Facility location under demand uncertainty:

Response to a large scale Bio-terror Attack

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Introduction

Deliberate release of viruses to cause illness

Key Decisions

- Where to open the facility

  (POD: points of disbursement)

- How many items to stock at each facility

Uncertainty

- ? Location of the emergency ?
- ? How many people are affected ?

“Capacitated Facility Location Model”

TO DECIDE: Which facilities to open and how many supplies to make available at each POD

IN ORDER TO: maximize the coverage of an unknown demand

IN: the event of a bio-terror attack

Assumptions

- Distance vs Fraction of people covered -- decreasing step function
- POD is capacitated -- maximum rate of service is limited
- Total amount of S -- scarce resources
- Demand is assigned to the facilities to maximize the covered demand

Coverage Bound Function

Distance (dij) ↑ Fraction of the people covered ↓

- This creates "multiple coverage levels"

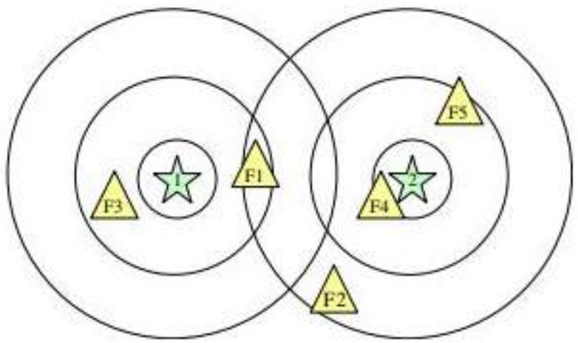


Fig. 1. Demand loss function.

Distances

$$0 = \delta_0 < \delta_1 < \delta_2 < \ldots < \delta_K$$

Proportion that can be reached

$$1 = f_1 > f_2 > \ldots > f_K > 0.$$

Covered demand

$$f_k D_i$$

Deterministic Model

- Which set of pre-specified facilities need to be opened
- I : set of demand points
- J: set of facility locations
- Each demand point has K levels of coverage
- Known demand

Deterministic Model

Parameters

S: total supply available during an emergency
N: total number of facilities that need to be opened
β_j: capacity of facility $j \in J$
D_i: demand for medical supplies from demand point $i \in I$
f_k: a fraction, $f_k D_i$ is the k-th level coverage bound on the demand of point i
δ_k: radius of the k-th coverage level from a demand point
d_{ij}: distance between demand point i and facility j

Deterministic Model

Decision Variables

x_j: takes a value of 1 if facility $j \in J$ is open and 0 otherwise
s_j: supply to be assigned to facility $j \in J$
t_{ij}: amount of medical supplies allocated to demand point i by facility j

Deterministic Model

Identify the locations to open facilities and their respective supplies ($s < S$)

Maximize Coverage

Under Certain Demand

Deterministic Model

Modeling

$$\max \sum_{i \in I, j \in J} t_{ij}$$

$$\text{s.t.} \quad \sum_{j \in J} x_j = N$$

$$\sum_{i \in I} t_{ij} \leq s_j \quad \forall j \in J$$

$$s_j \leq \beta_j x_j \quad \forall j \in J$$

$$\sum_{j \in J} s_j \leq S$$

$$\sum_{j \mid \delta_{k-1} < d_{ij} \leq \delta_k} t_{ij} \leq f_k D_i \quad \forall i \in I, k \in 1, \ldots, K$$

$$\sum_{j \in J} t_{ij} \leq D_i \quad \forall i \in I$$

$$\sum_{j \mid d_{ij} > \delta_k} t_{ij} = 0 \quad \forall i \in I$$

$$x_i \in \{0,\ 1\}; \quad s_j, t_{ij} \geq 0$$

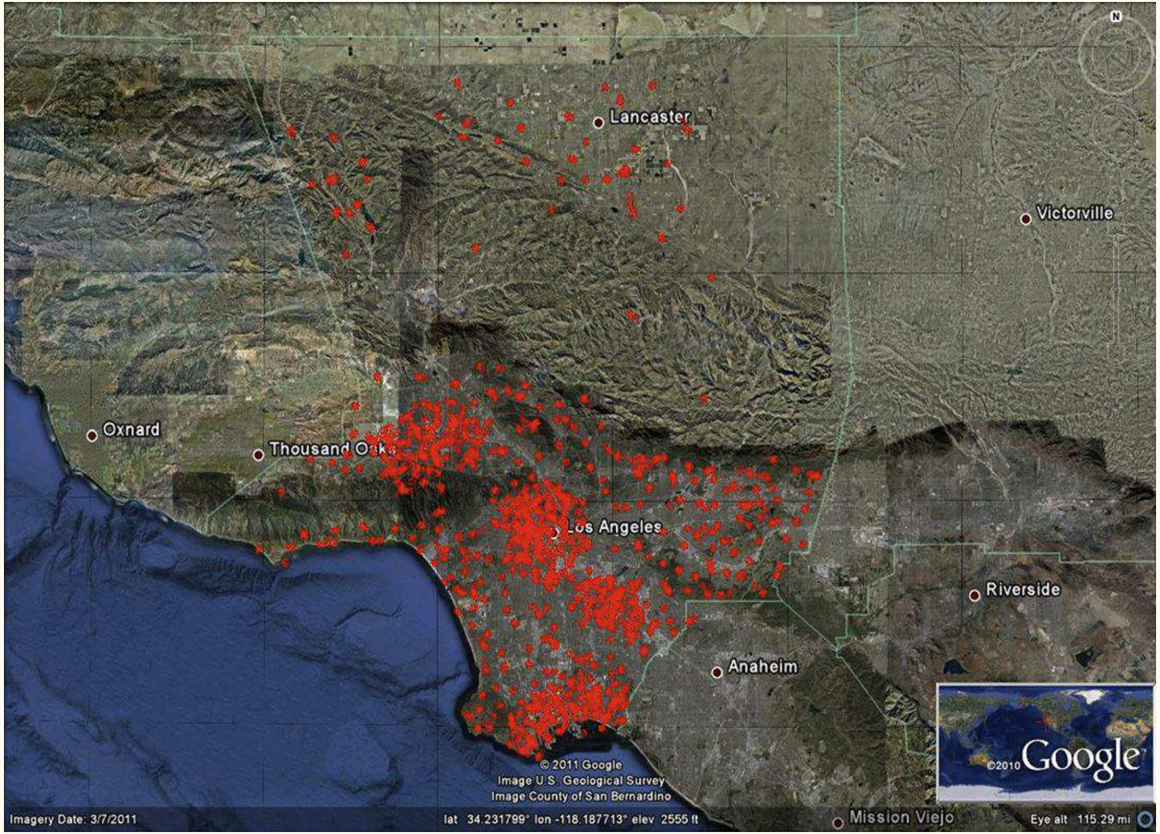
Experimental Analysis

Deterministic Model

- Anthrax attack on L.A. County
- Vaccines and Antibiotics are Required
- Fatal if left untreated

Experimental Analysis

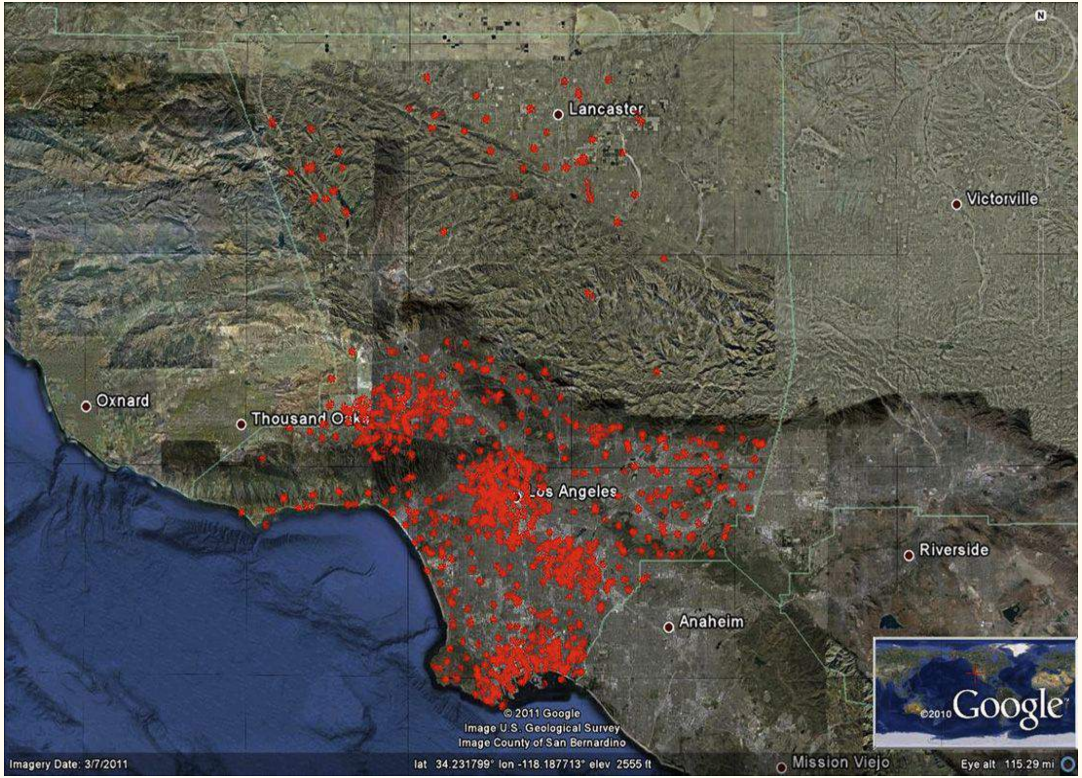
- 1939 Demand Points
- Demand: 0 to Aggregated Population
- 200 Candidate Facility Sites
- Loss Function Coverage Levels: 4, 8 and 12 miles
- f1 = 100%, f2 = 65%, f3 = 30%
- Facilities have a supply of 560,000 units
- S = some fraction of E[total D]



Experimental Analysis

Deterministic Model

- Mean Demand is Considered as Actual Demand
- Number of Facilities to be Opened and Service Levels are Varied
- Results are Compared with "Simulated Annealing Procedure"



Experimental Analysis

Deterministic Model

In a deterministic model, Locate-Allocate Heuristic outperforms Simulated Annealing Heuristic!

Coverage from the deterministic model using locate-allocate heuristic.

γ	100%	90%	80%	70%	50%	30%	20%
$N = 50$	94.19	89.99	80	70	50	30	20
$N = 40$	89.66	88.32	80	70	50	30	20
$N = 30$	84.96	83.68	77.01	70	50	30	20
$N = 20$	74.56	72.26	72.09	70	50	30	20

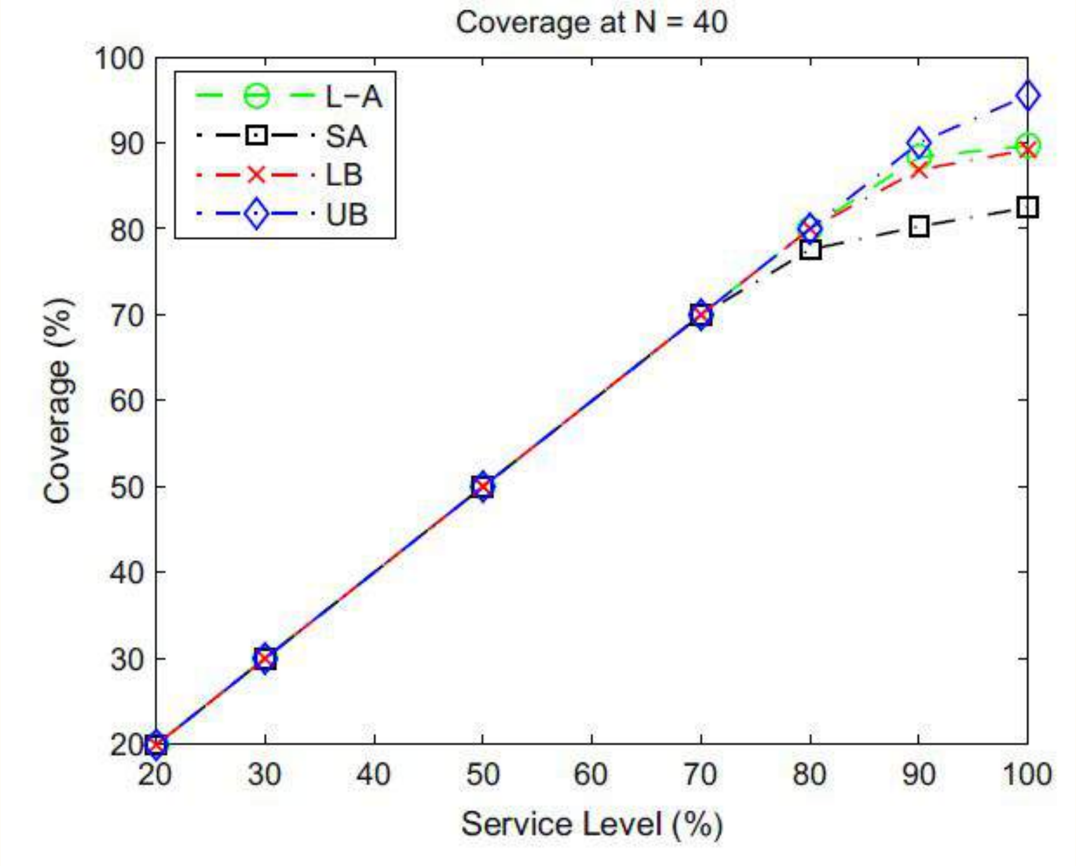
Coverage from the deterministic model using the simulated annealing heuristic.

γ	100%	90%	80%	70%	50%	30%	20%
$N = 50$	90.86	84.71	79.96	70	50	30	20
$N = 40$	82.45	80.24	77.58	69.97	50	30	20
$N = 30$	79.67	76.44	74.30	69.89	50	30	20
$N = 20$	71.34	69.91	68.83	68.67	50	30	20

Experimental Analysis

Deterministic Model

In a deterministic model, Locate-Allocate Heuristic outperforms Simulated Annealing Heuristic!



Chance-Constrained Model

We now consider the case when the demand for medical supplies at each demand point is <u>not known</u>. This means that constraints 5 and 6 of **DM** have some degree of uncertainty around them. We assume that the possible demand values due to an emergency event follow a random variable for which we know the probability distribution at each demand point. One solution in this case is to ignore the uncertainty, replace the random variable D_i with its expected value $E[D_i]$ and use the deterministic model **DM** to get a solution. However, ignoring the uncertainty can be risky in emergency situations.

Chance-Constrained Model

Modeling

$$\Pr\left(\sum_{j|\delta_{k-1}<d_{ij}\leq\delta_k} t_{ij} \leq f_k \xi^i\right) \geq 1-\varepsilon \quad \forall i \in I, k \in 1,\ldots,K$$

$$\Pr\left(\sum_{j \in J} t_{ij} \leq \xi^i\right) \geq 1-\varepsilon \quad \forall i \in I$$

ε is a small probability. $\varepsilon \in (0,1)$

The chance-constraint aims to identify an assignment of demand to facilities t_{ij} that could meet the demand at
the facilities with high probability. In other words, regardless of how the demand x_i changes, the assignment to facilities ensures that there is supply to satisfy some of this demand.

Experimental Analysis

Chance-Constrained Model

κ = the safety factor
ε = uncertainty
σ = standard deviation

We have already verified the quality of the locate-allocate heuristic for solving the deterministic model, we investigated the performance of the chance-constrained model and the heuristic under demand uncertainty.

Table 9
Coverage from the chance-constrained model.

κ	ε	$\sigma = 10\%$	$\sigma = 20\%$	$\sigma = 40\%$
1.96	0.025	62.52	52.57	41.62
1.64	0.050	63.47	53.16	42.11
1.44	0.075	64.08	53.38	42.65
1.28	0.100	64.66	54.09	44.83
1.15	0.125	64.94	55.13	45.54
1.04	0.150	65.18	56.69	46.73
0.93	0.175	65.27	57.21	48.44
0.84	0.200	65.78	59.32	49.23

We define κ to denote the Z value of the normal distribution corresponding to the confidence level ε.
$\phi(\kappa)=1-\varepsilon$.

For low values of ε (corresponding to high values of κ and low uncertainty), the constraint that $\sum_j t_{ij}$ should never exceed $f_k \xi^i_\varepsilon$ can be satisfied easily. Hence, $\sum_j t_{ij} \le f_k \xi^i_\varepsilon$ is a tight constraint. This explains the low coverage values in Table 9 above. As ε increases (that is, uncertainty increases), ξ^i_ε would need to be set to a higher value so that $\sum_j t_{ij}$ never exceeds $f_k \xi^i_\varepsilon$. As a result, coverage values increase.

Experimental Analysis

Chance-Constrained Model

Table 11 shows the merit of using the chance-constrained model to locate facilities, determine their supplies and allocate demand points to the open facilities. It also shows the relative gain in the coverage, achieved by using the chance-constrained model over the deterministic model, increases with a decrease in the safety factor and an increase in the standard deviation. The first is due to an increase in the numerator values as k decreases, and the second is due to a decrease in the denominator values as demand uncertainty increases. Under the best-case scenario, we see nearly a 20% gain in coverage by resorting to the locations given by the chance- constrained model.

Table 11

Ratio of the performance of the **CCM** to the **DM** in response to a random demand.

κ	ε	$\sigma = 10\%$	$\sigma = 20\%$	$\sigma = 40\%$
1.96	0.025	0.9799	0.9820	1.0152
1.64	0.050	0.9917	1.0048	1.0727
1.44	0.075	0.9962	1.0282	1.1082
1.28	0.100	1.0094	1.0390	1.1301
1.15	0.125	1.0138	1.0689	1.1353
1.04	0.150	1.0218	1.0797	1.1514
0.93	0.175	1.0242	1.0902	1.1675
0.84	0.200	1.0265	1.1058	1.1887

Heuristics -- much faster

- Locate N facilities
- Assign demand points to this facilities
- Create clusters of demand points served by each opened facility
- Try to reallocate the facilities to minimize the total distance in each cluster (supply stays the same)
- Stop when no more reallocation occurs

Alternative Actions

- Fair amount of literature on facility location exists
  - Locating disaster response facilities in Istanbul
  - Locating temporary shelter areas after an earthquake: A case for Turkey
- Uncertainty Aspect is a Common Trait of Both Natural Disasters and Terror Attacks
- A frequently used tool is “Maximal Covering Location Problem”, however, MCLP assumes full or no coverage, hence the previous usage of Coverage Bound Function (GMCLP)

Alternative Actions

- Berman and Krass (1998) ⟶ Competitive facility location
- Aboolian et al. (2007) ⟶ Facilities compete for demand

- Rawls and Turnquist (2010) ⟶ two-stage stochastic model, locate & supply
- Berman & Gavious (2007) ⟶ competitive locations responding terror attacks
- Jia et al. (2006) ⟶ Uncapacitated version of covering model

etc.

Alternative Actions

However…

There are not many papers on **maximal covering models with chance-constraints.**

Conclusions

The main contribution of the work is in designing a response strategy to distribute supplies in a large-scale emergency that considers distance-sensitive coverage, in addition to demand uncertainty. The constraints were:

- The facilities are capacitated by the service rate of a POD.
- The demand satisfied depends on the distance to the facility.
- Given the unpredictability as to when and where such an emergency scenario could occur and how many people would be affected, there is a significant uncertainty in demand values.

Conclusions

The aim was to identify locations and a way of distributing supplies that will be effective in meeting the uncertain demand. In an emergency situation an overriding objective is to <u>service as much of the demand as possible</u>. In such a situation, an effective placement of supplies so that the service demand is more important than an accurate model of how demand is distributed among facilities.

References

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Any Questions?