Weekly Scheduling Models for Traveling Therapists

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Contents

Big Picture

Current Situation

- There are many small towns in the world that do not have sufficient resources (time, money, settlement layout etc.) to open up hospitals or clinics.

- Moreover, there are many patients that need to be treated where they are currently at, either because they are unable to move (age, seriousness of condition etc.) or because they shouldn't be moved because of the risk of spreading the diseases.

The Census Bureau projects that… [1]

→ As the population ages, the need for healthcare professionals grows.

By 2030 there will be more than **70 M** Americans aged 65 and older, more than twice their number in 1995.

By 2040,

1 out of 5

Americans will be over 65.

Home healthcare is the business of providing professional healthcare services in the patient's place of residence. These healthcare services include nursing, physical therapy, speech therapy, occupational therapy, medical treatments, and general assistance.

To remain profitable, these organizations must better manage their resources.

➔ derivation of **cost-efficient schedules** for their therapists by constructing travel routes & assigning patients to therapists considering operational and logical constraints

Aim of the Study

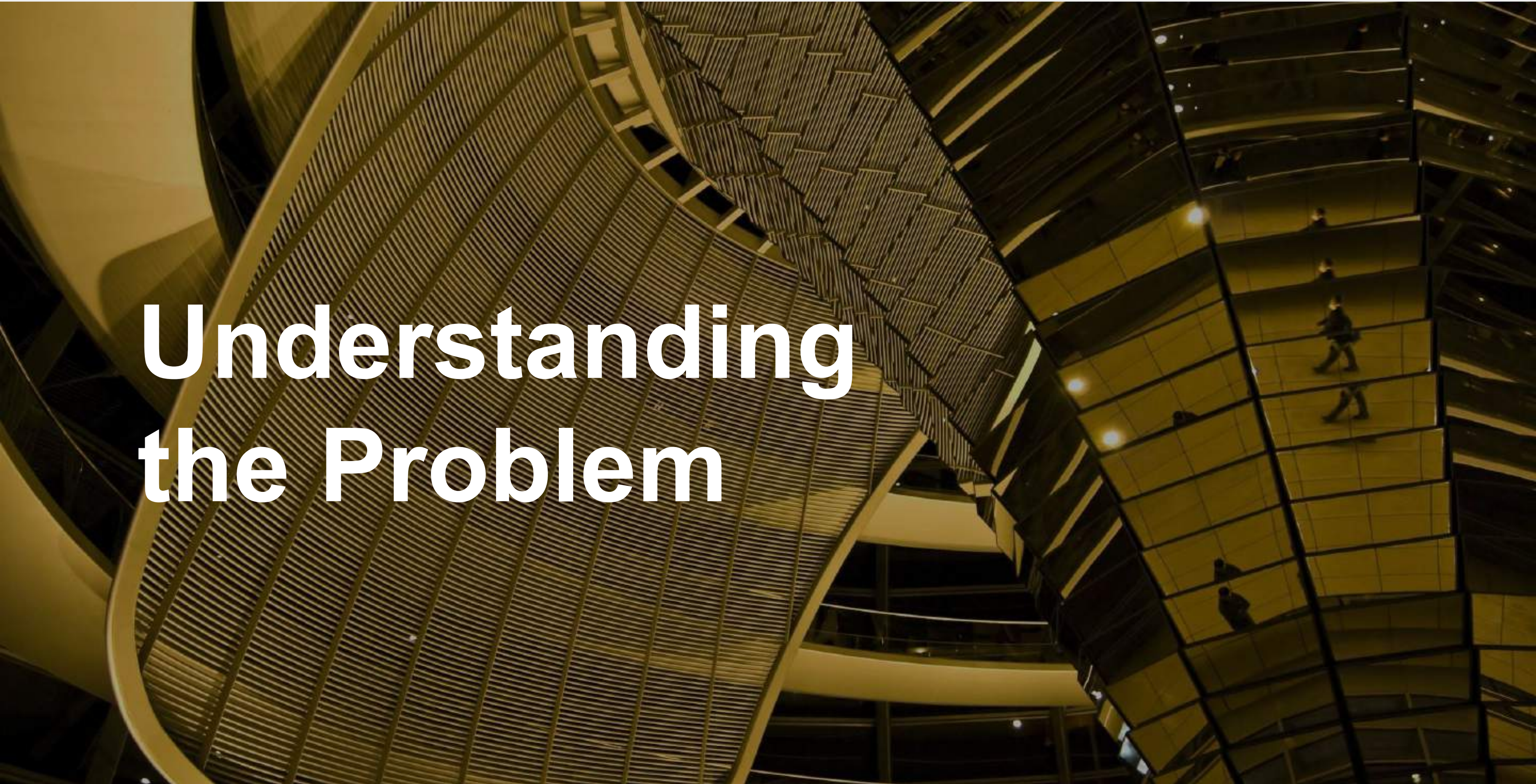
- The objective of the models is to **minimize the total cost of providing residential healthcare services for up to a week at a time** without violating availability constraints, the requirement for lunch breaks, and patient-appointment times.

General Assumptions

- “Residential healthcare” to reflect the fact that patients can be treated at locations other than their home.
- The type of organization that we have in mind is an independent agency that contracts with therapists to work on an hourly basis.
- Costs include wages for treatment and drive time, mileage reimbursement, and a premium for overtime.
- Because of different training levels and experience, therapists get paid at different rates.
- Travel costs arise from the fact that therapists must often travel for several hours between various locations to see their patients on any day of the week.

Possible Improvements

- ❖ Time efficiency
- ❖ Cost efficiency
- ❖ Therapist friendly (reimbursements, breaks etc.)
- ❖ Treatment effectiveness
- ❖ Minorities get access to healthcare
- ❖ More comfortable
- ❖ One-to-one attention
- ❖ Continuity
- ❖ Lower risk of infection
- ❖ Removal of social isolation [2] [3]



Understanding the Problem

Description of Operations and Assumptions

- Three categories: **physical**, occupational and speech.
- 40 h per → full-time
- They work on a fixed predefined schedule
- Full-timers are usually available five days a week between 8:00 am and 5:30 pm; however, they are not necessarily given schedules that fully cover this interval each day.
- PTs and PTAs could be either full-timers or part-timers, but the wages of the former are usually higher so it is better to schedule part-timers first whenever possible.
- Patients have fixed appointment times
- Each treatment lasts from 15 min to an hour.

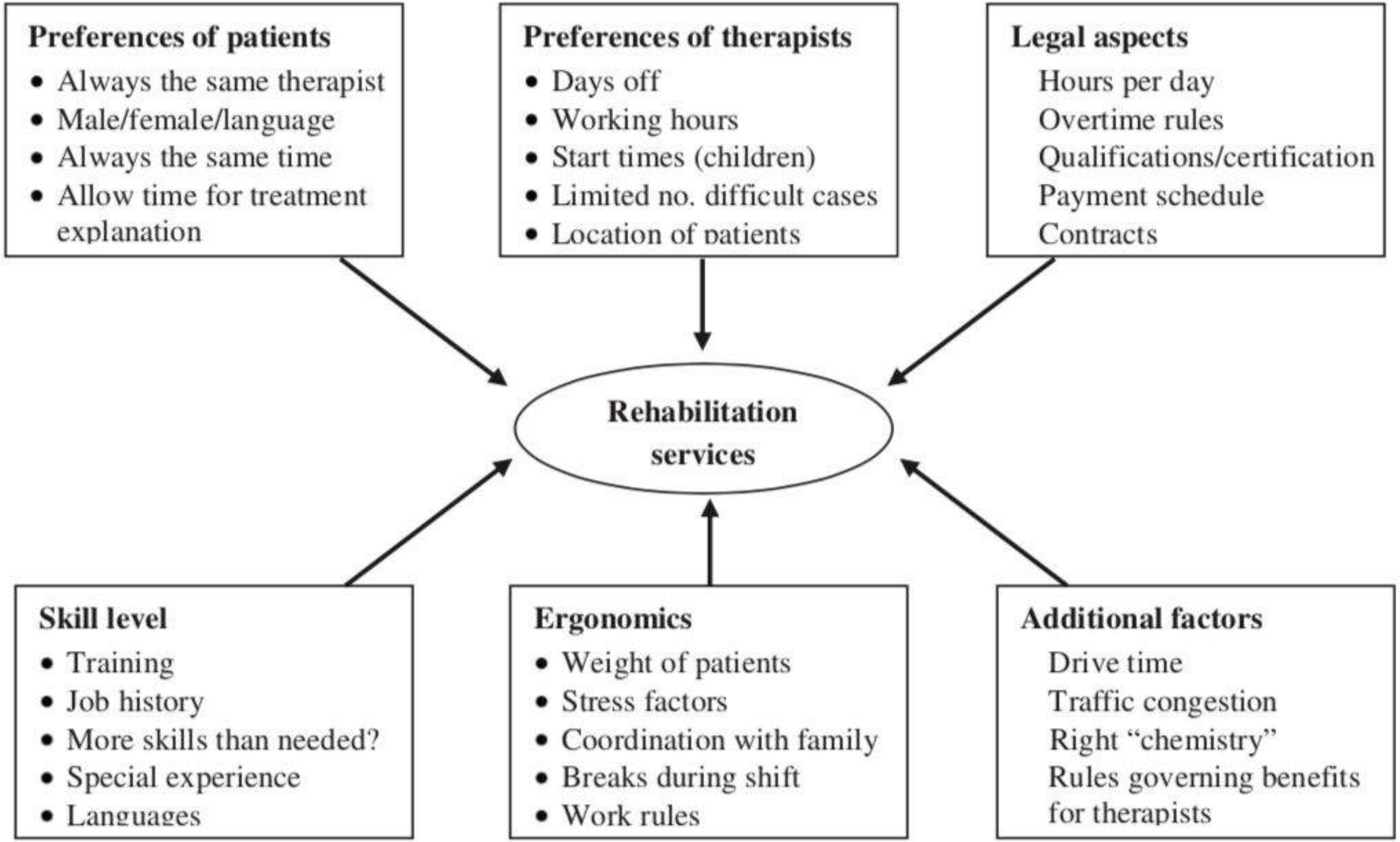


Fig. 1. Major issues in providing rehabilitation services.

Description of Operations and Assumptions

1 Scheduling Issues

- Therapists start their day at home, visit a series of patients at different sites and then return home after seeing their last patient.
- When a therapist is assigned 6 or more hours of work including driving time, a lunch break must be provided. We assume that the break is 30 minutes and must be provided between 11 a.m. and 1 p.m.
- After each task, administrative time is added.

2 Cost Issues

- Therapists are paid for a combination of driving time, treatment time and administrative task time.
- When a therapist works more than 40 h in a week, overtime begins to accumulate at an hourly rate equal to 1.5 times the regular wage rate. Idle time is not included.
- For mileage reimbursement, therapists are paid a supplement proportional to the number of miles drives in excess of 25.

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3 Travel Time Estimation

- Given (Xi,Yi) and (Xj,Yj) expressed in degrees of longitude and latitude, the distance in miles between i and j is calculated by:

$$D(i,j) = \sqrt{(53(X_i - X_j))^2 + (69.1(Y_i - Y_j))^2}$$

- In determining the driving speed for therapists, we use the following algorithm for computing vehicle velocity. Note that, the “Metro” flag is set to “N” for selected depots located in sparsely populated regions

Description of Operations and Assumptions

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3 Travel Time Estimation

If $(D(i,j) < 1)$ then $D(i,j) = 1$

If $(D(i,j) \leq 20$ and Metro = "Y") then

$\mathrm{MPH}\,(i,j) = 18.285 + 0.45159 \times D(i,j)$

ElseIf $(D(i,j) \leq 20$ and Metro = "N") then

$\mathrm{MPH}\,(i,j) = 5.447 + \log_e D(i,j) + 11.11$

ElseIf $(D(i,j) > 20)$ then

$\mathrm{MPH}\,(i,j) = 17.326 + 14.4356 \times \log_e D(i,j) + 11.11$

End If

$\mathrm{MPH}\,(i,j) = \min\{\mathrm{MPH}\,(i,j), 50\}$

Model Formulations

We described the problem in generic terms. In particular;

Vehicle = Therapists

Customer = Patient

Service = Treatment

Model Formulations

We described the problem in generic terms. In particular;

The full problem spans 5 days.

Model Formulations

Model Formulations

We described the problem in generic terms. In particular;

Our objective is to minimize the cost of:

- ➔ Treatment
- ➔ Driving
- ➔ Administrative
- ➔ Mileage
- ➔ Overtime

While observing our constraints:

- ➔ Flow balance
- ➔ Time window
- ➔ Resource

Model Formulations - Indices

Indices and sets

i,j: index for patients

k: index for therapists

d: index for days

$o(k)$: origin (and destination) of therapist k

Model Formulations - Sets

K: set of all therapists
I: set of patients to be seen over the planning horizon
$IF(i,d)$: set of patients that can follow patient i in a schedule on day d including $o(k)$
$IP(i,d)$: set of patients that can precede patient i in a schedule on day d including $o(k)$
$IL(d)$: set of patients that can precede a lunch break on day d
$IL(i,d)$: set of patients that can follow patient $i \in IL(d)$ in a schedule on day d

Model Formulations - Parameters

Data and parameters

c_{ij}^{k}: hourly traveling rate from patient i to patient j by therapist k

c_{id}^{k}: hourly treatment rate by therapist k to see patient i on day d

τ_{ij}: travel time between patient i and j

s_{id}: time required to provide treatment to patient i on day d

a_{id}: appointment time for patient i on day d

Model Formulations - Decision Variables

Decision variables

x^k_{ijd}: 1 if therapist k visits patients i and j in succession on day d,
0 otherwise

We have 3 cases:

Single Depot Vehicle Scheduling Problem (SDVSP)

All therapists are identical.

Multiple Depot Vehicle Scheduling Problem (MDVSP)

Therapists are not identical.

MDVSP With Overtime, Mileage and Lunch Break

MDVSP problem with the consideration of overtime, mileage and lunch break.

Single Depot Vehicle Scheduling Problem (SDVSP)

All therapists in the set K are identical.

- Which means that they have the same:
  - Working windows,
  - Treatment rates,
  - Travel rate

And they are based at the same clinic where they start and end their day. (Same depot)

Single Depot Vehicle Scheduling Problem (SDVSP)

$$\phi_d^{\mathrm{SDVSP}} = \text{Minimize} \sum_{i \in I \cup \{o\}} \sum_{j \in IF(i,d)} \bar{c}_{ijd} x_{ijd} \tag{1a}$$

$$\text{subject to} \sum_{j \in IF(i,d)} x_{ijd} = 1, \quad \forall\, i \in I \tag{1b}$$

$$\sum_{i \in IP(j,d)} x_{ijd} - \sum_{i \in IF(j,d)} x_{jid} = 0, \quad \forall\, j \in I \cup \{o\} \tag{1c}$$

$$\sum_{j \in I} x_{ojd} \leq |K| \tag{1d}$$

$$x_{ijd} \in \{0, 1\}, \quad \forall\, i \in I \cup \{o\}, \quad j \in IF(i, d) \tag{1e}$$

Single Depot Vehicle Scheduling Problem (SDVSP)

$$\phi_d^{\mathrm{SDVSP}} = \text{Minimize} \sum_{i \in I \cup \{o\}} \sum_{j \in IF(i,d)} \overline{c}_{ijd} x_{ijd} \tag{1a}$$

Minimize the cost of treating all patients on a specific day.

The coefficients cijd include the travel cost cij between location i and j and the cost of providing treatment cid for patient i on day d.

K: set of all therapists
I: set of patients to be seen over the planning horizon
$IF(i,d)$: set of patients that can follow patient i in a schedule on day d including $o(k)$
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Single Depot Vehicle Scheduling Problem (SDVSP)

$$\sum_{j \in IF(i,d)} x_{ijd} = 1, \quad \forall\, i \in I \tag{1b}$$

Ensure that **each patient is visited exactly once** while identifying her immediate successor.

$IF(i,d)$: set of patients that can follow patient i in a schedule on day d including $o(k)$

Single Depot Vehicle Scheduling Problem (SDVSP)

$$\sum_{i \in IP(j,d)} x_{ijd} - \sum_{i \in IF(j,d)} x_{jid} = 0, \quad \forall\, j \in I \cup \{o\} \tag{1c}$$

$$\sum_{j \in I} x_{ojd} \leq |K| \tag{1d}$$

(1c) guarantees **flow balance** at each location visited while (1d) **limits the number of schedules each day** to at most |K|, (one for each therapist).

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Single Depot Vehicle Scheduling Problem (SDVSP)

$$x_{ijd} \in \{0,1\}, \quad \forall\ i \in I \cup \{o\}, \quad j \in IF(i,d) \tag{1e}$$

Variable definitions.

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Single Depot Vehicle Scheduling Problem (SDVSP)

NOTICE:

There are no constraints in the formulation that prevent subtours that do not include the clinic; that is, there are no subtour elimination constraints that prevent a therapist from being assigned a route that cycles through a subset of patients and never returns to the clinic.

In our case:

Subtours can never arise because all patients have fixed appointment times.

Multiple depot vehicle scheduling problem (MDVSP) with lunch break

$$\phi_d^{\mathrm{MD-L}} = \text{Minimize} \sum_{k\in K} \sum_{i\in I\cup\{o(k)\}} \sum_{j\in IF(i,d)} \bar{c}_{ijd}^{k} x_{ijd}^{k} \tag{2a}$$

$$\text{subject to} \sum_{k\in K} \sum_{j\in IF(i,d)} x_{ijd}^{k} = 1, \quad \forall\, i\in I \tag{2b}$$

$$\sum_{i\in IP(j,k,d)} x_{ijd}^{k} - \sum_{i\in IF(j,k,d)} x_{jid}^{k} = 0, \quad \forall\, k\in K, \; j\in IK(k,d)\cup\{o(k)\} \tag{2c}$$

$$\sum_{j\in I} x_{o(k),j,d}^{k} \le 1, \quad \forall\, k\in K \tag{2d}$$

$$\sum_{i\in IL(d)} \sum_{j\in IL(i,k,d)} x_{ijd}^{k} \ge 1, \quad \forall\, k\in K \tag{2e}$$

$$x_{ijd}^{k} \in \{0,1\}, \quad \forall\, k\in K, \quad i\in I\cup\{o(k)\}, \quad j\in IF(i,k,d) \tag{2f}$$

Multiple depot vehicle scheduling problem (MDVSP) with lunch break

$$\sum_{i \in IL(d)} \sum_{j \in IL(i,k,d)} x_{ijd}^{k} \geq 1, \quad \forall\, k \in K \tag{2e}$$

Model (2) is the same as model (1) except for the reintroduction of the index k and the addition of (2e) which enforce the lunch break requirement for all therapists by insisting that each schedule include a lunch break arc (i,j), where the set IL(d) is limited to those patients whose appointment times are early enough in the day to satisfy the above two conditions on day d.

$IL(d)$: set of patients that can precede a lunch break on day d
$IL(i,d)$: set of patients that can follow patient $i \in IL(d)$ in a schedule on day d

Multiple depot vehicle scheduling problem (MDVSP) with lunch break

$$\sum_{i \in IL(d)} \sum_{j \in IL(i,k,d)} x_{ijd}^{k} \geq 1, \quad \forall\, k \in K \tag{2e}$$

Note that the depot or home base o(k) is an element in IL(i,k,d) if only to ensure that (2) is feasible.

In practice, the earliest appointment time is at 8:00 am so it is not possible for the lunch break to occur after the last patient assigned to therapist k is treated, given that the break must conclude no later than 1:00 pm. Constraints (2e) do not permit the break to precede the first appointment.

K: set of all therapists
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Multiple depot vehicle scheduling problem (MDVSP) with lunch break

$$x_{ijd}^{k} \in \{0,1\}, \quad \forall\, k \in K, \quad i \in I \cup \{o(k)\}, \quad j \in IF(i,k,d) \tag{2f}$$

In general, the binary requirements for the x-variables in (2f) cannot be relaxed. A special case exists, however, that preserves the MCFP (min-cost flow problem) characteristics of (2) when there is a single depot, that is, o(k) = o for all k.

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MDVSP With Overtime, Mileage and Lunch Break

The previous models decompose by day.

When a therapist works more than 40 h in a week, time and a half is paid for the overtime hours.

This factor ties the days of the week together and prevents a daily decomposition.

To incorporate overtime in the model, it is necessary to keep track of the **billable, travel, treatment and administrative time** that each therapist accumulates each day. Idle time, however, is not compensated.

MDVSP With Overtime, Mileage and Lunch Break

To incorporate overtime in the model, it is necessary to keep track of the **billable, travel, treatment and administrative time** that each therapist accumulates each day. Idle time, however, is not compensated.

To calculate the administration time:

$$\mathrm{adm}_{ikd} = (1/p_k - 1) \times c_{id}^{k}$$

where p_k is the productivity for therapist k.

MDVSP With Overtime, Mileage and Lunch Break

A second factor not yet considered is the nonlinear nature of mileage reimbursement.

For the first 25 miles, therapist k is not reimbursed.

But for all miles above that, he is paid at the rate of c_k^{miles}

MDVSP With Overtime, Mileage and Lunch Break

ADDITIONAL NOTATIONS: (Indices and Sets)

Indices and sets

$DU(i)$: set of days in the planning horizon that patient i is to be treated

$DK(k)$: set of days in the planning horizon that therapist k works

MDVSP With Overtime, Mileage and Lunch Break

ADDITIONAL NOTATIONS: (Data and Parameters)

Data and parameters

c_k^{reg}: hourly treatment rate for therapist k
c_k^{miles}: hourly travel cost for every mile that therapist k drives in a day beyond 25 miles (assumed to be constant at $0.55/mi)
adm_{ikd}: time required by therapist k to perform administrative functions after treating patient i on day d
e_{kd} earliest start time of therapist k on day d
f_{kd}: latest end time of therapist k on day d

MDVSP With Overtime, Mileage and Lunch Break

ADDITIONAL NOTATIONS: (Data and Parameters Cont'd)

$\tau_{dk}^{\max}$: upper bound on the number of working hours permitted for therapist k on day d

$\tau_{k}^{\text{max\_over}}$: upper limit on the amount of overtime permitted for therapist k

dist_{ij}: distance between locations i and j

$\overline{T}$: number of hours in a week above which overtime is paid; $\overline{T} = 40$

$\overline{D}$: number of miles driven in a day for which a therapist is not reimbursed; $\overline{D} = 25$

MDVSP With Overtime, Mileage and Lunch Break

ADDITIONAL NOTATIONS: (Decision Variables)

Decision variables

T^1_{dk}: time (hours) for therapist k to go from his home base to his first patient on day d

T^2_{dk}: time (hours) for therapist k to return to his home base after treating his last patient on day d

T_{dk}: total time for which therapist k is paid on day d

T^{over}_k: number of hours that therapist k works in a week beyond 40

D_{dk} number of miles that therapist k drives on day d

ΔD_{dk} number of miles above 25 that therapist k drives on day d

MDVSP With Overtime, Mileage and Lunch Break

Based on model (2), we add constraints for overtime and mileage reimbursement. This leads to model (3).

$$\phi^{\text{MD-L-O-M}} = \text{Minimize} \sum_{k \in K} \sum_{d \in DK(k)} \sum_{i \in IK(k,d) \cup \{o(k)\}} \sum_{j \in IF(i,k,d)} \times \left(c_{ij}^k + c_{id}^k\right) x_{ijd}^k + \sum_{d \in D} \sum_{k \in K} c_k^{\text{miles}} \Delta D_{dk} + \sum_{k \in K} 0.5 c_k^{\text{reg}} T_k^{\text{over}} \quad \text{(3a)}$$

$$\text{subject to} \quad \sum_{k \in K(i,d)} \sum_{j \in IF(i,k,d)} x_{ijd}^k = 1, \quad \forall\, i \in I, \quad d \in DU(i) \quad \text{(3b)}$$

$$\sum_{j \in IK(k,d)} x_{o(k),j,d}^k \le 1, \quad \forall\, k \in K, \quad d \in DK(k) \quad \text{(3c)}$$

$$\sum_{i \in IP(j,k,d)} x_{ijd}^k - \sum_{i \in IF(j,k,d)} x_{jid}^k = 0, \quad \forall\, k \in K, \quad d \in DK(k), \quad j \in IK(k,d) \cup \{o(k)\} \quad \text{(3d)}$$

$$\sum_{j \in IK(k,d)} \tau_{o(k),j} x_{o(k),j,d}^k = T_{dk}^1, \quad \forall\, k \in K, \quad d \in DK(k) \quad \text{(3e)}$$

$$\sum_{i \in IK(k,d)} \left(\tau_{i,o(k)} + s_{id} + \text{adm}_{ikd}\right) x_{i,o(k),d}^k = T_{dk}^2, \quad \forall\, k \in K, \quad d \in DK(k) \quad \text{(3f)}$$

$$T_{dk}^1 + T_{dk}^2 + \sum_{i \in IK(k,d)} \sum_{j \in IF(i,k,d)/\{o(k)\}} (\tau_{ij} + s_{id} + \text{adm}_{ikd}) x_{ijd}^k = T_{dk}, \quad \forall\, k \in K, \quad d \in DK(k) \quad \text{(3g)}$$

$$\sum_{d \in DK(k)} T_{dk} - \overline{T} \le T_k^{\text{over}}, \quad \forall\, k \in K \quad \text{(3h)}$$

$$\sum_{i \in I \cup \{o\}} \sum_{j \in IF(i,d)} \text{dist}_{ij} x_{ijd}^k = D_{dk}, \quad \forall\, d \in D, \quad k \in K \quad \text{(3i)}$$

MDVSP With Overtime, Mileage and Lunch Break

Based on model (2), we add constraints for overtime and mileage reimbursement. This leads to model (3).

$$D_{dk} - \overline{D} \leq \Delta D_{dk}, \quad \forall\ d \in D, \quad k \in K \tag{3j}$$

$$\sum_{i \in IK(k,d)} \sum_{j \in IL(i,k,d)} x_{ijd}^{k} \geq 1, \quad \forall\ k \in K, \quad d \in DK(k) \tag{3k}$$

$$\begin{aligned} & x_{ijd}^{k} \in \{0, 1\}, \quad T_{dk}^{1} \geq 0, \quad T_{dk}^{2} \geq 0, \quad T_{dk} \in \left[0, \tau_{dk}^{\max}\right], \\ & T_{k}^{\text{over}} \in \left[0, \tau_{k}^{\text{max\_over}}\right], \quad D_{dk} \geq 0, \quad \Delta D_{dk} \geq 0, \\ & \forall i \in I \cup \{o(k)\}, \quad d \in DU(i), \quad j \in IF(i,k,d), \quad k \in K(i,d) \end{aligned} \tag{3l}$$

MDVSP With Overtime, Mileage and Lunch Break

Number of miles above 25 that therapist k drives on day d.

$$\phi^{MD-L-O-M} = \text{Minimize} \sum_{k\in K}\sum_{d\in DK(k)}\sum_{i\in IK(k,d)\cup\{o(k)\}}\sum_{j\in IF(i,k,d)} \times \left(c_{ij}^{k} + c_{id}^{k}\right) x_{ijd}^{k} + \sum_{d\in D}\sum_{k\in K} c_{k}^{\text{miles}} \Delta D_{dk} + \sum_{k\in K} 0.5\, c_{k}^{\text{reg}} T_{k}^{\text{over}} \qquad (3a)$$

hourly treatment rate for therapist k

Hourly travel cost for every mile that therapist k drives in a day beyond 25 miles .

The objective function (3a) has three terms.

The first minimizes the cost of traveling between all pairs of patients + the cost of providing treatment. For those patients to be seen on day d, the set IK(k,d) must be defined accordingly.

The second and third terms minimize the mileage reimbursement and overtime payments, respectively

MDVSP With Overtime, Mileage and Lunch Break

$$\text{subject to} \quad \sum_{k \in K(i,d)} \sum_{j \in IF(i,k,d)} x_{ijd}^{k} = 1, \quad \forall\, i \in I, \quad d \in DU(i) \qquad \text{(3b)}$$

Constraints (3b) ensure that each patient has exactly one successor on a route on day d.

The first summation is over the set of therapists who are qualified to provide treatment to patient i and the second is over all patients who are compatible successors of i and hence can be on the same route.

(An ordered pair of patients (i,j) is said to be compatible on day d for therapist k if it satisfies some conditions.)

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MDVSP With Overtime, Mileage and Lunch Break

$$\sum_{j \in IK(k,d)} x^k_{o(k),j,d} \le 1, \quad \forall\, k \in K, \quad d \in DK(k) \tag{3c}$$

$$\sum_{i \in IP(j,k,d)} x^k_{ijd} - \sum_{i \in IF(j,k,d)} x^k_{jid} = 0, \quad \forall\, k \in K, \; d \in DK(k), \; j \in IK(k,d) \cup \{o(k)\} \tag{3d}$$

Constraints (3c) limit each therapist k $\in$ K to at most one schedule per day.

Constraints (3d) impose route continuity for each therapist k and each compatible patient j $\in$ IF(i,k,d) by requiring that tours (loops) are constructed rather than open paths.

MDVSP With Overtime, Mileage and Lunch Break

The time to go from the home base of therapist k to his first patient.

$$\sum_{j \in IK(k,d)} \tau_{o(k),j} x^k_{o(k),j,d} = T^1_{dk}, \quad \forall\, k \in K, \quad d \in DK(k) \qquad (3e)$$

time for therapist k to go from & return to his home on day d

$$\sum_{i \in IK(k,d)} \left(\tau_{i,o(k)} + s_{id} + \mathrm{adm}_{ikd}\right) x^k_{i,o(k),d} = T^2_{dk}, \quad \forall\, k \in K, \quad d \in DK(k) \qquad (3f)$$

The time required by therapist k to treat the last patient in his schedule on day d, perform the associated administrative functions, and then return to his home base.

Constraints (3e) and (3f) keep track of the amount of paid time accrued daily by therapist k.

Therapists may work for more than 8 h a day but overtime does not apply until their billable plus administrative plus travel time exceeds 40 hours in a week.

MDVSP With Overtime, Mileage and Lunch Break

#of hours
therapist k is paid
on day d

time for therapist k to go from & return to his
home on day d

$$T^1_{dk} + T^2_{dk} + \sum_{i\in IK(k,d)} \sum_{j\in IF(i,k,d)/\{o(k)\}} (\tau_{ij} + s_{id} + \mathrm{adm}_{ikd})x^k_{ijd} = T_{dk}, \quad \forall\, k\in K, \quad d\in DK(k) \tag{3g}$$

40 hours

$$\sum_{d\in DK(k)} T_{dk} - \bar{T} \le T^{over}_k, \quad \forall\, k\in K \tag{3h}$$

#of hours therapist k works in a
week beyond 40

$$\sum_{i\in I\cup\{o\}} \sum_{j\in IF(i,d)} \mathrm{dist}_{ij}x^k_{ijd} = D_{dk}, \quad \forall\, d\in D, \quad k\in K \tag{3i}$$

The total number of working hours that therapist k accrues on day d is calculated in (3g).
An upper bound is placed on this value in (3l).

Constraint (3h) sums the working hours over the week and subtracts T to provide a lower bound on the number of overtime hours for therapist k.

MDVSP With Overtime, Mileage and Lunch Break

$$D_{dk} - \overline{D} \leq \Delta D_{dk}, \quad \forall\ d \in D, \quad k \in K \tag{3j}$$

$$\sum_{i \in IK(k,d)} \sum_{j \in IL(i,k,d)} x_{ijd}^{k} \geq 1, \quad \forall\ k \in K, \quad d \in DK(k) \tag{3k}$$

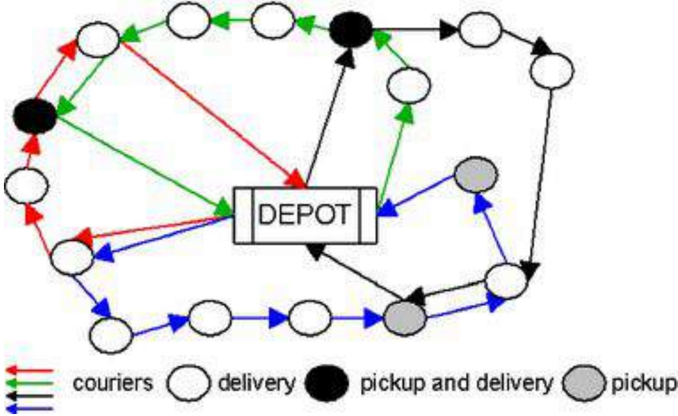
$$\begin{aligned} &x_{ijd}^{k} \in \{0,1\}, \quad T_{dk}^{1} \geq 0, \quad T_{dk}^{2} \geq 0, \quad T_{dk} \in \left[0, \tau_{dk}^{\max}\right], \\ &T_{k}^{\mathrm{over}} \in \left[0, \tau_{k}^{\mathrm{max\_over}}\right], \quad D_{dk} \geq 0, \quad \Delta D_{dk} \geq 0, \\ &\forall i \in I \cup \{o(k)\}, \quad d \in DU(i), \quad j \in IF(i,k,d), \quad k \in K(i,d) \end{aligned} \tag{3l}$$

Constraints (3i) are similar to those in (3g) and are used to determine the total distance that therapist k drives each day.

Whether mileage reimbursement is due is determined by (3j) in a manner similar to (3h). Constraints (3k) are written for those therapists whose time window on day d spans 6 or more hours and ensures that they traverse at least one arc (i,j) that admits sufficient time for the lunch break.

What have we learned in class?

1 - Vehicle Routing Problem (VRP)



$$\min \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

$$s.t. \sum_j x_{ij} = 1 \qquad \forall i \neq 1$$

$$\sum_j x_{ji} = 1 \qquad \forall i \neq 1$$

$$\sum_j x_{j1} = m$$

$$\sum_j x_{1j} = m$$

$$u_i - u_j + Q x_{ij} \leq Q - q_j$$

$$P_i \leq u_j \leq Q$$

$$x_{ij} \in \{0,1\} \qquad \forall i,j \in N$$

q_i: *demand of node i*
Q_k: *capacity of vehicle k*

x_{ij}: *1 if j is visited, 0 otherwise*

$Q_k = Q$

What have we learned in class?

2 - Time Windowed VRP

$$\min \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}$$

$$s.t. \sum_{j} x_{ij} = 1 \qquad \forall i \neq 1$$

$$\sum_{j} x_{ji} = 1 \qquad \forall i \neq 1$$

$$\sum_{j} x_{j1} = m$$

$$\sum_{j} x_{1j} = m$$

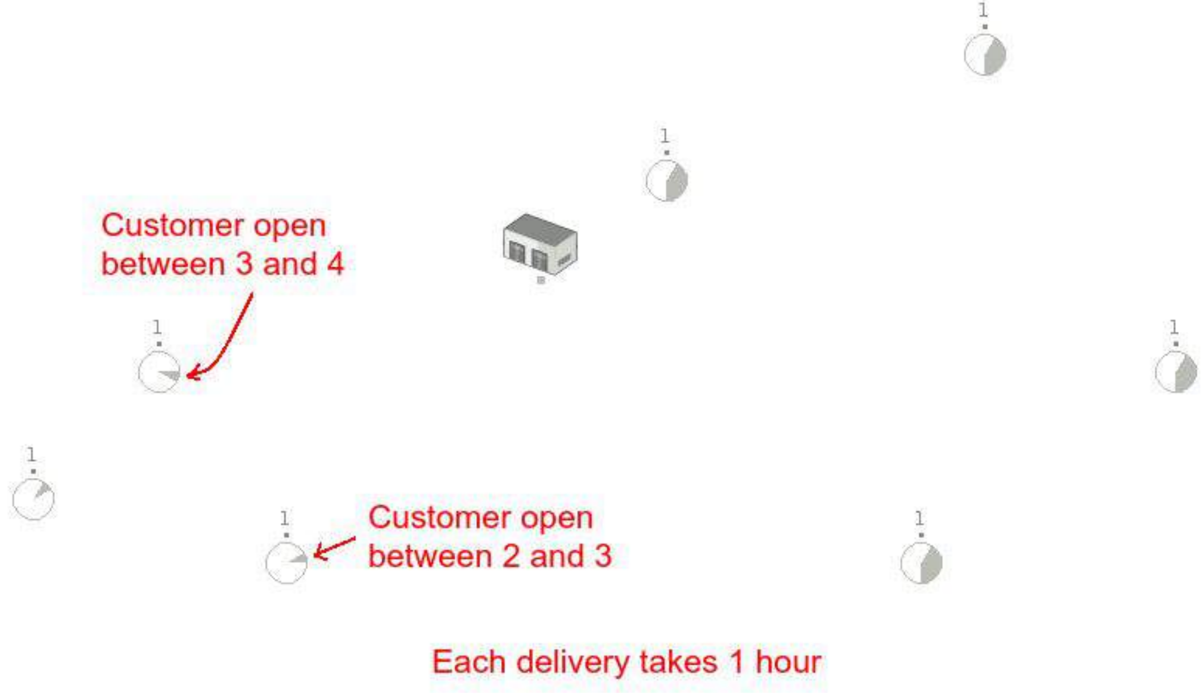
$$u_i - u_j + Q x_{ij} \leq Q - q_j$$

$$P_i \leq u_j \leq Q$$

$$t_i - t_j + M x_{ij} \leq M - t_{ij}$$

$$c_i \leq t_i \leq l_i$$

$$x_{ij} \in \{0,1\} \qquad \forall i, j \in N$$



Heuristic Method 1

It is important to note that for our problem, most full-time therapists are available for 45-50 hours and part-timers always work less than 40 hours. Through this information, it can be said that we can rule out overtime costs as the cost itself is not very impactful in the optimization.

Also, as reimbursement rate is a very low value, we can also remove the constraints regarding reimbursement to come up with a feasible solution in which we can add the overtime and reimbursement costs after the processing.

$$\phi^{IP}_{\text{actual}} = \phi^{IP}_{\text{relaxed}} + \sum_{k\in K} 0.5c_k^{\text{reg}} \cdot \max\{\overline{T}_k^{\text{over}} - 40, 0\}$$
$$- \sum_{d\in D} \sum_{k\in K} c_k^{\text{miles}} \cdot \min\{\overline{D}_{dk}, 25\}$$

Heuristic Method 2

As mentioned in heuristic 1, if we consider that overtime constraints rarely occur due to the problem characteristics. Considering this, it is possible to only relax the reimbursement constraints just like in the previous heuristic while keeping overtime constraints.

$$\phi_{\text{actual}}^{\text{IP}} = \phi_{\text{relaxed}}^{\text{IP}} - \sum_{d \in D} \sum_{k \in K} c_k^{\text{miles}} \cdot \min\{\overline{D}_{dk}, 25\}$$


Results

Results

Models (2) and (3) were tested in 40 randomly generated instances. Half of these tests featured a single starting point whereas in the other half therapists had their own point home location.

Each half was also divided into four subsection, with differentiating patient and therapist numbers.

Sample Schedules for Models (2) and (3)

Monday schedule for therapists from model (2).

Therapist ID	Route (patient-appointment time)
0	1098–8:00 am, 1105–1:00 pm, 1095–4:00 pm
1	1022–9:30 am, 1001–10:30 am, 1034–11:30 am, 1117–1:00 pm, 1136–1:30 pm, 1102–3:00 pm, 1074–4:00 pm
2	1075–1:30 pm, 1015–2:00 pm, 1073–4:00 pm
3	1013–8:00 am, 1042–10:30 am, 1007–1:00 pm, 1052–2:00 pm, 1090–2:30 pm, 1062–3:30 pm
5	1120–8:30 am, 1040–10:00 am, 1068–11:15 am, 1021–1:00 pm, 1059–4:00 pm
6	1139–8:30 am, 1091–9:00 am, 1036–10:00 am, 1033–10:30 am, 1100–11:00 am, 1030–1:00 pm, 1028–2:00 pm, 1106–3:00 pm, 1065–3:30 pm, 1116–4:30 pm
7	1064–9:00 am, 1101–10:00 am, 1088–11:00 am, 1038–2:00 pm, 1057–3:30 pm
8	1094–9:00 am, 1025–9:30 am, 1053–10:00 am, 1008–11:00 am, 1066–1:00 pm, 1147–2:00 pm, 1055–3:30 pm
10	1129–10:30 pm, 1110–11:00 am
11	1005–8:00 am, 1107–9:00 am, 1061–10:00 am, 1016–11:00 am, 1035–1:30 pm, 1138–2:00 pm, 1071–2:30 pm, 1115–3:30 pm, 1089–4:30 pm
12	1112–8:30 am, 1058–1:45 pm, 1039–3:30 pm
13	1081–7:30 am, 1097–8:30 am, 1029–10:30 am, 1027–11:00 am, 1054–1:00 pm, 1063–2:00 pm, 1087–2:45 pm, 1041–4:00 pm
14	1124–10:00 am, 1014–10:30 am, 1111–1:45 pm, 1032–4:00 pm

Monday schedule for therapists from model (3).

Therapist ID	Route (patient-appointment time)
0	1112–8:00 am, 1066–1:00 pm, 1147–2:00 pm, 1095–4:00 pm
1	1022–9:30 am, 1001–10:30 am, 1034–11:30 am, 1117–1:00 pm, 1136–1:30 pm, 1102–3:00 pm, 1074–4:00 pm
2	1075–1:30 pm, 1015–2:00 pm
3	1013–8:00 am, 1042–10:30 am, 1007–1:00 pm, 1063–2:00 pm, 1087–2:45 pm, 1062–3:30 pm
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7	1107–9:00 am, 1101–10:00 am, 1008–11:00 am, 1038–2:00 pm, 1055–3:30 pm
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12	1005–8:00 am, 1054–1:00 pm, 1041–4:00 pm
13	1081–7:30 am, 1097–8:30 am, 1029–10:30 am, 1027–11:00 am, 1021–1:00 pm, 1057–3:30 pm
14	1124–10:00 am, 1014–10:30 am, 1111–1:45 pm, 1032–4:00 pm

Distance and Cost Results

Through testing these models, it is seen that the number of miles above 25 travelled by therapists decrease from 212.37 to 176.47, which is due to the inclusion of mileage cost. It is also observed that the total cost of model with overtime and milage ($12,565.20) is higher that the total cost of the model without such constraints.

Specifically, it is found that when the total number of visits grows, the total cost grows linearly.

Table 3
Comparison of driving distance for all therapists on Monday.

Therapist ID	Model (2) distance	Model (2) distance above 25 miles	Model (3) distance	Model (3) distance above 25 miles
0	3.00	0.00	21.53	0.00
1	15.39	0.00	15.39	0.00
2	5.04	0.00	4.92	0.00
3	21.67	0.00	20.43	0.00
4	0.00	0.00	40.86	15.86
5	54.76	29.76	54.76	29.76
6	23.83	0.00	23.83	0.00
7	90.20	65.20	90.20	65.20
8	94.39	69.39	58.46	33.46
9	0.00	0.00	0.00	0.00
10	2.00	0.00	2.00	0.00
11	31.96	6.96	29.88	4.88
12	14.55	0.00	2.00	0.00
13	66.06	41.06	52.31	27.31
14	15.14	0.00	15.14	0.00
Total	438.00	212.37	431.72	176.47

Results of Heuristics

- Even though all solutions provided satisfactory solutions, as the optimality gap is often lesser than 1%. In terms of the solutions quality, for small samples CPLEX is providing better solutions, but with increased sample size, the best method shifts from CPLEX to the 2nd Heuristic Method (in which the overtime constraints are intact but reimbursement is removed).

- However, this is mainly due to the parameters we have for this problem, as increased costs for overtime and reimbursement would change this situation.

Table for Heuristic Results

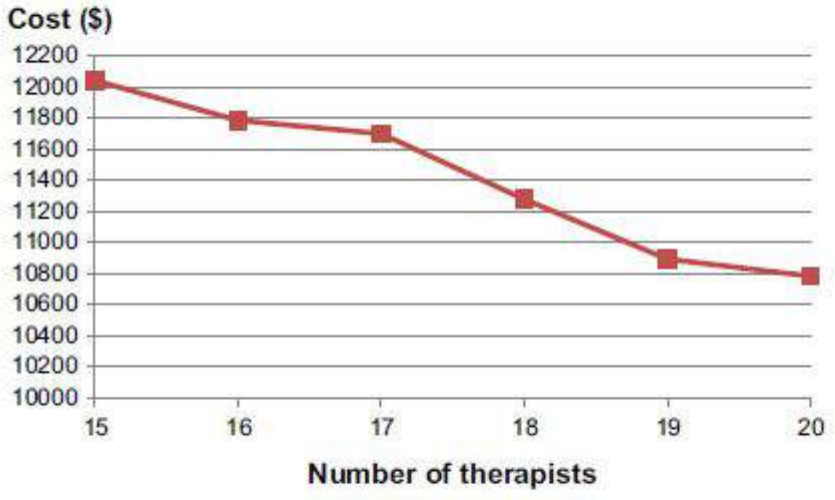
Table 5
Algorithmic comparisons.

Data set	Relaxed MIP I				Relaxed MIP II				CPLEX			LB[b] ($)
	$\phi^{IP}_{relaxed}$	Cost ($)	Time (s)	Gap[a] (%)	$\phi^{IP}_{relaxed}$	Cost ($)	Time (s)	Gap[a] (%)	Cost ($)	Time (s)	Gap[a] (%)	
1	5459	5182	0.15	0.06	5459	5182	0.13	0.06	5180	0.52	0	5179
2	5391	5103	0.19	0.03	5391	5103	0.19	0.03	5102	0.52	0	5102
3	5964	5657	0.22	0.03	5964	5657	0.21	0.03	5655	0.42	0	5655
4	6723	6397	0.33	0.09	6723	6397	0.40	0.09	6391	0.47	0	6391
5	7020	6694	0.37	0.04	7020	6694	0.36	0.04	6691	0.64	0	6691
6	9106	8635	2.47	0.32	9106	8630	1.59	0.27	8616	51.64	0.10	8607
7	10,402	9912	3.66	0.37	10,402	9906	3.96	0.31	9888	90.65	0.12	9876
8	11,004	10,561	6.13	1.04	11,013	10,484	6.62	0.30	10,466	134.50	0.13	10,453
9	11,785	11,363	9.12	1.45	11,795	11,249	9.35	0.43	11,225	226.24	0.21	11,201
10	12,302	11,820	10.71	0.96	12,307	11,741	19.95	0.29	11,722	259.94	0.13	11,707
11	12,721	12,088	27.29	0.95	12,721	12,088	23.39	0.95	12,066	573.50	0.76	11,977
12	13,575	12,912	29.70	0.89	13,576	12,907	32.39	0.85	12,880	774.51	0.65	12,798
13	15,330	14,726	66.06	1.46	15,334	14,636	80.77	0.87	14,619	1,329.20	0.75	14,510
14	16,246	15,577	53.41	1.13	16,250	15,533	78.50	0.85	15,511	1,900.90	0.70	15,403
15	17,908	17,266	76.32	1.39	17,916	17,182	143.64	0.90	17,160	2,946.21	0.76	17,029
16	16,206	15,427	135.17	1.21	16,205	15,427	183.95	1.21	15,434	2,066.10	1.25	15,243
17	17,324	16,553	120.46	1.46	17,325	16,506	165.17	1.20	16,507	5,555.80	1.21	16,310
18	20,026	19,335	185.88	2.01	20,035	19,154	375.84	1.08	19,151	6,228.13	1.07	18,949
19	21,656	20,971	304.66	1.91	21,680	20,787	965.70	1.04	20,816	10,494.91	1.19	20,572
20	23,249	22,515	659.63	1.81	23,263	22,347	8,251.18	1.06	22,384	15,689.97	1.22	22,113

Analysis for the Number of Therapists

- Considering Model (3) [The model with overtime, mileage and lunch breaks], an analysis was performed using a sample with 150 patients, 306 visits and 15 therapists. The number of therapists was then incremented by 1 up to 20.

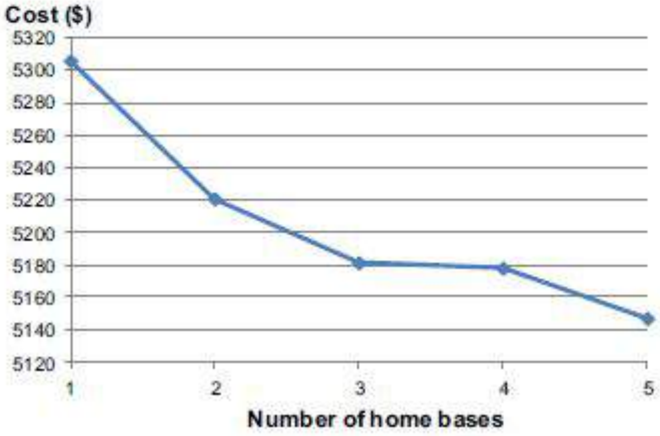
- With 15 therapists, the initial total cost was 12.045. It is seen that number decreased by 10.5% to 10.783 when we increase the number of patients to 20.



Analysis for Number of Home Bases

- Initial scenario features 112 patient visits during the week and five therapists randomly drawn from the database.

- In the initial scenario all therapists do share a common home base with average cost of $5.306. Increasing the number of home bases, it was seen that the extra cost due to having a common home base is only 159$.



Alternative Actions

Questioning	What can be done
Can all therapists provide same level of treatment to patients	Include level constraints
Do therapists really need to return back to their home location after last patient	T2dk may be relaxed or removed

Additional Examples

-Using SDVSP and MVDSP that is usually modelled as Multi- Commodity Network Flows (MCNF), a Set-Partitioning Problem (SPP), or Time-Space Networks (TSN) in

- Distribution networks (ETİ in Turkey/ Henkel in Germany, Turkey, Hong Kong, India/ Coca-Cola in US, UK, China/ Zara in Spain, Portugal etc.) [4]
- Sales Networks (Unilever in Netherlands, Turkey, China/ P&G in US, India etc.)
- After Sales Services (Bosch in Germany,Turkey,India/ Siemens in Germany, Turkey, UK etc.)
- Daily Care Service Provider Companies (cleaning, cooking, laundry, nursery etc.) (Rinse in US, Helping Hands in UK)
- Bill delivery [5]
- Waste collection [6]
- Relief item distribution [7]
- Newspaper distribution [8]

Comparison in Turkey

1.How to solve this problem in Turkey?

★ Mobile Workplace Doctors

★ Some hospitals also provide this service (Söke State Hospital, Keçiören Hospital, Selçuk University Hospital etc.)

2. How are same/similar models used to solve other problems in Turkey?

- ➔ İstanbul Elektrik, Tramvay ve Tünel (IETT) İşletmeleri Genel Müdürlüğü- Metrobus System [9]
- ➔ MDVSP is modelled as MCNF
- ➔ Assigning routes and times to vehicles parking in multiple garages.
- ➔ Trying to use minimum number of vehicles and minimize the dead distance travelled

İETT
İŞLETMELERİ GENEL MÜDÜRLÜĞÜ

Decision Variables

□ $L_{d,i}$: Binary variable takes a value of 1 if trip i is the □rst trip of a bus schedule
and covered by a vehicle emanating from depot d and 0 elsewhere.

□ $x_{i,j,d}$: Binary variable takes a value of 1 if a dead-running trip between two compatible trips i and j is covered by a vehicle emanating from depot d and 0 elsewhere.□ $A_{i,d}$: Binary variable takes a value of 1 if trip i is the last trip of a bus schedule and covered by a vehicle emanating from depot d and 0 elsewhere.
□
$y_{i,d}$: Binary variable takes a value of 1 if trip i is covered by a vehicle emanating from depot d and 0 elsewhere.

Parameters

$l_{d,i}$: Cost of a pull-out trip from depot d to the starting station of trip i.

$c_{i,j,d}$: Cost of a dead-running trip from ending station of trip i to the starting station of trip j. Please note that the value of the parameter is independent from source depot of the vehicle covers trip j immediately after trip i.

$a_{i,d}$: Cost of pull-in trip from ending station of trip i to depot d.

r_d: Maximum number of vehicles emanating from depot d (depot capacities).

$$\min \quad \sum_d \sum_i l_{d,i} L_{d,i} + \sum_i \sum_j \sum_d c_{i,j,d} x_{i,j,d} + \sum_d \sum_i a_{i,d} A_{i,d}$$

$$\text{s.t.} \quad \sum_j L_{d,i} \leq r_d \qquad \forall d,$$

$$L_{d,i} + \sum_j x_{j,i,d} - y_{i,d} = 0 \qquad (j,i) \in C \qquad \forall i, d,$$

$$A_{i,d} + \sum_j x_{i,j,d} - y_{i,d} = 0 \qquad (i,j) \in C \qquad \forall i, d,$$

$$\sum_j A_{i,d} \leq r_d \qquad \forall d,$$

$$\sum_d y_{i,d} = 1 \qquad \forall i,$$

All variables are binary.

References

[1] Super N. Who will be there to care? The growing gap between caregiver supply and demand. National health policy forum, 1e19. Washington DC: George Washington University, http://www.nhpf.org/library/background- papers/BP_Caregivers_1-02.pdf; 2002.

[2] “The Benefits Of Residential Rehab Facilities.” *Residential Treatment Benefits*, www.treatment4addiction.com/treatment/types/residential/.

[3] Bauman, John. “The Three Benefits of Residential Treatment.” *Bradford Health Services - Drug and Alcohol Rehabilitation Center*, Bradford Health Services - Drug and Alcohol Rehabilitation Center, 2 June 2018, bradfordhealth.com/three-benefits-residential-treatment/.

[4] S. Rath and W. J. Gutjahr, “A math heuristic for the warehouse location routing problem in disaster relief,” Computers and Operations Research, vol. 42, pp. 25 – 39, 2014.

[5] C. K. Y. Lin, C. K. Chow, and A. Chen, “A location-routing-loading problem for bill delivery services,” Computers and Industrial Engineering, vol. 43(1- 2), pp. 5–25, 2002.

References

[6] T. Kulcar, “Optimizing solid waste collection in Brussels,” European Journal of Operational Research, vol. 90(1), pp. 71–77, 1996.

[7] A. Moreno, D. Alem, and D. Ferreira, “Heuristic approaches for the multi- period location transportation problem with reuse of vehicles in emergency logistics,” Computers and Operations Research, vol. 69, pp. 79 – 96, 2016.

[8] S. K. Jacobsen and O. B. G. Madsen, “A comparative study of heuristics for a two-level routing-location problem,” European Journal of Operational Research, vol. 5(6), pp. 378–387, 1980.

[9] Sevim, İsmail. *A State Space Reduction Heuristic for Multiple Depot Vehicle Scheduling Problem*. earsiv.sehir.edu.tr:8080/xmlui/bitstream/handle/11498/37315/000130142002.pdf?sequence=1&isAllowed=y.

Thank You!