.79	4.30	18.66	4.52	13,698	5975.20	1.31%	10,000
15	200	426	0.59	20	37.73	0.78	4.10	20.55	4.26	14,955	6876.39	0.79%	10,000
16	280	554	0.59	20	38.57	0.75	4.80	26.48	5.54	19,048	56,382.74	1.25%	10,000
17	280	567	0.59	20	36.24	0.76	4.75	27.32	5.67	18,850	47,451.51	1.09%	10,000
18	280	540	0.59	20	38.95	0.76	4.55	26.47	5.40	19,865	33,404.91	1.26%	10,000
19	280	539	0.60	20	37.07	0.80	4.60	25.39	5.39	18,139	26,397.81	1.15%	10,000
20	280	532	0.60	20	39.85	0.77	4.70	26.03	5.32	20,144	27,554.21	1.11%	10,000

Table B4
Results from model (3) for different therapists at different locations.

Data set	Fixed patients			Therapists						Results			
	No.	No. visits	Avg. treatment time/visit	Input			Output			Cost ($)	CPU (s)	Gap (%)	No. nodes
				No.	Avg. wage/h	Avg. prod. (%)	Avg. no. days/wk	Avg. hrs/wk	Avg. no. visits/ therapist per day				
1	150	306	0.60	15	39.33	0.74	4.60	20.81	4.08	12,045	1566.60	0.81%	10,000
2	150	325	0.59	15	37.33	0.75	4.40	21.07	4.33	11,270	2133.87	1.09%	10,000
3	150	307	0.60	15	38.87	0.78	4.40	19.53	4.09	10,442	1575.07	0.66%	10,000
4	150	312	0.61	15	41.20	0.75	4.40	21.06	4.16	12,565	1705.30	0.68%	10,000
5	150	306	0.62	15	41.40	0.76	4.33	20.99	4.08	13,076	1487.60	0.59%	10,000
6	195	386	0.59	15	39.73	0.74	4.73	26.05	5.28	14,991	7689.80	0.88%	10,000
7	195	383	0.60	15	40.65	0.76	4.20	24.34	5.11	14,446	2512.21	0.54%	10,000
8	195	387	0.60	15	37.39	0.75	4.53	25.59	5.16	13,740	5841.48	0.99%	10,000
9	195	376	0.60	15	36.93	0.78	4.40	23.89	5.01	13,016	3383.83	0.71%	10,000
10	195	351	0.60	15	35.79	0.76	4.53	22.97	4.68	11,952	1404.21	0.74%	10,000
11	200	402	0.60	20	41.75	0.76	4.20	19.63	4.02	15,914	5817.41	0.98%	10,000
12	200	419	0.60	20	39.47	0.76	4.45	20.42	4.19	15,363	6235.48	1.39%	10,000
13	200	421	0.59	20	38.84	0.77	4.25	19.72	4.21	14,333	7645.86	1.16%	10,000
14	200	432	0.60	20	40.05	0.76	4.50	20.58	4.32	15,653	8726.33	1.45%	10,000
15	200	416	0.61	20	43.55	0.77	4.60	19.71	4.16	16,490	8602.79	0.84%	10,000
16	280	504	0.60	20	36.54	0.79	4.65	23.17	5.04	15,920	37,649.13	2.04%	20,000
17	280	524	0.60	20	40.50	0.77	4.45	25.14	5.24	20,003	39,894.68	0.87%	20,000
18	280	540	0.60	20	35.58	0.76	4.55	25.76	5.40	17,695	46,769.14	1.14%	20,000
19	280	549	0.60	20	38.61	0.75	4.80	26.39	5.49	19,391	109,289.64	1.44%	20,000
20	280	534	0.61	20	40.55	0.74	4.65	26.42	5.34	21,148	63,705.99	1.12%	20,000

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