Socio-Economic Planning Sciences 48 (2014) 175–188

Contents lists available at ScienceDirect

Socio-Economic Planning Sciences

journal homepage: www.elsevier.com/locate/seps

Scheduling food bank collections and deliveries to ensure food safety and improve access

Lauren B. Davis [a,*], Irem Sengul [b], Julie S. Ivy [b], Luther G. Brock III [a], Lastella Miles [b]

[a] Department of Industrial and Systems Engineering, North Carolina A&T State University, College of Engineering, 1601 East Market Street, Greensboro, NC 27411, United States

[b] Edward P. Fitts Department of Industrial and Systems Engineering, North Carolina State University, 111 Lampe Street, Raleigh, NC 27695-7906, United States

ARTICLE INFO

Article history:
Available online 24 April 2014

Keywords:
Food distribution
Hunger relief
Humanitarian logistics
Set covering
Vehicle routing

ABSTRACT

Food banks are privately-owned non-profit organizations responsible for the receipt, processing, storage, and distribution of food items to charitable agencies. These charitable agencies in turn distribute food to individuals at risk of hunger. Food banks receive donated food from national and local sources, such as The Emergency Food Assistance Program (TEFAP) and supermarkets. Local sources with frequent high-volume donations justify the use of food bank vehicles for collection. Food bank vehicles are also used to deliver food to rural charitable agencies that are located beyond a distance safe for perishable food to travel without spoilage. Due to limited funds, food banks can only afford to sparingly use their capital on non-food items. This requires exploring more cost effective food delivery and collection strategies. The goal of this paper is to develop transportation schedules that enable the food bank to both (i) collect food donations from local sources and (ii) to deliver food to charitable agencies. We identify satellite locations, called food delivery points (FDPs), where agencies can receive food deliveries. A set covering model is developed to determine the assignment of agencies to an FDP. Both vehicle capacity and food spoilage constraints are considered during assignment. Using the optimal assignment of agencies to FDPs, we identify a weekly transportation schedule that addresses collection and distribution of donated food and incorporates constraints related to food safety, operator workday, collection frequency, and fleet capacity.

1. Introduction

The number of individuals with limited or uncertain access to safe food is growing in the United States. This condition, formally known as *food insecurity*, has affected more than 10% of U.S. households every year since 1998 [1]. Many food insecure individuals receive assistance from federal programs and/or charitable agencies (e.g., soup kitchens or food pantries) focused on hunger relief. The success of these charitable agencies, however, is highly dependent on upstream sources of food supply from food banks. Food banks solicit, receive, inspect, inventory and distribute food and grocery products to charitable agencies. Essentially, they function as aggregators and distributors of donated food supply from various private and federal sources [2]. National donors, food gleaning organizations, local sponsors, and other food banks are among the primary sources for donated food items. In addition to managing the distribution of food to charitable agencies, food banks also manage collection of donations generated by local sponsors. The focus of this paper is this challenge of managing food collection and distribution.

A food bank's ability to balance both delivery and collection requirements is challenging, particularly when transportation resources are limited, the delivery and collection network is diverse, and characteristics of the donated food items influence the collection and delivery strategy. Food banks differ by size based on pounds of food received, the number of agencies served, and the size of their service area. Many food banks serve more than 100 charitable agencies, with some agencies located as far as 80 miles or more from their warehouse [2]. Since food banks primarily make food available to agencies through on-site warehouse shopping, distance limits the ability of an agency to shop on a regular basis. For example, the Good Shepherd Food Bank of Maine cites "lack of transportation as a common and significant barrier for its food pantries" [3]. Distance is not the only barrier that agencies face. Many have limited access to refrigerated vehicles for transporting food. Food banks are actively seeking distribution strategies in an

* Corresponding author. Tel.: +1 336 285 3724; fax: +1 336 334 7729.
E-mail address: lbdavis@ncat.edu (L.B. Davis).

http://dx.doi.org/10.1016/j.seps.2014.04.001
0038-0121/

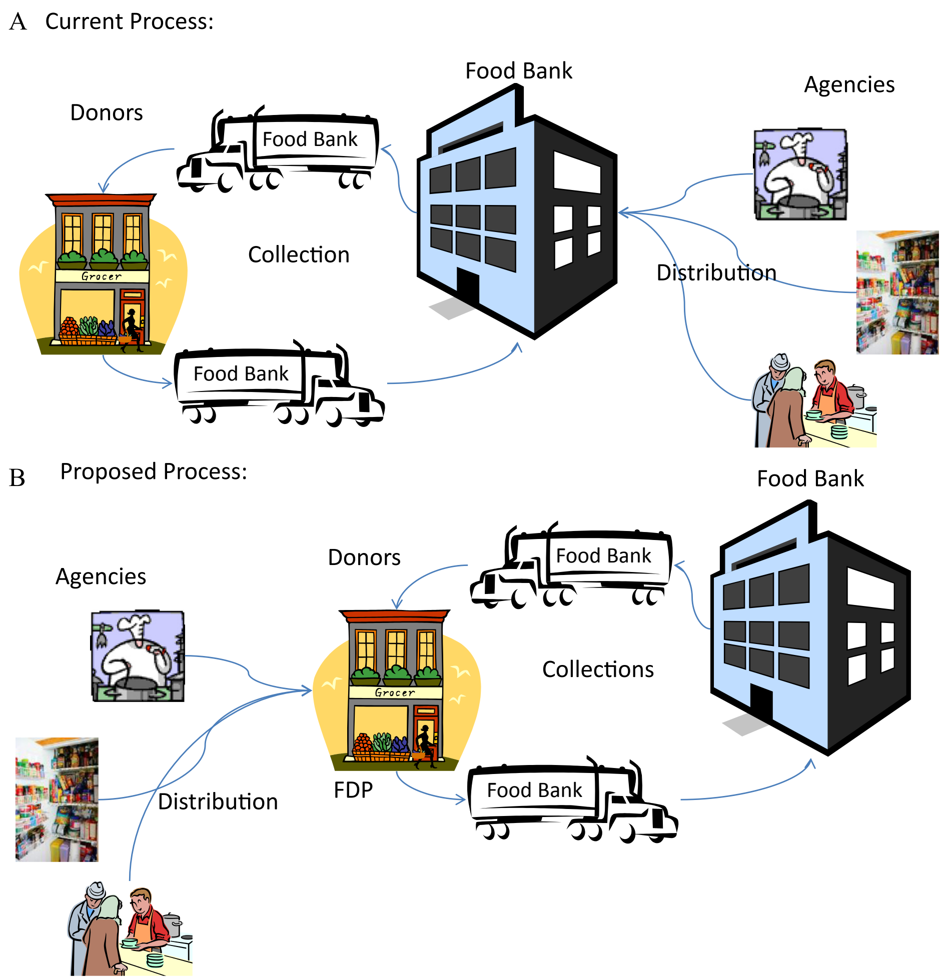

Fig. 1. The food bank's current (A) and FDP-based distribution processes (B).

effort to increase the access to donated food, particularly for agencies located in remote parts of the service area. Some strategies employed are direct delivery to charitable agencies from retail donors, establishing mobile food pantries in remote counties, and offering delivery services.

Delivery services involve direct delivery of food to agencies that are unable to shop at a food bank due to distance (e.g., rural) and/or resource (i.e. transportation) constraints. The frequency of the deliveries may be monthly [4] or bi-monthly. In addition, only a limited number of agencies may be able to take advantage of this service, as food banks often charge a small fee to recover the cost associated with fuel and vehicle usage. Providing access to donated food in an equitable manner can result in significant operational costs when the service area is large and agencies are distributed throughout the service area. Furthermore, providing regular and frequent delivery to rural agencies can be difficult if food bank vehicles are also used for collections.

The development of efficient transportation schedules is not only influenced by the distribution strategy and network topology, but also the characteristics of food items donated and the specifications of the donor. For example, highly perishable food items have short lead times and must be distributed quickly due to perishability. In contrast, food items with a longer shelf life, such as uncooked meats received from supermarkets, can be stored and distributed at a future date. Most perishable items are considered unsafe and must be discarded if left unrefrigerated (above 40 °F) for more than 2 h. Food safety related to the handling and distribution of perishable food is always of concern, particularly when charitable agencies are sparsely distributed throughout a food bank's service area. While agencies in close proximity to a food bank can safely transport food to their site without requiring delivery assistance, those outside of a safe travel distance must receive their donations through either food bank managed dispatches or incur costs associated with providing refrigerated transport.

While profit is not their objective, food banks, like other non-profit organizations, must efficiently use their existing resources in order to best serve their communities. For an organization that operates with a limited fleet of vehicles, this translates to a need to identify low-cost approaches for performing food collections and deliveries. This paper presents an approach to managing the collection and delivery of high-volume food items for food banks with service areas that are extensive in distance covered and agencies supported. In particular, the delivery network is characterized by areas that are both densely populated with charitable agencies serving urban counties, and sparsely populated with charitable agencies serving rural counties. The supply network is characterized by high-volume collections from retail donors several times per week. Given that a food bank must provide access to donated food to each agency it serves in an equitable manner, we propose a model that uses food delivery points (FDPs) to increase food access to charitable agencies. FDPs are satellite delivery locations activated only when deliveries are required to supplement an agency's access to food bank supply. Charitable agencies can pick up food at an FDP rather than traveling to the food bank warehouse. A distinguishing feature of an FDP is that it is a site where an existing collection also occurs (i.e., collection and delivery sites are co-located). Fig. 1 illustrates the current distribution process and the FDP-based distribution process.

The focus of this work is to determine which locations to use as FDPs, as well as how to schedule both collection from food donors and deliveries to charitable agencies via the selected FDPs. Specifically, the questions this research seeks to address are:

Research Question 1. *What benefit can a FDP-based distribution strategy bring over a non-FDP-based distribution strategy?* Benefit is quantified in terms of the improvement in the total distance traveled and total number of tours required. The number of tours is directly related to the number of vehicles and provides a measure of resources expended per distance traveled.

Research Question 2. *How robust are the proposed pickup and delivery schedules with respect to certain distribution policy parameters?* The policy parameters investigated are food delivery duration, available vehicles, driver availability, and collection amount. We hypothesize that driver availability (operating time) and vehicle capacity are the primary policy parameters that affect the optimal schedule. Therefore, we want to characterize the impact of these two parameters on efficiency and access. We would also like to explore the computational requirements of the PVRPB model when the available operating time is varied.

A capacitated set covering problem (CSCP) is used to determine the location of FDPs and assignment of charitable agencies to FDPs. We identify the set of feasible FDPs based on time constraints induced by food spoilage risk and constrain the assignment of charitable agencies based on the capacity of the delivery vehicle. Next, a periodic vehicle routing problem with backhauls is formulated to determine the collection and delivery schedule. The collection and delivery schedule incorporates constraints associated with the available operating time for dispatched vehicles and requirements for weekly collection and delivery. In addition, food safety in the form of safe travel distances as well as restrictions regarding comingling of food for delivery and pick-up are addressed. A numerical study is performed using data from a food bank that motivated this study. This particular food bank covers a rural and urban service area, where some rural agencies' access to food is constrained by travel time and availability of adequate transportation resources. Using the proposed model, we illustrate how a food bank can improve the quantity and access to food within its service area in a cost effective manner through the use of FDPs.

The remainder of this paper is outlined as follows. Section 2 provides a review of related literature. Section 3 describes the problem and solution approach. The experimental design and results are presented in Section 4. Section 5 provides concluding remarks and identifies areas for future research.

2. Related literature

2.1. Vehicle routing

The vehicle routing problem (VRP) has been applied to problems that consider a set of customers requiring service from a set of vehicles over a single period. The key assumptions for this problem are (1) all customers experience demand in the form of a delivery or collection, (2) customers are served by the arrival of exactly one vehicle, (3) all vehicles start and end their tours (routes) at a depot, and (4) the vehicles used to serve customers must have adequate capacity (with respect to some constrained attribute) to satisfy the demand for that attribute specified by the customers served. All of these assumptions except the last are shared with the traveling salesperson problem (TSP), thus when the resource capacity is negligible, this problem can be simplified to a TSP problem. When the location of depots is unknown and the location of the customers is known *a priori*, the VRP generalizes to a location routing problem [5]. The formulations for this problem are similar to the VRP except that the actual location of the depot is a decision variable.

The periodic vehicle routing problem (PVRP) is a generalization of the VRP that relaxes the assumption that all customers are visited daily [6]. Instead, this problem identifies the period in which customers are visited in addition to developing a set of feasible tours. Variations of the PVRP include instances where customers can only be served within specified time windows (PVRPTW) [7,8]; vehicles utilized can either originate at more than one depot (MDPVRP) or deposit commodities at intermediate deposit locations [9], and instances where greater emphasis is placed on employee workload factors (PVRP-SC) [10,11]. The reader is referred to Francis et al. [12] for a comprehensive review of recent VRP research.

Another generalization of the VRP that is similar to the research presented in this paper is the vehicle routing problem with backhauls (VRPB). This generalization is applicable when there are two sets of customers in a network. The first set of customers is served through linehauls or the delivery of commodities from the depot. The second set of customers is served through backhauls or the collection of commodities received from customers for their eventual return to the depot. The model formulation is based on the following assumptions: (1) vehicles originate their tour at the depot; (2) each linehaul (or backhaul) customer is serviced by only one vehicle; (3) vehicles assigned to both linehaul and backhaul customers must serve all linehaul customers before servicing backhaul customers; (4) vehicle capacity when servicing linehaul (or backhaul) customers is not exceeded. Other related extension of the VRPB includes problems where there are time windows in which customers can be serviced [13–16]; problems where routes are asynchronous or the travel time associated with traversing an arc is time dependent [17]; and multiple depots or vehicle types are utilized [18].

2.2. Related transportation scheduling problems for charitable food assistance

Prior research addressing logistical challenges experienced by charitable food assistance programs considers the dispatch of pre-cooked and uncooked food items. The earliest work in the area of pre-cooked meal delivery addresses route construction for a charitable agency implementing a Meals-on-Wheels program [19]. In this problem, there is a limited workforce available to deliver meals and the client list changes frequently. As a result, the approach presented is able to not only determine the routes assigned to each driver but also accommodate the dynamic nature of the delivery network. Since food is prepared and delivered by an outside caterer, the collection of donated food does not need to be addressed. Yildiz et al. [20] also address the delivery of pre-cooked meals. However, their problem considers the case where several kitchens are used to prepare meals. As a result, the location of these kitchens, as well as the routing of meals to customers must be determined. It is assumed that each potential kitchen can serve a limited number of customers and each route is served through only one kitchen.

Location and routing in the context of uncooked food delivery is similarly considered by Solak et al. [21]. However, the locations that need to be determined are food delivery sites where agencies will pick up their food. As a result an approach for determining the location of the delivery sites, the allocation of agencies to the sites, and the resulting vehicle routes that visit each site is presented.

Their problem is also motivated by conversations with a food bank but does not address the collection of donated food.

Phillips et al. [22] consider the collection of food donated from food rescue programs. A simulation–optimization model is presented that incorporates uncertainty in daily food supply, and uncertainty in daily demand to generate a pickup schedule as a function of the available supply. A single period linear programming model is used to generate the pickup schedule. The LP model is solved iteratively based on the available supply at each donor on a given day. The model determines the cost of picking up food, the number of days demand can be met, and the impact on demand fulfillment as a result of adding more donors. Lien et al. [23] similarly focus on the delivery of scarce supply obtained through food recovery programs. However, the objective is to allocate the supply given uncertainty in food demand by the agencies, rather than the generation of transportation schedules. A dynamic approach is presented to allocate supplies given the sequence in which the charitable agencies are visited. The optimal allocation schedule promotes equity in service by maximizing the proportion of demand satisfied.

2.3. Research contribution

The problem presented in this paper is most similar to the VRPB. First, there are two distinct customer classes requiring service; customers from whom food is collected (donors) or backhauls; customers to whom food is delivered or linehauls. In our problem, FDPs serve as delivery nodes rather than each individual charitable agency. Second, due to food safety constraints, linehauls must occur before backhauls. Furthermore, in our model linehaul and backhaul customers are co-located, backhaul customers require more frequent service than linehaul customers and we consider both the routing and scheduling problem simultaneously. We identify an optimal schedule over a finite planning horizon (weekly schedule) rather than a single planning period because the collection and delivery frequencies are not identical. In particular, our problem addresses the case where there are multiple collection times during a week while deliveries only occur once due to equity considerations. To the best of our knowledge the current literature does not address the challenges associated with adequately serving both linehaul and backhaul customers over a multi-period time horizon without violating the requirement that commodities associated with the two customer types remain segregated.

The related research in charitable food assistance has primarily focused on single period problems considering various context driven constraints (e.g. pre-cooked, highly perishable food items). Our problem extends the other charitable food assistance transportation problems in the literature in several ways. We model collections as well as deliveries, which were not directly considered in Refs. [21,23]. While Phillips et al. [22] address supply collection and demand, delivery to charitable agencies is not directly considered. The problem presented in our study is constrained by the fact that food items must be verified at the food bank before it is distributed, negating the applicability of vehicle routing variations that perform direct transports between customers providing supply and those that request supply (e.g., the pickup and delivery problem). In addition, food quality concerns are also addressed in the assignment of agencies to an FDP. Food spoilage in the form of travel time impacts the selection of delivery sites as well as the agencies that can be assigned there.

It is important to note that this research addresses instances where customer service is not limited to a specified time interval. Time windows require that transactions between the customer and the arriving vehicle only occur within pre-determined time frames. While time windows may be useful in some practical applications, many realistic routing problems do not have these additional model constraints. Tarantilis et al. [24] highlight many of the applications of single-period vehicle routing problems without time constraints. Furthermore, of the papers reviewed in their comprehensive review of the periodic vehicle routing problems (PVRP), Francis et al. [12] identify that only a fraction of the publications consider time windows.

3. Mathematical formulation

3.1. Assumptions and notation

We consider a single warehouse that stores donated food. Food donations come from a fixed set of locations called collection sites. The set of all collection sites is denoted by N. Each collection site $i \in N$ donates Q_i pounds of food. The number of required collections per week (R_i) is fixed and known in advance. Furthermore to reflect food availability, the number of consecutive site collections is limited to β days. For example, β equal to 2 implies that collections can occur Monday, Tuesday, and Thursday but not Monday, Tuesday, and Wednesday.

Collection sites can also serve as food delivery points (FDPs) where charitable agencies served by the food bank can receive food. If a collection site serves as an FDP, it can only satisfy the demand of a charitable agency if the one way travel time between locations does not exceed $T_{\max}$. This time restriction reflects the maximum amount of time unrefrigerated items remain safe for consumption.

The set of charitable agencies is denoted by M. We assume each charitable agency $m \in M$, is assigned to one FDP and is allowed exactly one delivery per week of q_m pounds. All agencies assigned to an FDP receive deliveries on the same day at the FDP site. If a delivery is scheduled for a set of agencies served by an FDP, it must be the first stop on the route. This is to ensure the products are not comingled as the food being delivered has already been through a quality control process, while the food that is being collected has not.

Each day, vehicle travel is constrained to an operating window ($W_{\max}$) associated with the standard operator driving time. When a delivery is scheduled for a particular FDP site, trucks must remain for a period of time that is sufficient to complete both FDP and food collection activities. The time required to complete FDP-related activities at site j is denoted as T_j^{D}. This reflects the fact that delivery activities can vary by site based on the number of charitable agencies being served and the poundage of food to be delivered. The parameter, T^{C}, denotes the time to complete collection-related activities at a site. It should be noted that the time spent at a collection site is longer if both a collection and a delivery are scheduled at the site on a particular day.

The objective is to determine which collection sites should also serve as FDPs and to determine a delivery and collection vehicle schedule over a finite planning horizon of $P_{\max}$ days.

3.2. Solution approach

The proposed solution approach consists of two phases. The first phase determines the set of active FDPs and associated rural agency assignments using a capacitated set covering problem (CSCP). The set covering model incorporates capacity constraints imposed by the delivery vehicle as well as covering constraints based on food spoilage. The second phase develops vehicle schedules for each workday. These schedules identify both the period when a predetermined site is used as an FDP, as well as the sequence and time to visit each of the collection sites.

The decision to decouple the location/allocation/routing problem is motivated by several features of the problem. First, the phased approach provides a computationally efficient way to address large scale network delivery problems. In particular, the problem motivating this study has 126 delivery locations. Second, agencies are responsible for collecting food donations from specified locations therefore direct delivery to agencies is not a part of this problem. Deliveries to charitable agencies are achieved through assignment to an FDP. Since all collection sites will be visited, and a subset of collection sites will serve as FDPs, it is sufficient to decouple the agency assignment/delivery location determination decision from the routing/scheduling decision. Lastly, since the agency–FDP matching solution obtained from the first phase is independent from a specific schedule, this gives us the flexibility to solve the second phase for a variety of distribution policy parameters (e.g., drop off time, number of available vehicles) without having to change the agency–FDP assignments. A flow chart of the solution approach is shown in Fig. 2.

3.2.1. Phase 1: FDP selection

Given a set of collection sites (N) which represent potential FDPs, and a set of charitable agencies (M), we seek an optimal selection of sites as FDPs and allocation of charitable agencies to FDP sites. The FDP selection and agency assignment are formulated as a capacitated set covering problem. Capacity constraints are introduced to reflect the maximum amount of food that can be carried by the delivery vehicle. The mathematical model is as follows.

Index sets:

N, the set of collection sites $N = \{1, \ldots, n\}$.
M, the set of all charitable agencies $M = \{1, \ldots, m\}$.

Decision variables:

$a_{mi} \in \{0, 1\}$, where 1 indicates agency $m \in M$ is assigned to site $i \in N$.
$f_i \in \{0, 1\}$, where 1 indicates site $i \in N$ is selected as an FDP.

Parameters:

q_m, delivery quantity
t_{mi}, travel time to site i for agency m.
$\gamma_{mi} \in \{0, 1\}$, where 1 indicates $t_{mi} \le T_{\max}$
K, delivery capacity of the truck.

$$\min \sum_{i \in N} f_i \tag{1}$$

Subject to

$$\sum_{i \in N} \gamma_{mi} a_{mi} = 1 \quad \forall m \in M \tag{2}$$

$$\sum_{m \in M} q_m \gamma_{mi} a_{mi} \le K f_i \quad \forall i \in N \tag{3}$$

$$a_{mi} \in \{0, 1\} \quad \forall m \in M, \ i \in N \tag{4}$$

$$f_i \in \{0, 1\} \quad \forall i \in N \tag{5}$$

The objective function (1) minimizes the number of activated FDPs. Constraints (2) ensure that each charitable agency is served by exactly one FDP. The binary covering matrix Γ has entries $\{\gamma_{mi}\}$ set to 1 if agency m is covered by FDP location i. Here an agency is considered covered if the one-way travel time to the FDP location does not exceed $T_{\max}$. Constraints (3) ensure the delivery quantity (q_m) associated with all agencies served by an FDP does not exceed the delivery capacity (K) of the truck. Constraints (4) and (5) represent the binary constraints on decision variables indicating the assignment of an agency m to an FDP i (a_{mi}) and the selection of a site i as an FDP (f_i).

The set covering problem is known to be NP-hard [25], though in practice straightforward to solve to optimality. It should also be noted that the set covering problem can produce multiple optimal solutions. However, in the context of our problem, we define the covering matrix by assigning agencies to potential FDP sites based on the agency's ability to pick up the food and return to their location without the food spoiling. So, while several possible pickup locations may exist, the set covering model ensures the minimum number of sites are chosen such that assignment of agencies to locations does not violate the capacity of the truck or the travel time.

3.2.2. Phase 2: periodic vehicle pick-up and delivery model with backhauls (PVRPB)

Based on the optimal solution ($\boldsymbol{f}^*, \boldsymbol{A}^*$) obtained from the set covering model, we define the set of all collection sites selected as FDPs as $N_{\text{fdp}} = \{i \in N | f_i^* = 1\}$. Given the set of all sites (N), we develop a PVRPB formulation to determine the optimal collection and delivery schedule over a finite planning horizon. The model presented is an extension of the Miller–Tucker–Zemlin [26] formulation of the CVRP which incorporates lifted subtour elimination constraints of Kara et al. [27]. A similar formulation appears in Schrage [28]. However, we include the following extensions: (i) a

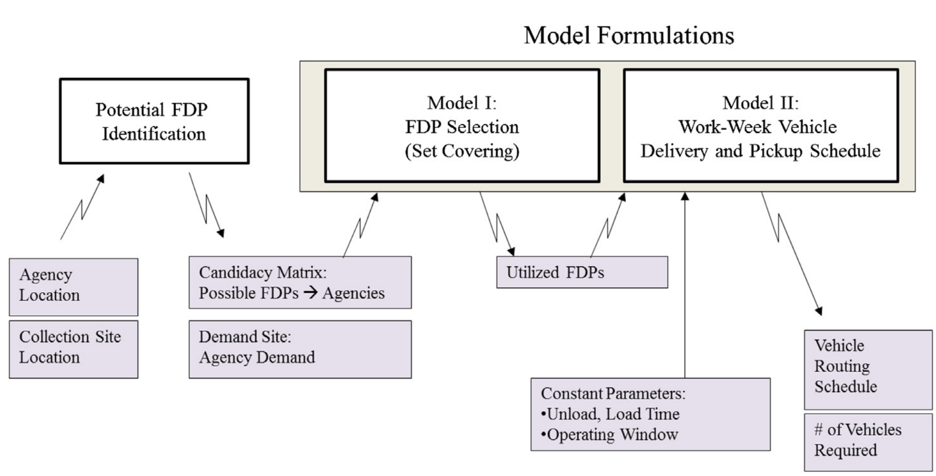

Fig. 2. Two-phased methodology for FDP identification and selection and delivery-pickup schedule.

time index (p) to reflect the multi-period nature of the problem; (ii) the calculation of the accumulated departure time as well as load since we must ensure that the travel time of a tour is within the operating window of the driver; and (iii) new decision variables and associated constraints to represent the scheduling of visits to FDP and non-FDP sites. The following notation is used to model and formulate the PVRPB.

Index sets:

N, the set of collection sites $N = \{1,\ldots,n\}$ with the depot denoted as $N_0 = \{0\}$.
N_{fdp}, the set of all collection sites that also serve as food delivery points. $N_{\text{fdp}} \subset N$.
P, the set of operator workdays; $p = \{1,\ldots,P_{\max}\}$.

Decision variables:

$x_{ijp} \in \{0,1\}$, where 1 indicates vehicle travel on day $p \in P$ between sites i and j where $i,j \in N \cup N_0$.
$y_{ip} \in \{0,1\}$, where 1 indicates site $i \in N$ is scheduled for collections on day $p \in P$.
$z_{ip} \in \{0,1\}$, where 1 indicates site $i \in N_{\text{fdp}}$ is scheduled for delivery on day $p \in P$
u_{ip}, accumulated departure time on day $p \in P$ given site $i \in N \cup N_0$ is visited.
s_{ip}, accumulated departure load on day $p \in P$ given site $i \in N \cup N_0$ is visited.

Parameters:

$W_{\max}$, maximum operating time per period per driver.
Q_i, amount collected at site $i \in N$.
β, the maximum number of consecutive periods a site can be visited for collections.
δ, the maximum number of FDP visits per period.
V, the number of vehicles available per period.
R_i, required number of collections at site $i \in N$ during the planning horizon.
d_{ij}, travel distance between sites i and j; $i,j \in N \cup N_0$.
t_{ij}, travel time (inclusive of collection time at site j) between sites i and j;$i,j \in N \cup N_0$.
T_j^{D}, service time allotted for agency pickup at FDP j;$j \in N_{\text{fdp}}$.
T^{C}, service time allotted for collection.
K, delivery capacity of the truck.

Model formulation:

$$\min Z = \sum_{p \in P} \left[\sum_{i \in N \cup N_0} \sum_{j \neq i} d_{ij} x_{ijp} \right] \tag{6}$$

Subject to

$$\sum_{j \in N - \{i\}} x_{ijp} = y_{ip} \quad \forall i \in N; \ \ p \in P \tag{7}$$

$$\sum_{j \in N - \{i\}} x_{jip} = y_{ip} \quad \forall i \in N; \ \ p \in P \tag{8}$$

$$\sum_{j \in N} x_{0jp} \leq V \quad \forall p \in P \tag{9}$$

$$\sum_{p \in P} y_{ip} = R_i \quad \forall i \in N \tag{10}$$

$$\sum_{p = p'}^{\beta + p'} y_{ip} \leq \beta \quad \forall i \in N; \ \ p' \in \{1 \ldots P_{\max} - \beta\} \tag{11}$$

$$\sum_{j \in N_{\text{fdp}}} z_{jp} \leq \delta \quad \forall p \in P \tag{12}$$

$$\sum_{p \in P} z_{jp} = 1 \quad \forall j \in N_{\text{fdp}} \tag{13}$$

$$x_{0jp} \geq z_{jp} \quad \forall j \in N_{\text{fdp}}, \ \ p \in P \tag{14}$$

$$u_{ip} - u_{jp} + W_{\max} x_{ijp} + (W_{\max} - t_{ij} - t_{ji}) x_{jip} \leq W_{\max} - t_{ij} \quad \forall i,j \in N, \ \ p \in P \tag{15}$$

$$u_{jp} \geq t_{0j} x_{0jp} + T_j^{\text{D}} z_{jp} \quad \forall j \in N_{\text{fdp}}, \ \ p \in P \tag{16}$$

$$u_{jp} \leq W_{\max} - (W_{\max} - t_{0j}) x_{0jp} + T_j^{\text{D}} z_{jp} \quad \forall j \in N_{\text{fdp}}, \ \ p \in P \tag{17}$$

$$u_{jp} \leq W_{\max} - \left(t_{j0} - T^{\text{C}} \right) x_{j0p} \quad \forall j \in N, \ \ p \in P \tag{18}$$

$$s_{ip} - s_{jp} + K x_{ijp} + (K - Q_i - Q_j) x_{jip} \leq K - Q_i \quad \forall i,j \in N, \ \ p \in P \tag{19}$$

$$s_{jp} \leq K - (K - Q_j) x_{0jp} \quad \forall j \in N, \ \ p \in P \tag{20}$$

$$s_{jp} \geq Q_j + \sum_{i \in N - \{j\}} Q_i x_{ijp} \quad \forall i \in N; \ \ p \in P \tag{21}$$

$$x_{iip} = 0 \quad \forall i,j \in N \cup N_0; \ \ i \neq j; \ \ p \in P \tag{22}$$

$$x_{ijp} \in \{0,1\} \quad \forall i \in N; \ \ p \in P \tag{23}$$

$$y_{ip} \in \{0,1\} \quad \forall i \in N; \ \ p \in P \tag{24}$$

$$z_{ip} \in \{0,1\} \quad \forall i \in N_{\text{fdp}}; \ \ p \in P \tag{25}$$

$$u_{ip} \geq 0 \quad \forall i \in N \cup N_0; \ \ p \in P \tag{26}$$

$$s_{ip} \geq Q_i y_{i,p} \quad \forall i \in N \cup N_0; \ \ p \in P \tag{27}$$

$$s_{ip} \leq K \quad \forall i \in N \cup N_0; \ \ p \in P \tag{28}$$

Equation (6) minimizes the total distance traveled over the planning horizon. Distance minimization has been chosen as it is more amenable to quantify costs (e.g., fuel, maintenance) in terms of distance traveled. If site i is scheduled for collection, constraints (7) and (8) ensure that a vehicle must enter and exit that site. Constraints (9) ensure the number of tours scheduled per day (reflected by initiation of the route at the depot) does not exceed the number of available vehicles (V) at the depot. Constraints (10) define the required number of collections for a site during the time horizon. Constraints (11) represent limits on food availability at the collection site. In particular, the number of successively scheduled visits at a collection site cannot exceed β. This reflects the situation where it may take time for donated food to accumulate for pickup.

Constraints (12) restrict the maximum number of FDP sites that can be visited in a day, which reflect food and resource availability constraints at the food bank. Constraints (13) ensure charitable agencies receive scheduled deliveries once during the time horizon through their assigned FDP. The restriction on delivery frequency ensures that food bank resources are used efficiently while also providing routine service. Constraints (14) ensure that FDP stops are a) performed on the first leg of a vehicle tour, and b) consistent with the selected food delivery day. This constraint enforces the quality control aspect of food delivery and collection by ensuring that the delivered food (which has been inspected) and collected food (which has not been inspected) are not co-mingled.

Constraints (15) are subtour elimination constraints. They are the time based equivalent of the vehicle capacity subtour elimination constraints in Desrochers and Laporte [29]. For example, if site j follows site i, then the total time associated with this tour after collection at site j is the total time from the depot to site i (u_{ip}) plus the travel time from site i to site j (t_{ij}). Constraints (16) and (17) are first stop constraints and ensure that the accumulated tour time at site j (u_{jp}) reflects collection as well as delivery time (T_j^{D}) if an FDP visit is scheduled ($z_{jp} = 1$). This is an extension of the first stop constraints formulated previously in Schrage [28]. Constraints (18) are last stop constraints. In our model, the travel time is inclusive of the collection time, since every stop on a tour requires collection. As a result, the collection time (T^{C}) must be subtracted from the travel time since there is no collection upon returning to the depot. Constraints (19)–(21) are the capacity based equivalent of constraints (15)–(18). Constraints (22) restrict the possible of a vehicle cycling. Constraints (23)–(28) are bounds on the decision variables.

It should be noted, vehicle routing problems are based on the traveling salesman problem, an NP-complete problem where a single vehicle visits each node (i.e. customers) exactly once [30]. Vehicle routing problems are much more difficult to solve as they involve multiple salesmen (i.e. vehicles), each of which has added restrictions on tow capacity and/or maximum tour duration. As such, obtaining a feasible solution to these problems becomes increasingly more difficult as the number of decision variables increases (see e.g. Francis et al. [12] and Laporte [31]). The PVRPB presents an additional dimension of complexity to the routing problem when compared to either the PVRP or VRPB.

4. Case study

4.1. Background

To illustrate the benefits that can be achieved with FDP-based delivery in predominantly rural areas, we construct a numerical study based on our discussion with the Second Harvest Food Bank of Northwest North Carolina (NWNCFB). North Carolina has the fifth highest rate of food insecurity in the nation [32]. NWNCFB is a one of seven food banks serving the food insecure population in North Carolina and is a member of Feeding America. Feeding America is a national hunger relief organization that secures surplus food from manufacturers, retailers, and the government. The donated food is subsequently distributed through members in their food bank network.

NWNCFB distributes food in an 18-county service area consisting of over 400 charitable agencies located in rural and urban counties in western North Carolina. Of the seven Feeding America food banks in the state, it is the third largest in terms of counties served and second largest in terms of emergency food assistance programs supported (www.feedingamerica.org). While many of the charitable agencies served by this particular food bank are in close proximity to the warehouse, more than 100 of the agencies reside in 13 remote counties. Few, if any, of these agencies are able to travel to the Food Bank with refrigerated transport to pick up food safely. The Food Bank offers fee-based transportation services to ensure remote agencies have access to food. However, at the time of this study, the frequency of the transportation services was constrained by the limited vehicle fleet and the significant distance/time required for traveling to some of the remote counties located in the mountainous region of the state. As a result, these rural agencies received food deliveries only every other week.

A new agreement with a national supermarket chain presents an opportunity for the Food Bank to offer increased delivery options to rural agencies. The supermarket chain will provide high-volume food items at each of their franchise locations multiple times per week. Franchises of this sponsor are in close proximity to many of the rural agencies and can therefore serve as satellite delivery locations. Our goal is to identify which franchise locations should serve as FDPs and construct an efficient schedule which increases rural agency access and hence equity while performing the required pickups. We hypothesize that the distribution of and access to donated food can be improved by using a subset of the franchise collection sites as satellite delivery locations.

4.2. Experimental design

The service area consists of 28 collection sites (in rural and urban areas) and 123 rural charitable agencies. In order to understand the size of the network, a histogram of the one-way travel time between the Food Bank and the charitable agencies is presented in Fig. 3. Over 90% of the agencies are more than 1 h from the Food Bank. Similarly, 11 of the 28 collection sites are 50 min or more from the Food Bank (Table 1).

Table 2 summarizes the input parameters for the CSCP and PVRPB models with the corresponding range of values evaluated. For the CSCP, the input parameters are the weekly delivery quantities for each agency in pounds, the vehicle capacity, and the assignment of agencies to potential FDP sites via the covering matrix. The entries in the covering matrix are determined by the exogenous model variables: (i) travel time between the charitable agency and the FDP sites; and (ii) the maximum one-way travel time before food spoilage ($T_{\max}$). If a collection site serves as an FDP, it can only satisfy the demand of a charitable agency if the one way travel time is no more than 40 min. The delivery quantity to an FDP is based on historical demand of the agencies to be served which is estimated by the distribution volume.

For the PVRPB model, the vehicle capacity in pounds (K), delivery duration (in minutes) at FDP site j (T_j^{D}), collection time (T^{C}),

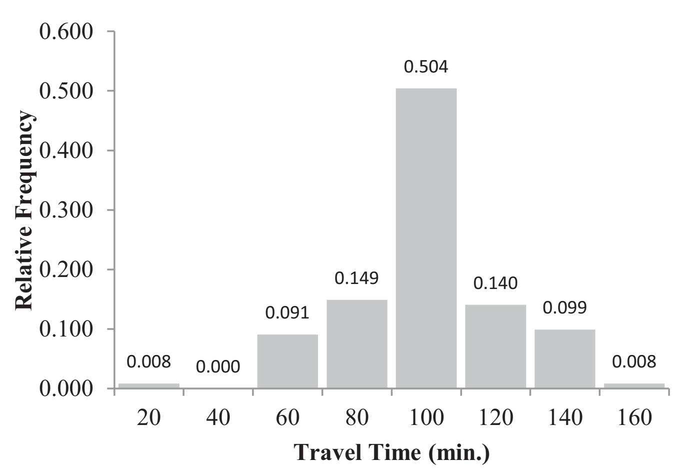

Fig. 3. Frequency distribution of one-way travel time (min) from charitable agencies to the food bank warehouse.

Table 1
Location of food collection sites and the food bank.

Site #	Location	Distance (mi.) from NWNCFB	One way travel time (min)
0	NWNCFB	–	–
1	Mayodan	37	53
2	Mocksville	26	29
3	Mount Airy	42	45
4	Winston-Salem	8	15
5	Winston-Salem	12	18
6	Winston-Salem	7	15
7	Asheboro	51	52
8	Burlington	43	44
9	Burlington	50	56
10	High Point	21	24
11	High Point	13	16
12	Kernersville	10	18
13	Eden	63	78
14	Lexington	26	33
15	Mebane	57	57
16	Randleman	39	41
17	Reidsville	55	62
18	Thomasville	17	29
19	Greensboro	32	35
20	Greensboro	29	32
21	Greensboro	20	21
22	Elkin	44	53
23	Statesville	44	45
24	Granite Falls	78	79
25	Lenoir	86	93
26	Taylorsville	64	67
27	West Jefferson	92	99
28	Wilkesboro	58	60

and collection quantity (Q_i) are based on information obtained from the Food Bank. For the base case we assume that (i) FDP-related tasks require 60 min, (ii) food collections require 30 min, and (iii) the collection quantity at each site is 450 pounds. The maximum number of consecutive days during which collections can occur (β) is 2 to allow the collection sites to accumulate food donations.

Our sensitivity analysis explores the effect of the delivery duration, collection quantity, number of available vehicles and operating window on the optimal schedule, FDP assignment and computation time. Initially, we assume delivery durations are 60 min. However, in practice delivery durations may be affected by the number of agencies assigned to an FDP. Two additional delivery durations are defined to reflect this situation. We assume a full truck load (10,000 lbs) corresponds to 60 min of delivery time; or equivalently that approximately 166 pounds of food can be unloaded per minute. Given an unloading rate of 166 pounds per

Table 2
Input parameters.

Parameter	Values [base case]	Units	PVRPB	CSCP
$T_{\max}$	[40]	Minutes		•
K	[10,000]	Pounds	•	•
δ	[3]	Days	•	
β	[2]	Days		
Q_i	[450], 1000, 1250, 1500	Pounds	•	
R_i	[3]	Days	•	
T^C	[30]	Minutes	•	
T_j^D	[60], Variable according delivery amount at site j, Average according to delivery amount over all sites	Minutes	•	
V	[20]	Vehicles	•	
$W_{\max}$	[360], 420, 480, 540, 600, 660, 720	Minutes	•	

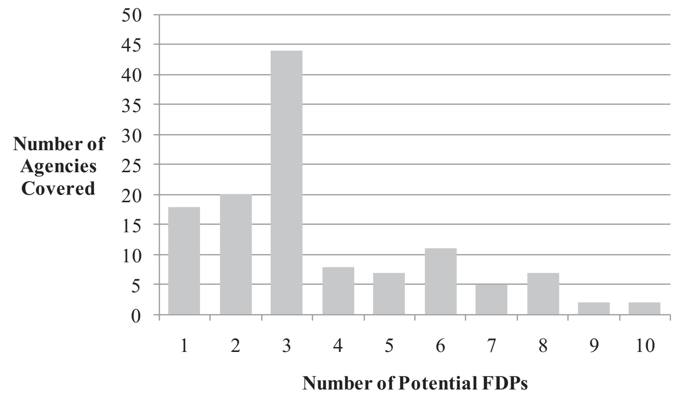

Fig. 4. Distribution of covering matrix.

minute and the optimal assignment of agencies to FDP j determined from the CSCP (a^*_{mj}), equation (29) defines the total delivery duration in terms of the pounds delivered for each agency m (q_m). Equation (29) allows variable delivery durations to be generated at each FDP site by dividing the total poundage of food delivery to FDP j by the unloading rate. Equation (30) defines constant delivery durations based on the average of the variable FDP site durations generated in equation (29). The number of sites selected as FDPs ($\sum_j f^*_j$) is the optimal solution from the CSCP.

$$T_j^{\mathrm{D}} = \frac{\sum_m a^*_{mj} q_m}{166} \tag{29}$$

$$T_j^{\mathrm{D}} = \frac{\sum_j \sum_m a^*_{mj} q_m}{166 \sum_j f^*_j} \quad \forall j \tag{30}$$

The collection amounts Q_i for all collection sites are varied in order to explore the tradeoff between time being the binding constraint versus capacity being the binding constraint in the PVRPB model. We will solve this problem as an unconstrained vehicle routing problem so the number of vehicles will not influence the schedule. (This will enable us to determine the number of trucks required as well.) Therefore for the base case, the number of available vehicles (V) is initially set to 20. This number is a conservative estimate which would allow for a single vehicle to visit each of the collection sites every day. This allows us to understand the fleet size that would satisfy the travel requirements for the optimal schedule. The resulting number of required fleet capacity can be used by the Food Bank as an estimate for generating proposals for potential capacity increase. We vary the number of available vehicles in order to characterize the relationship between constrained vehicle resources and route efficiency. Route efficiency is measured by the total travel distance, the number of vehicles required, and the collection-delivery schedules. Varying both the available vehicles and operator time also allows us to compare the benefit gained and cost incurred from either adding a new vehicle to the fleet or requiring overtime for the operators.

Both the CSCP and PVRP are solved using GAMS/CPLEX on a 64-bit Intel Core2 Quad 2.40 GHz CPU with 8 MB cache. The CSCP is solved to optimality. The PVRP is solved using the GAMS/CPLEX mixed integer linear programming solver with relative termination tolerance set to 0.05. In addition, we allow a maximum processing time of 2400 s.

4.3. Phase 1 results: agency allocation and FDP selection

Fig. 4 summarizes the number of agencies covered by one or more potential FDPs corresponding to the entries in the covering matrix for CSCP. There are eighteen agencies covered by exactly one site. Four of the 18 agencies were found to be beyond the desired travel time threshold (Table 3). Based on a recommendation from the Food Bank, these agencies were assigned to the closest collection site. It should be noted that the travel time to the closest site for these four agencies is no more than 8 min longer than the desired travel time. Fourteen of the remaining 18 agencies are covered by one of four possible FDP sites: Reidsville, West Jefferson, Lenoir and Wilkseboro. It should be noted that there are a total of 123 agencies plus one dummy agency. One agency had an average weekly demand that exceeded the capacity of a single truck; therefore, a dummy node was introduced so that the demand could be split between the two nodes. This agency was located in Rockingham County and the agency was assigned to the FDPs [13,17] both of which are in Rockingham County.

The results from the first phase of the solution approach (capacitated set covering problem) are displayed in Table 4. The collection sites activated as FDPs along with the delivery volume and the number of rural agencies served are presented. As suggested by Fig. 4, there are multiple agencies that can be satisfied by more than one potential FDP. Note, that several FDPs could accommodate a larger delivery volume as the capacity of the truck is not reached. In addition, the site that has one agency assigned to it corresponds to the agency that was split into two nodes due to the distribution volume. Effectively this agency is served by two FDPs and receives deliveries twice a week. Fig. 5 depicts the FDP sites relative to their geographic service area (i.e., those sites that act as both linehaul and backhaul customers) and the non-FDP sites (i.e., those sites that are only backhaul customers).

4.4. Phase 2 results: collection and FDP delivery schedule

The solution for the PVRPB problem including the total number of tours per day and the total travel distance is summarized in Table 5. The number of tours is directly correlated with the number of vehicles. Since each tour is assigned to one vehicle, the number of tours indicates the number of vehicles required for food collection and deliveries. The pounds processed per mile are determined by dividing the total pounds (delivered and collected) by the distance traveled in miles. The results illustrate that travel distance, along with the number of vehicles required decreases as the operating window increases. This behavior is expected since allowing more time in a day allows more collections to occur with fewer vehicles. This increase in efficiency with increasing operating windows can also be seen from the pounds processed per mile (PPM), since fewer miles are traveled to perform collection and delivery tasks. However, the decrease in vehicle usage can come at a price. Namely, the Food Bank may incur additional expense associated with drivers covering a route which exceeds their contracted work schedule. Although we do not incorporate overtime costs directly into our model, labor costs are typically proportional to labor time. In our model, labor time is directly accounted for by the tour duration constraints (i.e. operating time).

Table 3
Agencies exceeding travel time threshold.

Agency number	Agency location	Nearest collection site/id	Weekly volume (lbs)	Travel time (min)
1	Laurel Strings	West Jefferson/28	28	48
2	Boone	West Jefferson/28	550	45
3	Sugar Grove	West Jefferson/28	152	42
4	Milton	Reidsville/17	160	48

Table 4
Selected FDP sites.

Site #	Poundage distributed (weekly)	# of assigned agencies
3	5354	3
13	5884	1
15	7466	19
17	8741	9
19	7981	18
22	8562	17
24	9297	11
25	8599	5
26	9985	19
27	9955	13
28	9630	9
Grand total	91,453	124

The results in Table 5 provide intuition regarding the tradeoff between the available capacity and total travel distance. We see that six trucks are required to support the six and 7 h time windows, whereas five trucks are required for the 8 h time window and four trucks are sufficient for time windows between nine to 12 h. Hence, increasing the operating window from 6 h to 9 h results in a reduction of two trucks in the required fleet size and also an approximately 19% improvement on the total travel distance. Therefore, by increasing the operating window by 3 h, a significant improvement in both the total travel distance and the fleet capacity is achieved. However, if we increase the operating window beyond 9 h, to a 12 h schedule, we only achieve an improvement of approximately 2% as compared to the 9 h schedule. More significant improvement can be achieved by increasing the operating window from six to 9 h. Increasing the operating window beyond 9 h yields limited improvement while significantly increasing the driver workday.

Table 5 also shows the corresponding optimality gaps for the different solutions. We see that the optimality gap increases as the operating window decreases. The PVRPB model is NP-hard and has high complexity with 9268 constraints and 4686 decision variables of which 4400 are binary decision variables. Furthermore, the operating window adds an additional level of computational burden such that if the allowed window is too tight, the particular problem instance becomes more difficult to solve, i.e., the gap between the value of the best feasible solution and the dual lower bound remains too large even after a long run time [33]. This happens due to time being the binding constraint. Fig. 6 illustrates the change in model run time for the range of operating windows (W_{max}) with an optimality gap of 5%. The run times decrease exponentially as the operating window increases, ranging from 108,661.42 s for $W_{max} = 9$ h, 506.3 s for $W_{max} = 10$ h, 348.77 s for $W_{max} = 11$ h, to 25.41 s for $W_{max} = 12$ h. It becomes increasingly more difficult to satisfy all the delivery and collection requirements in such short time windows. However, if one would be willing to sacrifice solution quality, feasible solutions can be obtained for very tight operating windows with large optimality gaps.

The best vehicle schedules obtained from the model for all operating windows are summarized in Table 6. As shown in Table 6, it is interesting to note that schedules are either front-loaded (i.e., Monday–Tuesday, refer to the seven and 9 h operating windows), back-loaded (i.e., Thursday–Friday, refer to the six and 8 h operating windows), or both (refer to the 10, 11 and 12 h operating windows) with the fewest deliveries occurring during the middle of the week. This is reasonable given the constraint that food donations are not available three consecutive days in a row at a collection site.

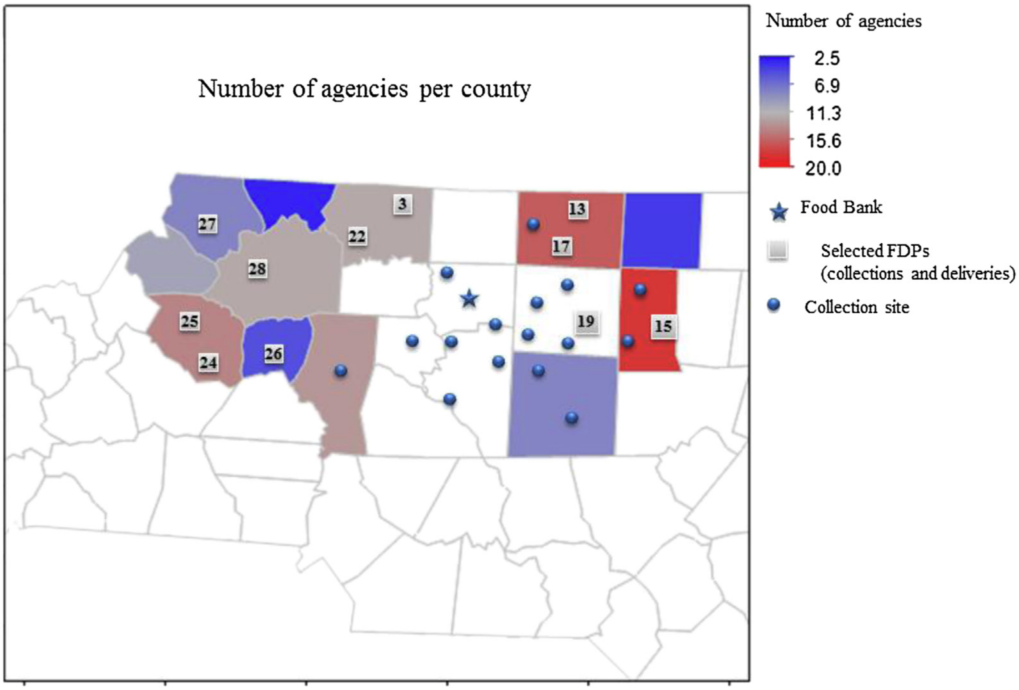

Fig. 5. Map of FDP and non-FDP sites. Rural agencies are located in the shaded regions.

Table 5
Total number of tours per day by day of week.

Operating window (h)	Weekday					Total distance traveled (mi.)	Route efficiency (lbs/mi.)	Minimum optimality gap
	Monday	Tuesday	Wednesday	Thursday	Friday			
6	6	2	2	6	5	2841	36.63	24%
7	4	4	2	2	6	2612	39.84	18%
8	5	1	4	4	2	2399	43.37	10%
9	3	3	1	2	4	2304	45.16	5%
10	4	3	0	3	4	2302	45.20	5%
11	3	3	1	2	4	2287	45.50	5%
12	4	3	0	3	2	2258	46.08	5%

The impact of clustering and the extreme points in the network is highlighted by the distance traveled in the schedule, as shown in parentheses in Table 6. For example, the tour associated with the 7 h schedule on Tuesday (Tour 2) has 6 collection points covering 104 miles. Whereas, on Wednesday, Tour 1 only visits three sites covering 236 miles. A similar pattern is exhibited for the 9 h schedule. Table 6 along with Fig. 5 demonstrate that the model attempts to cluster the extreme points where appropriate given the

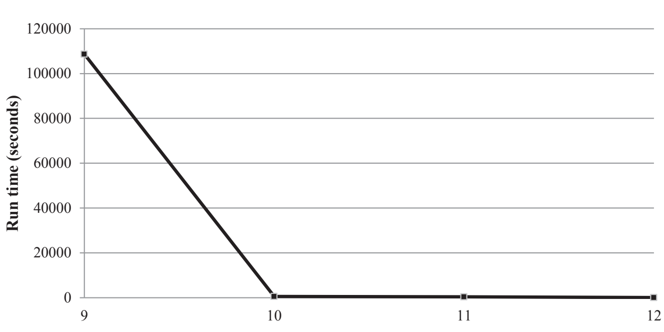

Fig. 6. Computational complexity of the PVRPB model.

constraints on operator workday. The time constraints combined with the collection and delivery frequency can lead to unbalanced schedules. A schedule is unbalanced if there is a significant variability in the number of tours per day. Note that 11 h operating window in Table 6 has the smallest variability, and hence is the most balanced schedule. As a result, there are some schedules that have no deliveries on Wednesday (10 and 12 h schedules) and the number of sites visited varies significantly across tours.

It is important to note that one of the goals of these schedules is to improve service to rural agencies. Route efficiency is a significant challenge faced by the Food Bank. Direct delivery to all agencies is infeasible given the number of agencies and the Food Bank's vehicle capacity and resource constraints. In addition, the current weekly demand and proximity of the agencies to the main warehouse would preclude many of the agencies from receiving food every week. Using co-located sites increases rural agency access with minimal investment. It would be cost prohibitive for rural agencies to provide their own refrigerated transportation since many are local shelters or food pantries. In addition, the idea of co-location also allows for the possibility of rural agencies assigned to a specific drop-off point to pool resources and invest in a single truck to provide local delivery if desired. Thus the co-located site acts as an informal cross docking facility.

Table 6
Total number of collection sites visited per tour (distance traveled) by operating window and weekday.

Operating window (h)	Weekday	T1	T2	T3	T4	T5	T6	FDP visits per day
6	Monday	3 (131)[a]	4 (128)	5 (98)	4 (75)[a]	4 (138)[a]	4 (194)	3
	Tuesday	5 (131)	3 (181)[a]					1
	Wednesday	6 (134)	2 (185)[a]					1
	Thursday	2 (84)[a]	6 (104)	4 (124)[a]	6 (116)	4 (181)	2 (185)[a]	3
	Friday	6 (104)	4 (114)[a]	3 (113)[a]	3 (181)[a]	4 (140)		3
7	Monday	7 (115)	4 (114)[a]	5 (183)[a]	4 (204)[a]			3
	Tuesday	2 (84)[a]	6 (104)	6 (134)[a]	5 (142)			3
	Wednesday	3 (236)	4 (160)[a]					1
	Thursday	6 (164)	4 (75)[a]					1
	Friday	6 (104)	4 (132)[a]	6 (134)	3 (113)[a]	6 (185)	3 (229)[a]	3
8	Monday	3 (64)	7 (165)[a]	6 (116)	6 (187)[a]	5 (219)[a]		3
	Tuesday	7 (130)						0
	Wednesday	2 (23)	5 (147)[a]	5 (198)[a]	5 (229)[a]			3
	Thursday	2 (84)[a]	8 (183)	4 (194)[a]	6 (190)[a]			3
	Friday	5 (130)[a]	8 (140)[a]					2
9	Monday	8 (187)[a]	7 (176)[a]	6 (228)[a]				3
	Tuesday	4 (132)[a]	7 (118)[a]	7 (194)[a]				3
	Wednesday	6 (240)[a]						1
	Thursday	10 (173)	5 (229)[a]					1
	Friday	4 (114)[a]	10 (191)	3 (113)[a]	7 (209)[a]			3
10	Monday	4 (114)[a]	11 (195)	7 (209)[a]	5 (229)[a]			3
	Tuesday	2 (84)[a]	5 (156)[a]	7 (255)[a]				3
	Wednesday							0
	Thursday	9 (178)	6 (123)[a]	7 (263)[a]				2
	Friday	6 (104)	7 (137)[a]	3 (113)[a]	5 (142)[a]			3
11	Monday	9 (185)	6 (205)[a]	5 (229)[a]				2
	Tuesday	2 (84)[a]	11 (202)[a]	8 (259)[a]				3
	Wednesday	4 (114)[a]						1
	Thursday	8 (151)[a]	3 (95)[a]					2
	Friday	4 (132)[a]	12 (208)	7 (281)[a]	5 (142)[a]			3
12	Monday	6 (104)	10 (209)[a]	3 (113)[a]	9 (266)[a]			3
	Tuesday	10 (226)[a]	9 (214)[a]	5 (229)[a]				3
	Wednesday							0
	Thursday	2 (84)[a]	8 (140)[a]	7 (209)[a]				3
	Friday	8 (201)[a]	7 (263)[a]					2

[a] Denotes first stop in tour is an FDP visit.

4.5. Sensitivity analysis

4.5.1. Policy robustness with respect to delivery durations

In the solutions explained so far, we have assumed the delivery time at the FDP locations to be 60 min independent of the amount delivered. We now explore the effect of this assumption on the optimal solution when delivery times are varied. We considered three cases: the base case ($T_j^D = 60$ min), the variable case using Equation (29) (T_j^D varies by location according to poundage) and the average case using Equation (30) (T_j^D is set according to the average delivery per FDP site, 49.884 min). We found that the impact of the delivery durations is marginal in terms of the total distance, route efficiency and the optimality gap. As expected, the base case results in a worse objective function value than the variable and average cases since the delivery duration for the base case is greater than the delivery durations for the variable and average cases. In terms of resources expended, using variable delivery times requires the same number of trucks overall. So the worst case scenario of delivering fully loaded trucks increases the distance traveled by 29 miles and decreases route efficiency by 1.2%.

4.5.2. Policy robustness with respect to collection quantity

In our analysis, we have observed that time is the binding constraint in all the instances solved so far. Here we explore the tradeoff between time being the binding constraint versus capacity being the binding constraint by varying the collection amounts. In the base case, the collection amount at all locations, Q_i, is set to 450 pounds. We see from Table 6 that, for an operating window of 12 h, the maximum number of collection sites observed has 10 collections in the second tour on Monday and the first tour on Tuesday. This corresponds to a load on the truck of 4500 pounds. Since the capacity of the truck is 10,000 pounds, we can directly see that this schedule is valid for collection amounts, Q_i, up to $10000/10 \cong 1000$ pounds. Table 7 illustrates the number of tours and total distance traveled for an operating window of 12 h when Q_i takes the values of 1000, 1250, and 1500 pounds for an optimality gap of 10%. We see that the total distance traveled increases when the collection amounts increase. This is expected since more tours are required to perform all collections due to the truck's capacity. The computational requirements of the model also increase exponentially as the collection amounts increase since it becomes more difficult to satisfy all the collections with the truck's capacity limit in addition to the time constraint.

4.5.3. Policy robustness with respect to transportation resources

In our analysis, we set the number of available trucks to a large number in order to obtain the required fleet size as an output from the results of the model. In this section, we analyze the robustness of the PVRPB model to changes in the fleet capacity. Table 8 shows the resulting total distance traveled and the required number of trucks for different operating windows for an optimality gap of 5% and a maximum processing time of 2400 s. Notice that due to the

Table 7
Total number of tours per day and total distance traveled by collection amounts.

Collection amounts, Q_i (lbs)	Weekday					Total distance traveled	Run time (s)
	Monday	Tuesday	Wednesday	Thursday	Friday		
1000	3	3	1	2	2	2354	10.53
1250	4	3	2	3	4	2379	15.40
1500	4	5	2	1	4	2388	522.87

Note: the optimality gap is 10%.

structure of the PVRPB model, at least 11 tours per week (corresponding to the number of FDPs to be visited) are required. For this reason, the smallest feasible fleet size is three because any smaller value would result in fewer than 11 tours per week.

Table 8 shows that the results of the PVRPB model are very robust to changes in the fleet capacity. By constraining the number of trucks, we constrain the total number of tours in the week. However, if the resulting number of total tours in a week for the base case solution remains feasible for the constrained case, we can obtain an objective function that is not significantly different by changing the days of some tours. For example, for an operating window of 12 h, in the base case, we need a minimum of four trucks and that results in an objective of 2258 miles. When we constrain the fleet size to three trucks, we get an objective of 2269 miles which is not significantly different. For the base case, in total, we have 12 tours in a week and when we constrain the fleet capacity to three trucks, we again have a total of 12 tours in a week; only the distribution of the tours over the weekdays is different. Furthermore, we notice that in some cases, no feasible solutions could be found in the allowed time. The fleet capacity constraint becomes too constraining in terms of the total number of tours allowed per week and increases the solution time of the problem.

4.6. Quantifying the benefits of FDPs: measuring equity

In the context of food delivery, access translates to servicing *more* of the customer base *more* effectively and reflects the ability to improve service delivery with fewer resources. We discuss two techniques that illustrate how our proposed FDP strategy is a more equitable solution in terms of food access. The first equity improvement is quantified relative to the capacitated set covering problem. The second equity improvement is quantified relative to the PVRPB.

There are several ways in which equity can be measured for location problems. We adopt one of the measures presented in Ref. [34], defined in terms of the deviation between the best and worst off groups. Let E_i represent the effect of the location decision on a group i. In site location problems, a typical measure of the location decision is distance or time-dependent. In our problem, we define the effect on group i as the total travel time from agencies in group i to the closest food delivery point. When no FDPs are used, the closest food delivery point is the food bank warehouse. When FDPs are used, the closest food delivery point is the FDP assigned to the particular agency. A formal description of the equity measure is presented in equations (31) and (32). Agencies located in the same county are assigned to the same group. FDP(m) denotes the pickup location for agency m.

$$\max_i E_i - \min_j E_j \tag{31}$$

where

$$E_i = \sum_{m \in \{\text{Group}i\}} t_{m,\text{FDP}(m)} \tag{32}$$

In the original problem, when all agencies have to travel to the food bank warehouse to obtain food, the deviation between the best and worst groups (equation (31)) is 1325 min. When FDPs are used, the deviation is reduced to 545 min. This implies the level of inequity associated with access to food is reduced.

In order to quantify the improvement in the level of access associated with the proposed routing and scheduling policy (using FDPs), two cases are considered. The first case identifies the number of vehicles required to deliver to rural agencies and pickup from the collection sites by decoupling the pickup and delivery problems. The delivery problem is estimated based on a single period version of the PVRPB with time constraints and no collection constraints. Because there are 124 agencies, estimates of access are based on a clustered approach and therefore agencies are clustered by city resulting in 37 nodes in the delivery network. The collection problem is addressed by the PVRPB model presented in Section 3 without deliveries. By setting the right hand side of constraint (13) to zero, we can use the model to determine collection schedules only. The policy determined by this approach is referred to as the decoupled pickup and delivery problem (DPD).

The second case considers the combined pickup-delivery problem using a larger set of FDP locations. The PVRP model presented in Section 3 is used with a larger delivery network constructed under the assumption that agencies are clustered by FDP location. However, in contrast to our original FDP problem, all nodes are used as FDPs rather than the minimum number selected via the set covering problem. This equates to 28 delivery nodes in the network. The policy determined by this approach is referred to as the combined pickup and delivery problem (CPD). It should be noted that direct to agency delivery was not an element of the problem, however these two cases allow us to benchmark our current solution against alternative classifications of agency access to food.

Table 9 summarizes the total number of tours, the total travel distance, and the standard deviation of the number of tours per day for each policy. The policy determined by the DPD increases the vehicle requirements as this scenario does not take advantage of the potential efficiencies associated with co-locating pickup and delivery sites. In order to satisfy the operator workday constraints, more vehicles are required to do all deliveries in one day. When a schedule is created that combines collections and deliveries, then the total number of tours is reduced indicating fewer vehicles are

Table 8
Comparison of alternative distribution policies.

Operating window (h)	Number of trucks available			
	20 (base case)	5	4	3
6	2841 (6 trucks)	2950 (5 trucks)	No solution	No solution
7	2612 (6 trucks)	2662 (5 trucks)	2542 (4 trucks)	No solution
8	2399 (5 trucks)		2477 (4 trucks)	No solution
9	2304 (4 trucks)			2351 (3 trucks)
10	2302 (4 trucks)			2289 (3 trucks)
11	2287 (4 trucks)			2289 (3 trucks)
12	2258 (4 trucks)			2269 (3 trucks)

Table 9
Comparison of alternative distribution policies.

Time constraint	Policy	Number of agency clusters	Total number of tours			Total distance traveled			Std. dev. daily tours
			Delivery	Pickup	Total	Delivery	Pickup	Total	
6	DPD	37	20	18	38	2620	2520	5140	–
	CPD	28	–	–	28	–	–	3114	2.07
	BCPD	11	–	–	13	–	–	2841	1.14
8	DPD	37	10	17	27	1856	2437	4293	–
	CPD	28	–	–	28	–	–	3026	1.94
	BCPD	11	–	–	13	–	–	2399	1.14

utilized. While delivery to each agency cluster may increase access, it does not do so in an efficient or cost-effective manner. The policy determined by PVRPB model that uses the minimum number of FDP locations (BCPD) uses fewer vehicles, travels fewer miles, and has less variability between daily route schedules. These results suggest that the service provided by the policy BCPD which integrates the pickup and delivery aspects while efficiently considering the food safety restrictions has the potential to increase reach (access) with lower vehicle utilization and fewer miles traveled, while also increasing the ability to obtain more food.

5. Conclusion

This paper addresses the growing concern for non-profit hunger relief organizations to efficiently and effectively satisfy the needs of those with food insecurity under food safety and geographic constraints. Some interesting, unique, and challenging characteristics of the Food Bank's problem include: time constraints imposed by perishability that dictate feasible service areas, a sparsely populated network with clusters and extreme points, different schedules associated with linehauls and backhauls, and requirements that linehaul and backhaul customers share physical space but cannot have comingled product. The approach presented in this paper addresses this challenge through the identification of satellite locations for the rural food delivery program, rural agency assignment to the satellite locations, and the generation of a weekly schedule for both collection and delivery of donated food items. Our problem considers constraints on operator workday, collection frequency, delivery frequency and fleet capacity. We present a two-phased approach consisting of a capacitated set covering problem that identifies the minimum number of food delivery sites necessary to satisfy serviceable agencies, and a periodic vehicle routing problem that identifies both truck tour configurations, as well as the departure time of vehicles from sites visited in the network.

This approach was applied to the NWNCFB case study with 28 sites servicing 123 rural agencies. The solution alleviated the need for trucks to deliver to individual agencies and increased the access of the rural agencies to donated food. This model incorporates both time and distance. Time is the primary concern of food banks like NWNCFB due to the time-perishable nature of the delivered food and restrictions on the operator workday. We consider both constant delivery durations to the FDP sites and variable delivery durations based on the delivery amounts on a specific site. Our numerical study illustrates the benefits obtained from a route efficiency and food access perspective by using a minimal set of co-located collection and delivery sites. We also quantify the impact to the optimal transportation schedule for a variety of distribution policy parameters.

While our problem has been largely motivated by the operations of a particular food bank, our models can be applied to any other food bank that has a similar network structure (i.e., a mix of urban and rural charitable agencies) and service objectives. When implementing this model in practice, particular care must be given to the selection of the maximum allowable time under which perishable food can travel without refrigeration. In our numerical study, there were four agencies for which this travel time threshold was not met. While we were directed to assign the agencies to the closest FDP, we recognize that this policy may not be a practical one and raises concerns regarding food safety. The maximum time for which perishable food items can remain unrefrigerated before spoilage varies based on the type of food. For example, highly perishable food like dairy items may need to be refrigerated quickly (e.g. within 30 min), while frozen meats may remain unrefrigerated for up to an hour. Clearly, the type of food transported should be an important consideration when determining feasible agency assignment to an FDP. Furthermore, the selection of potential FDP sites for a charitable agency can be limited to the county in which the agency resides and adjacent counties. This intuitively makes sense and also reduces the computational burden of having to compute travel times for all agency–FDP pairs in the network.

There are many opportunities to extend this problem in the future. One of the most apparent is the modification of this model formulation to accommodate cost. The objective function does not consider the cost of vehicle rentals, which is the primary cost factor in many VRP objective functions (see Ref. [31]). The objective function can be modified to consider overall system costs in addition to travel distance.

Our problem is based on the assumption that a fully loaded truck will go to a single drop-off point. However, a fully loaded truck going to multiple food delivery points is an interesting area for future work. This extension is a more general form of the PVRPB that accommodates instances where some locations require direct delivery because the closest FDP exceeds the maximum distance required to avoid spoilage of unrefrigerated food items.

The requirement that prohibits food collection on three consecutive days suggests that collection frequency may have some effect on available supply. Reviewing this problem after historic data is available may provide opportunities to develop vehicle routes that consider the reward of different collection volumes as a result of pickup frequency.

Lastly, the network of this food bank is constantly expanding. Agency expansion can be accommodated by assigning the agency to the closest FDP. As long as the site is not fully loaded in terms of vehicle capacity, additional agency demands can be accommodated. However, the addition of new collection sites will change the routing schedule. Therefore, route construction heuristics, as well as meta-heuristic formulations should be explored to accommodate expansion to the transportation network.

Acknowledgments

We would like to express our sincere appreciation to Paul Jones, the former operations manager at the Northwest North Carolina Second Harvest Food Bank for his valuable input into this project.

We would also like to thank graduate students, Daniel Mota and Husniyah Abdus-Salaam, for their assistance. We would also like to thank the anonymous referees whose comments greatly improved the presentation of this work. This research was supported by NSF grants #CMMI-1000018 and #CMMI-1000828.

References

[1] Coleman-Jensen A, Nord M, Andrews M, Carlson S. Household food security in the United States in 2011 [Economic research report no. ERR-141]. U.S. Department of Agriculture, Economic Research Service; 2011. September 2012.
[2] Ohls J, Saleem-Ismail F, Cohen R, Cox B. The emergency food assistance system – findings from the provider survey, vol. II. U.S. Department of Agriculture; 2002 [Food Assistance and Nutrition Research Report No. 16-2].
[3] White B. The cupboard collective. Retrieved September 11, 2013 from: http://www.gsfb.org/how-we-help/programs/cupboard-collective/; 2013.
[4] Food bank of the Albermale – rural delivery program. Retrieved September 11, 2013 from: http://www.afoodbank.org/AboutUs/Services/RuralDeliveryProgram.aspx; 2013.
[5] Min H, Jayaraman V, Srivastava. Combined location-routing problems: a synthesis and future research directions. Eur J Oper Res 1998;108(1):1–15.
[6] Coene S, Arnout A, Spieksma F. On a periodic vehicle routing problem. J Oper Res Soc 2010;61(12):1719–28.
[7] Desrochers M, Desrosiers J, Solomon S. A new optimization algorithm for the vehicle routing problem with time windows. Oper Res 1992;40(2):342–54.
[8] Alonso F, Alvarez M, Beasley J. A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility restrictions. J Oper Res Soc 2008;59:963–76.
[9] Angelelli E, Speranza M. The periodic vehicle routing problem with intermediate facilities. Eur J Oper Res 2002;137(2):233–47.
[10] Francis P, Smilowitz K, Tzur M. The period vehicle routing problem with service choice. Transp Sci 2006;40(4):439–54.
[11] Smilowitz K, Francis P. Modeling techniques for periodic vehicle routing problems. Transp Res Part B 2006;40(10):872–84.
[12] Francis P, Smilowitz K, Tzur M. The vehicle routing problem: latest advances and new challenges. Oper Res/Comput Sci Interf Ser 2008;43(1):73–102.
[13] Thangiah SR, Potvin JY, Sun T. Heuristic approaches to vehicle routing with backhauls and time windows. Comput Oper Res 1996;23:1043–57.
[14] Duhamel C, Potvin J, Rousseau J. Tabu search heuristic for the vehicle routing problem with backhauls and time windows. Transp Sci 1997;31:49–59.
[15] Cheung RK, Hang D. Multi-attribute label matching algorithms for vehicle routing problems with time windows and backhauls. IIE Trans 2003;35:191–205.
[16] Eglese RW, Mercer A, Sohrabi B. The grocery superstore vehicle scheduling problem. J Oper Res Soc 2005;56:902–11.
[17] Toth P, Vigo D. Heuristic algorithm for the symmetric and asymmetric vehicle routing problems with backhauls. Eur J Oper Res 1999;113:528–43.
[18] Wang X, Sun J, Ren C. Study on hybrid genetic algorithm for multi-type vehicles vehicle routing problem with backhauls; 2009. Paper presented at the 2009 6th international conference on service systems and service management, ICSSSM '09, June 8–10, 2009, Xiamen, China.
[19] Bartholdi III J, Platzman L, Collins R, Warden III W. Minimal technology routing system for meals on wheels. Interfaces 1983;13:1–8.
[20] Yildiz H, Johnson M, Roehrig S. A genetic algorithm for the home-delivered meals location-routing problem [Working paper]; 2009.
[21] Solak S, Scherrer C, Ghoniem A. The stop-and-drop problem in nonprofit food distribution networks. Ann Oper Res; 2012. http://dx.doi.org/10.1007/s10479-012-1068-7.
[22] Phillips C, Hoenigman R, Higbee R, Reed T. Understanding and optimizing food recovery [Working paper]; 2012.
[23] Lien RW, Iravani S, Smilowitz K. Sequential resource allocation for nonprofit operations [Working paper]. Northwestern University; 2007.
[24] Tarantilis CD, Ioannou G, Prastacos G. Advanced vehicle routing algorithms for complex operations management problems. J Food Eng 2005;70(3):455–71.
[25] Lorena L, de Souza Lopes L. Genetic algorithms applied to computationally difficult set covering problems. J Oper Res Soc 1997;48(4):440–5.
[26] Miller CE, Tucker AW, Zemlin RA. Integer programming formulations of traveling salesman problems. J Assoc Comput Mach 1960;7(4):326–9.
[27] Kara I, Laporte G, Bektas T. A note on the lifted Miller–Tucker–Zemlin subtour elimination constrains for the capacitated vehicle routing problem. Eur J Oper Res 2004;158(3):793–5.
[28] Schrage L. Optimization modeling with LINGO. Chicago: LINDO Systems, Inc.; 2006.
[29] Desrochers M, Laporte G. Improvements and extensions to the Miller–Tucker–Zemlin subtour elimination constraints. Oper Res Lett 1991;10(1):27–36.
[30] Karp RM. Reducibility among combinatorial problems. In: Junger M, Liebeling T, Nemhauser G, Pulleyblank W, Reinelt G, Rinaldi G, et al., editors. 50 Years of integer programming 1958–2008. Berlin: Springer; 2010. pp. 219–41.
[31] Laporte G. What you should know about the vehicle routing problem. Naval Res Logist 2007;54(8):811–9.
[32] Mitchell T. Shrinking at the SNAP of a finger: more cuts to food aid loom as North Carolina faces high rates of food hardship. Retrieved from: http://www.ncjustice.org/?q=budget-and-tax/btc-brief-shrinking-snap-finger-more-cuts-food-aid-loom-north-carolina-faces-high; 2013.
[33] Wolsey LA. Integer programming. New York: John Wiley; 1998.
[34] Marsh M, Schilling D. Equity measurement in facility location analysis: a review and framework. Eur J Oper Res 1994;74(1):1–17.

Lauren Berrings Davis is an Associate Professor in the Department of Industrial and Systems Engineering at North Carolina A&T State University. She is a member of the Institute for Operations Research and Management Science (INFORMS), the Institute of Industrial Engineers, and serves as the 2013–2014 president for the INFORMS Minority Issues Forum. Her research interests are in stochastic optimization particularly in the areas of supply chain management and humanitarian logistics. Her research has been funded by NSF and the Department of Homeland Security.

Irem Sengul is a doctoral candidate in the Edward P. Fitts Department of Industrial and Systems Engineering at North Carolina State University. Prior to starting her doctoral studies, she received her B.S. degrees in Industrial Engineering and Mechanical Engineering from Bogazici University, Istanbul, Turkey in 2010. Her research interests are focused on the mathematical modeling of complex supply chains with conflicting objectives. She is interested in using mathematical modeling, robust optimization and stochastic programming to address long-term humanitarian issues and public health problems to generate applicable policies for improving people's circumstances. She is a member of INFORMS and IIE.

Julie Simmons Ivy is an Associate Professor in the Edward P. Fitts Department of Industrial and Systems Engineering and Fitts Faculty Fellow in Health Systems Engineering. She is a member of Institute of Operations Research and Management Science (INFORMS) and the Institute of Industrial Engineers. She served as the 2007 Chair (President) of the INFORMS Health Applications Society (HAS) and the 2012–13 President for the INFORMS Minority Issues Forum. Her research interests are mathematical modeling of stochastic dynamic systems with emphasis on statistics and decision analysis as applied to health care, public health, and humanitarian logistics. Her research has been funded by the NSF and the Centers for Disease Control and Prevention.

Luther Brock is a doctoral candidate in the Department of Industrial & Systems Engineering at North Carolina A&T State University. His research interests include the application of systems engineering and supply chain management to enhance the efficiency of non-profit and other community-serving organizations. He obtained both a Bachelor of Science and a Master of Science in Industrial & Systems Engineering from North Carolina A&T State University.

LaStella Miles, MS is a Research Health Economist at RTI-Health Solutions. She assists in the development of economic models, literature reviews, modeling reports, and manuscripts. She also has experience developing computer integrated database systems and simulation models.