An optimization model for the location of disaster refuges

Presentation Outline

- Introduction
- Assumptions
- Parameters & Decision Variables
- Model
- Extensions
- Augmented Model
- Results & Analysis
- Contemporary Examples
- Q&A

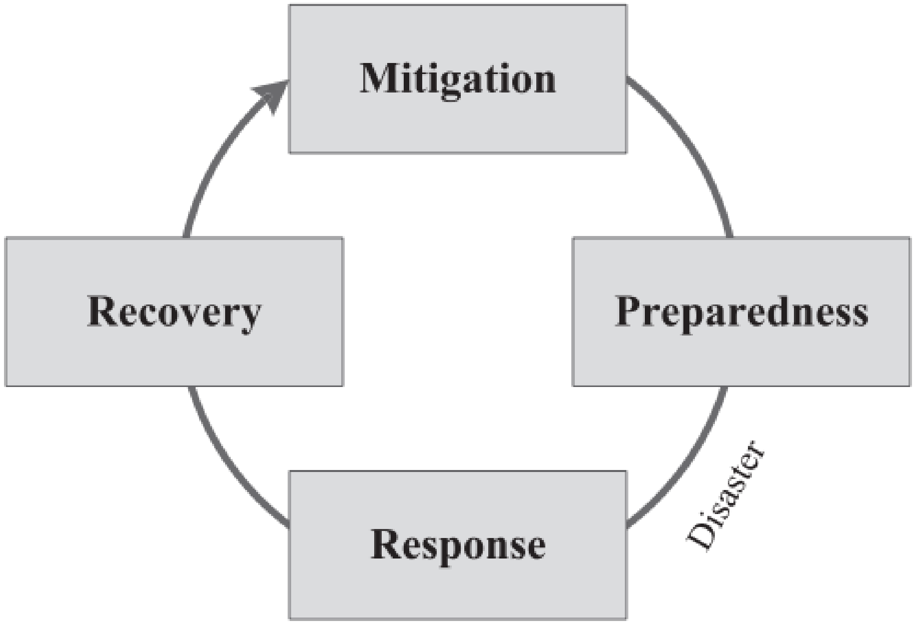
Disaster Management

- Despite technological changes, disasters still lead to huge monetary and life costs.
- December 2004 Indonesia earthquake – 266,000 dead and 500,000 injured
- March 2011 Japan Tsunami – 18,000 killed and nuclear fallout

Chile one of the Hotspots

- 2 of the worst earthquakes in history in Chile.
- Valdiva (1960, 9.5 magnitude)
- Cobquecura (2010, 8.8 magnitude)
- 1655 killed, 3000 injured, 2,000,000 left homeless & 550 million USD in damages in Southern Chile alone!

Disaster Management Steps



Disaster Management Steps

- Mitigation: To prevent the disaster from occurring/minimizing its effects if it occurs.
- Preparedness: Organize communities to prepare for incoming disasters.
- Response: Using emergency procedures to deliver necessary resources to the affected regions.
- Recovery: Stabilizing and restoring affected areas.

2-type Refuge Problem

- Service Level 1: Shelter and basic necessities
- Service Level 2: Medical and Psychological Care
- Type A refuge: Refuges that provide only Service Level 1
- Type B refuge: Refuges that provide both Service Levels 1 & 2
- Multistage optimization is required as we need to integrate decisions (localization of refuges & assignment of selected homes to refuges)

2-type Refuge Problem

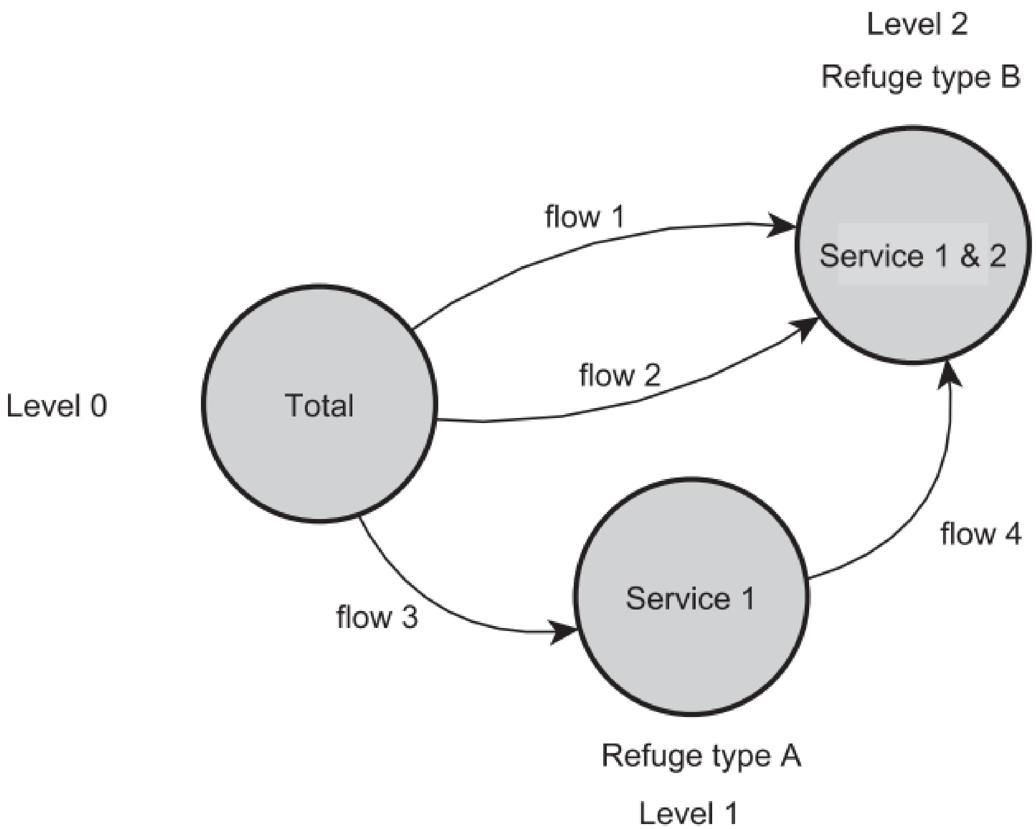


Fig. 2. Flow of disaster victims.

Parameters of the Model

W_i: Total number of households in square $i \in I$.
β_{is}: Proportion of households in square $i \in I$ requiring service $s \in S$.
d_{ij}: Rectangular distance between demand point $i \in I$ and $j \in J$.
d_{jk}: Rectangular distance between the refuge $j \in J$ and the refuge $k \in J$.

Parameters of the Model

α: Factor adjusting the importance of the two types of distance from a refuge of A type to a refuge of B type.
θ: Proportion of households moved from an A type to a B type refuge due to needing service 2.
N_t: Maximum number of refuges of type $t \in T$.
Q_{js}: Capacity of the refuge $j \in J$ for providing the service $s \in S$.
ρ_t: Level of minimum utilization of the refuge of type $t \in T$.
σ: Percentage of the capacity of B type refuges available for sheltering victims that do not require medical and psychological services.

Decision Variables

x_{ij}^{s}: Number of victims going from square $i \in I$ to the refuge $j \in J$ requiring service $s \in S$.

z_{jk}: Number of victims going from a refuge $j \in J$ providing service 1 to a refuge $k \in J$ providing service 2.

y_j^t: 1 if the refuge $j \in J$ is selected as type $t \in T$ and 0 otherwise.

Capacitated hierarchical model - HMCA

$$\text{Min } Z = \sum_{i \in I} \sum_{j \in J} \sum_{s \in S} d_{ij} \left(x_{ij}^{s} \right) + \alpha \sum_{j \in J} \sum_{k \in J} d_{jk} z_{jk} \quad (1)$$

Subject to

$$\sum_{j \in J} x_{ij}^{s} = \beta_{is} W_i, \quad \forall i \in I, \ \forall s \in S \quad (2)$$

$$\sum_{k \in J} z_{jk} \geq \theta \sum_{i \in I} x_{ij}^{1}, \quad \forall j \in J \quad (3)$$

$$\sum_{i \in I} x_{ij}^{1} \leq Q_{j1} y_j^1 + \sigma Q_{j2} y_j^2, \quad \forall j \in J \quad (4)$$

$$\sum_{i \in I} x_{ij}^{2} + \sum_{k \in J} z_{kj} \leq (1 - \sigma) Q_{j2} y_j^2, \quad \forall j \in J \quad (5)$$

$$\sum_{t \in T} y_j^t \leq 1, \quad \forall j \in J \tag{6}$$

$$\sum_{j \in J} y_j^t \leq N_t, \quad \forall t \in T \tag{7}$$

$$\sum_{i \in I} x_{ij}^1 \geq \rho_1 Q_{j1} y_j^1, \quad \forall j \in J \tag{8}$$

$$\sum_{i \in I} x_{ij}^2 + \sum_{k \in J} z_{kj} \geq (1 - \sigma) \rho_2 Q_{j2} y_j^2, \quad \forall j \in J \tag{9}$$

$$x_{ij}^s, \quad z_{jk} \in Z^+, \quad \forall i \in I, \forall j \in J, \forall k \in J, \forall s \in S \tag{10}$$

$$y_j^t \in \{0, 1\}, \quad \forall j \in J, \forall t \in T \tag{11}$$

Capacitated hierarchical model with refuge attribute selection ($HMCA_s$)

$$\sum_{j \in J} y_j^t \leq N_t, \quad \forall t \in T \tag{7}$$

$$\sum_{j \in J_t} y_j^t \leq N_t, \quad \forall t \in T$$

$$\sum_{j \in J \setminus J_t} y_j^t = 0, \quad \forall t \in T$$

Capacitated hierarchical model with refuge attribute selection and threshold of maximum distance ($HMCA_sD_t$)

$$x_{ij}^s(d_{ij} - d_{max}^s) \leq 0, \forall i \in I, \forall j \in J, \forall s \in S$$

$$z_{jk}\left(d_{jk} - dz_{max}\right) \leq 0, \forall j \in J, \forall k \in J$$

Capacitated hierarchical model with refuge attribute selection, threshold of maximum distance, and quality of service $HMCA_sD_tQ$

$$\max F = F_1 - F_2$$

$$F_1 = \frac{\omega}{H}\left(\sum_{i\in I}\sum_{j\in J}\sum_{s\in S} d'_{ij}x^s_{ij} + \alpha\sum_{j\in J}\sum_{k\in J} d'_{jk}z_{jk}\right)$$

$$F_2 = \frac{(1-\omega)}{|J|}\sum_{j\in J}\sum_{t\in T} c_j y^t_j$$

$$H = \sum_{i\in I}\sum_{s\in S}\beta_{is}W_i + \alpha\frac{\sum_{j\in J}\theta_j}{|J|}\sum_{i\in I}\beta_{i1}W_i$$

$$d'_{ij} = \frac{d_{max} - d_{ij}}{d_{max} - d_{min}}$$

Flow of victims to refuges

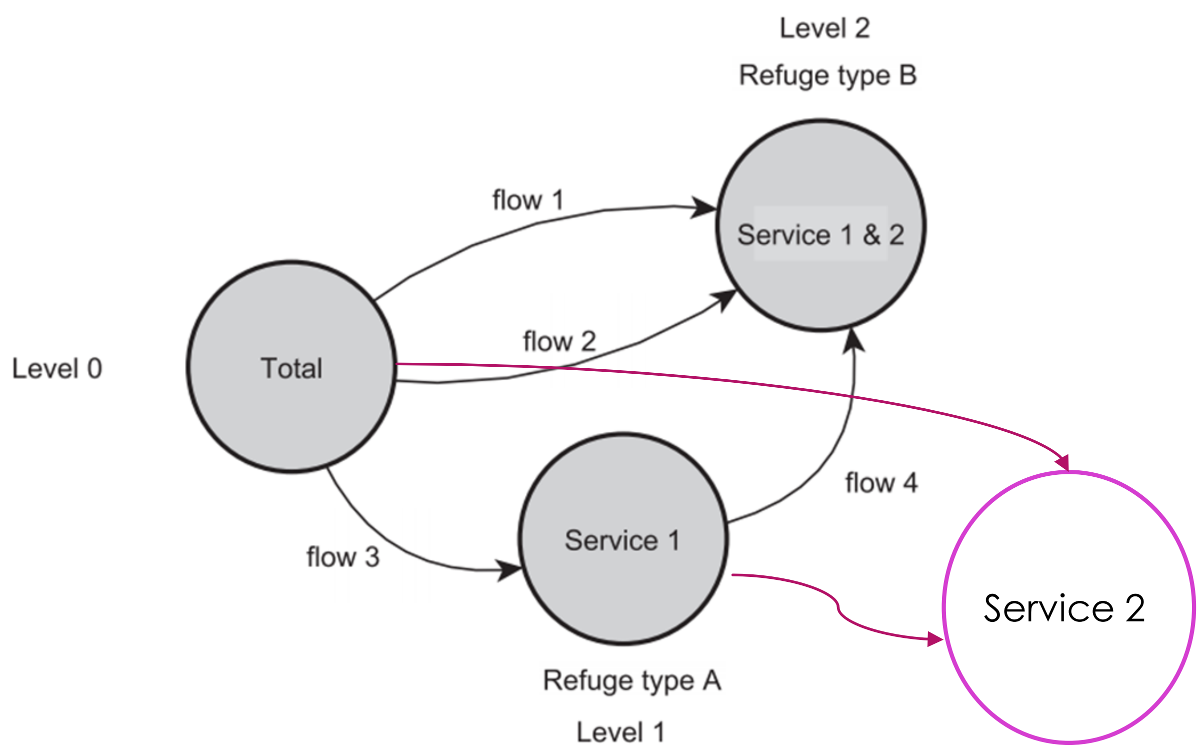


Fig. 2. Flow of disaster victims.

Model of 2 Interservice Referrals with refuge attribute selection

$$Min\ \ z = \sum_{i\epsilon I} \sum_{j\epsilon J} \sum_{s\epsilon S} \sum_{t\epsilon T} d_{ij} x_{ij}^{st} + \alpha_B \sum_{j\epsilon J} \sum_{k\epsilon K} d_{jk} z_{jk}^{B} + \alpha_C \sum_{j\epsilon J} \sum_{k\epsilon K} d_{jk} z_{jk}^{C} \qquad (1)$$

Subject to:

$$\sum_{j\epsilon J} \sum_{t\epsilon T} x_{ij}^{st} = \beta_{is} W_i \ , \quad \forall\, i \in I \, , \ \forall\, s \in S \ \ (1)$$

$$\sum_{k\epsilon J} z_{jk}^{t} \le \theta_t \sum_{i\epsilon I} x_{ij}^{1A} \ , \quad \forall\, j \in J \ , \forall\, t \in \{B, C\} \ \ (2)$$

$$\sum_{i\epsilon I} x_{ij}^{1A} \le Q_{jA} y_j^{A} \ , \quad \forall\, j \in J \ \ (3)$$

$$\sum_{i\epsilon I} x_{ij}^{1B} \le \sigma Q_{jB} y_j^{B} \ , \quad \forall\, j \in J \ \ (4)$$

$$\sum_{i\epsilon I} x_{ij}^{2B} + \sum_{k\epsilon J} z_{kj}^{B} \le (1-\sigma) Q_{jB} y_j^{B} \ , \quad \forall\, j \in J \ \ (5)$$

$$\sum_{i\epsilon I} x_{ij}^{2C} + \sum_{k\epsilon J} z_{kj}^{C} \le Q_{jC} y_j^{C} \ , \quad \forall\, j \in J \ \ (6)$$

$$\sum_{j\epsilon J_t} y_j^{t} \le N_t \ , \quad \forall\, t \in T \ \ (7)$$

Model of 2 Interservice Referrals with refuge attribute selection

$$\sum_{j\epsilon J/J_t} y_j^t = 0 \;, \quad \forall\, t \in T \quad (8)$$

$$\sum_{t\epsilon T} y_j^t \le 1 \;, \quad \forall\, j \in J \quad (9)$$

$$\sum_{i\epsilon I} x_{ij}^{1A} \ge \rho_A Q_{jA} y_j^A \;, \quad \forall\, j \in J \quad (10)$$

$$\sum_{i\epsilon I} x_{ij}^{2B} + \sum_{k\epsilon J} z_{kj}^B \ge \rho_B (1-\sigma) Q_{jB} y_j^B \;, \quad \forall\, j \in J \quad (11)$$

$$\sum_{i\epsilon I} x_{ij}^{2C} + \sum_{k\epsilon J} z_{kj}^C \ge \rho_C Q_{jC} y_j^C \;, \quad \forall\, j \in J \quad (10)$$

$$x_{ij}^{st}, z_{jk}^B \in Z^+ \qquad , \forall\, i \in I, \forall j \in J, \forall k \in J, \forall s \in S, \forall t \in T \quad (11)$$

$$y_j^t \in [0,1] \qquad , \forall j \in J, \qquad \forall t \in T \quad (12)$$

Result and Analysis

- When the number of installed refuges is minimum, the average distance traveled is maximum

Table 1

HMCA Results. SC: Scenario, α: Factor considering different travel costs, N_1: Maximum number of refuges A type, N_2: Maximum number of refuges B type, TSR (TC): Total selected refuges (Total capacity), NRS1 (C): Number of refuges A type selected (Capacity), NRS2 (C): Number of refuges type 2 selected (Capacity), ADT: Average distance traveled (m), SR: Selected refuges (A type), (B type).

SC	α	N_1	N_2	TSR (TC)	NRS1 (C)	NRS2 (C)	ADT	SR(Atype, Btype)
1	0.1	15	15	13 (1.191)	7 (804)	6 (387)	396.310	(2,3,5,6,8,11,14), (1,4,9,10,13,15)
2		6	6	11 (1.164)	5 (832)	6 (332)	418.672	(1,8,9,11,15), (3,4,5,7,13,14)
3		5	5	10 (1.149)	5 (832)	5 (317)	423.846	(1,8,9,11,15), (3,4,7,13,14)
4		4	4	8 (1.134)	4 (791)	4 (343)	625.146	(7,9,10,14), (1,4,12,13)
5	0.5	15	15	13 (1.160)	7 (825)	6 (335)	448.183	(2,3,5,6,8,9,14), (1,4,7,10,11,15)
6		6	6	12 (1.172)	6 (827)	6 (345)	481.188	(3,5,7,8,11,14), (1,2,4,6,13,15)
7		5	5	10 (1.149)	5 (832)	5 (317)	518.442	(1,8,9,11,15), (3,4,7,13,14)
8		4	4	8 (1.134)	4 (791)	4 (343)	693.650	(7,9,10,14), (1,4,12,13)
9	0.9	15	15	13 (1.160)	7 (825)	6 (335)	489.751	(2,3,5,6,8,9,14), (1,4,7,10,11,15)
10		6	6	12 (1.172)	6 (827)	6 (345)	530.771	(3,5,7,8,11,14), (1,2,4,6,13,15)
11		5	5	10 (1.183)	5 (823)	5 (360)	588.942	(3,5,7,10,14), (1,2,4,12,13)
12		4	4	8 (1.134)	4 (791)	4 (343)	759.022	(7,9,10,14), (1,4,12,13)

Result and Analysis

For HMCAsDtQ;

Table 4

$HMCA_SD_TQ$ Results. *SC*: Scenario, *TSR* (*TC*): Total selected refuges (Total capacity), *NRS1* (*C*): Number of refuges A type selected (Capacity), *NRS2* (*C*): Number of refuges B type selected (Capacity), *ADT*: Average distance traveled (m), *AQI*: Average quality penalty index of selected refuges, *OF*: Objective function value, *SR*: Selected refuges (A type), (B type).

SC	*ω*	*TSR* (*TC*)	*NRS1* (*C*)	*NRS2* (*C*)	*ADT*	*AQPI*	*OF*	*SR*
1	0.9	12 (1227)	4 (886)	8 (341)	429.582	0.333	0.55036	(1,7,8,14), (3,4,5,9,10,11,12,15)
2	0.8	11 (1212)	4 (886)	7 (326)	434.558	0.309	0.46378	(1,7,8,14), (3,4,9,10,11,12,15)
3	0.7	11 (1182)	4 (834)	7 (348)	441.302	0.291	0.37797	(1,6,11,14), (3,4,7,9,10,12,15)
4	0.6	10 (1176)	4 (855)	6 (321)	452.113	0.290	0.29482	(1,6,9,14), (3,4,7,10,11,12)
5	0.5	10 (1176)	4 (855)	6 (321)	452.113	0.290	0.21346	(1,6,9,14), (3,4,7,10,11,12)
6	0.4	10 (1134)	4 (813)	6 (321)	472.678	0.270	0.13280	(1,9,14,15), (3,4,7,10,11,12)
7	0.3	10 (1134)	4 (813)	6 (321)	472.678	0.270	0.05460	(1,9,14,15), (3,4,7,10,11,12)
8	0.2	9 (1194)	3 (868)	6 (326)	554.988	0.256	−0.01677	(1,13,15), (3,4,7,9,10,12)
9	0.1	9 (1194)	3 (868)	6 (326)	554.128	0.256	−0.08505	(1,13,15), (3,4,7,9,10,12)

Computational performance of the optimization model

- 135 squares
- 15 refuges
- set of 4 groups

Table 5
Statistics of the running times and gap MILP on CPLEX 12.5. Each group is composed of 10 instances $p-q$. p represents the number of potential refuges and q represents the number of blocks.

Model	Instance p-q	Gap [%]			Time [s]		
		min	av	max	min	av	max
HMCA	15—100	0,00	0,00	0,02	5,00	309,60	901,00
	30—400	0,02	0,19	0,67	900,00	900,00	900,00
	90—900	1,70	3,40	4,80	900,00	900,00	900,00
	160—1600	1,66	38,33	67,14	900,00	900,00	900,00
$HMCA_SD_TQ$	15—100	0,00	0,03	0,09	0,65	630,34	900,00
	30—400	0,00	0,56	5,00	3,98	373,25	900,00
	90—900	0,00	1,54	6,68	91,97	810,22	900,00
	160—1600	0,13	6,58	55,70	900,00	900,00	900,00

Locating disaster response facilities in Istanbul

- Provide warehouses for the pre-disaster storage of relief items such as tents, sleeping bags, blankets, medical first aid kits, dry food and water.

Locating disaster response facilities in Istanbul Model

Parameters

$D_{i,j}$: Distance between neighbourhood i and neighbourhood j

R_i: Refugee population in neighbourhood i

F: Number of TFs to locate in the district

C: Capacity of a TF

Decision variables

f_i: Number of TFs in neighbourhood i

$x_{i,j}$: Number of people in neighbourhood i who are served by the TFs in neighbourhood j

- TF:temporary facilities

Model

$$\text{Min} \sum_{i,j} D_{i,j} x_{i,j} \tag{1}$$

subject to

$$\sum_i f_i = F \tag{2}$$

$$\sum_j x_{i,j} = R_i \; \forall i \tag{3}$$

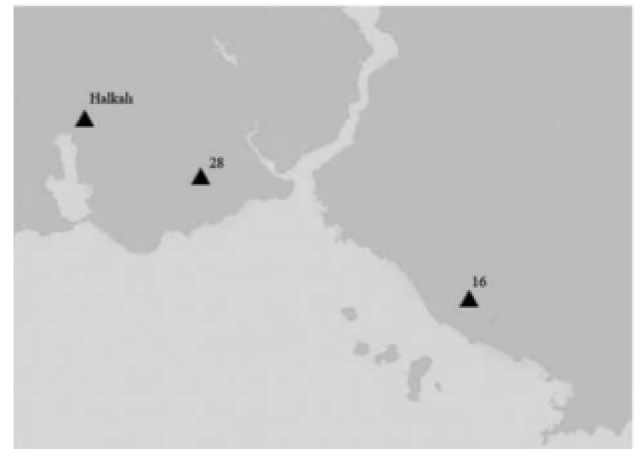
$$\sum_i x_{i,j} \leqslant C f_j \; \forall j \tag{4}$$

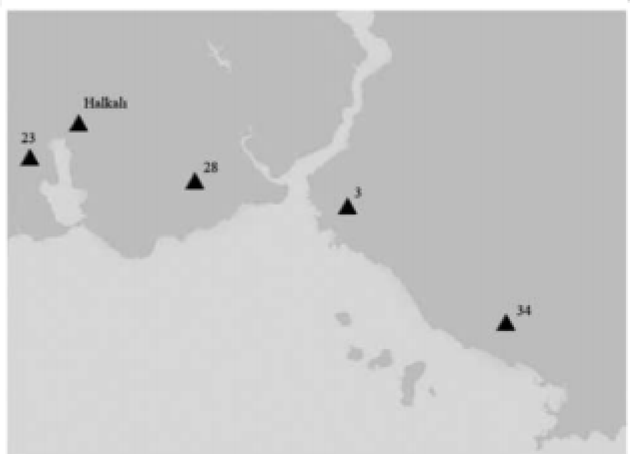
$$x_{i,j} \geqslant 0 \; \forall i, j \tag{5}$$

$$f_i \geqslant 0, \text{ integer } \; \forall i \tag{6}$$

Table 1 Summary of solutions for different *MaxFac* values

	Maximum number of facilities (MaxFac)				
	1	2	3	4	5
Average distance (km)	21.75	13.67	9.29	7.95	7.05
Maximum distance (km)	89.54	71.11	71.11	62.74	62.74
Open facilities	41	16, 41	16, 28, 41	16, 23, 28, 41	3, 23, 28, 34, 41





Additional Example; Locating temporary shelter areas after an earthquake: A case for Turkey

Sets

I: set of candidate locations
J: set of districts

Parameters

w_i: weight of candidate location i, between 0 and 1
d_i^{health}: distance between candidate location i and nearest health center
d_i^{road}: distance between candidate location i and nearest main road
$Demand_j$: total demand of district j in terms of square meters
cap_i: capacity of candidate location i
$distSorted_{ij}$: ith closest candidate location index to demand point j
DistHealth: threshold value for shelter area–health center distance
DistRoad: threshold value for shelter area–main road distance
α: threshold value for pair wise utilization difference of candidate shelter areas
β: threshold value for minimum utilization of open shelter areas

Decision variables

$$x_i: \begin{cases} 1 & \text{if candidate location } i \text{ is chosen as a shelter area} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij}: \begin{cases} 1 & \text{if district } j \text{ is assigned to location } i \\ 0 & \text{otherwise} \end{cases}$$

w_{min}: minimum weight of operating candidate shelter areas

Formulation

Formulation

$$\text{maximize } w_{min} \tag{1}$$

$$w_{min} \leq x_i w_i + (1 - x_i) \quad \forall i \in I \tag{2}$$

$$\sum_{j \in J} y_{ij} \times Demand_j \leq cap_i \times x_i \quad \forall i \in I \tag{3}$$

$$\sum_{i \in I} y_{ij} = 1 \quad \forall j \in J \tag{4}$$

$$d_i^{health} \times x_i \leq \text{DistHealth} \quad \forall i \in I \tag{5}$$

$$d_i^{road} \times x_i \leq \text{DistRoad} \quad \forall i \in I \tag{6}$$

$$\frac{\sum_{j \in J} y_{ij} \times Demand_j}{cap_i} \geq \beta x_i \quad \forall i \in I \tag{7}$$

$$\frac{\sum_{j \in J} y_{ij} \times Demand_j}{cap_i} - \frac{\sum_{j \in J} y_{kj} \times Demand_j}{cap_k} \leq \alpha + (1 - x_i) + (1 - x_k) \quad \forall i \in I, k \in I, i \neq k \tag{8}$$

$$y_{distSorted(1,j),j} = x_{distSorted(1,j)} \quad \forall j \in J \tag{9a}$$

$$y_{distSorted(i,j),j} \geq x_{distSorted(i,j)} - \sum_{k=1}^{i-1} x_{distSorted(k,j)} \quad \forall j \in J, \; i = 2 \ldots |I| \tag{9b}$$

$$x_i \in \{0, 1\} \quad \forall i \in I \tag{10a}$$

$$y_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J \tag{10b}$$

Result

Table 9
Comparison of Turkish Red Crescent's solution and our solution.

	TRC's solution	Our solution
The maximum distance (m)	5486.645	5486.645
The minimum utilization	0.095	0.277
The maximum utilization	0.767	0.967
The average utilizaton	0.375	0.569
Total distance (m)	110607.517	107317.701

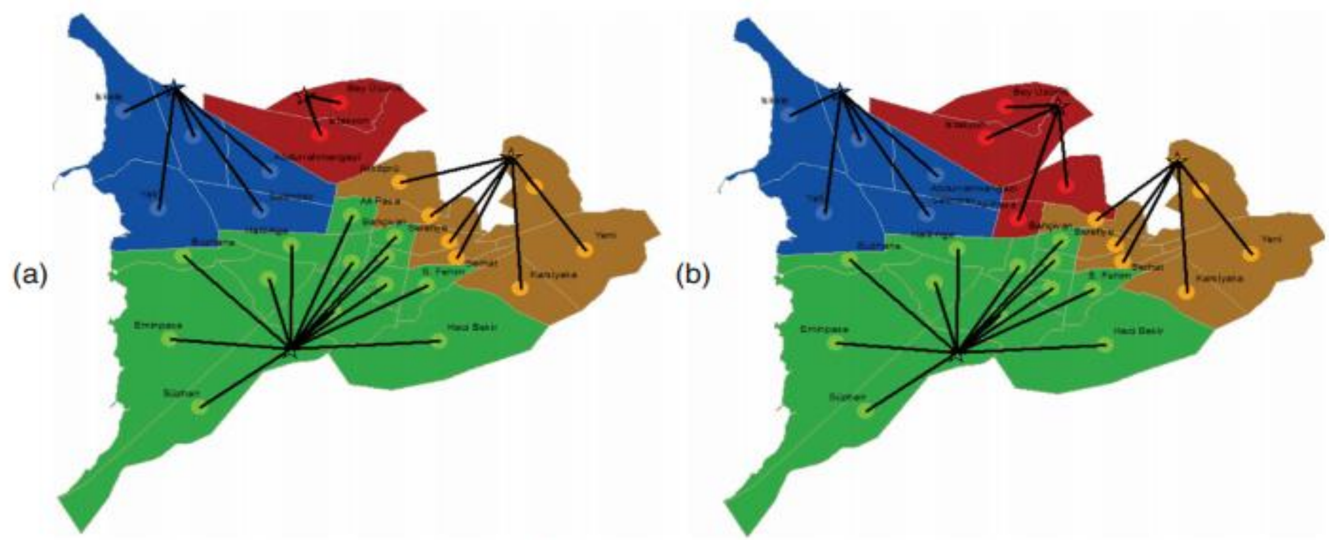


Fig. 3. (a) Turkish Red Crescent's solution. (b) Our solution.

References

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- Kilci F, Kara B, Bozkaya B. Locating temporary shelter areas after an earthquake: a case for Turkey. Eur J Oper Res 2015;

Q/A

- Any question ?