Hanan Ismail
Gözde Uğur
Şerife Yetişkin

# Location-Routing Problem for Used Oil

# Purpose of the Paper

To optimize

- ❑ facility location
- ❑ capacity acquisition
- ❑ vehicle routing decisions

For used-oil waste

## Contribution of the Paper

- ❑ Incorporation of vehicle routing problem (VRP)
  - ■ Adaptation of two commodity flow formulation
- ❑ Integration of an advanced assessment of environmental risk

# The Used Oil Management Framework

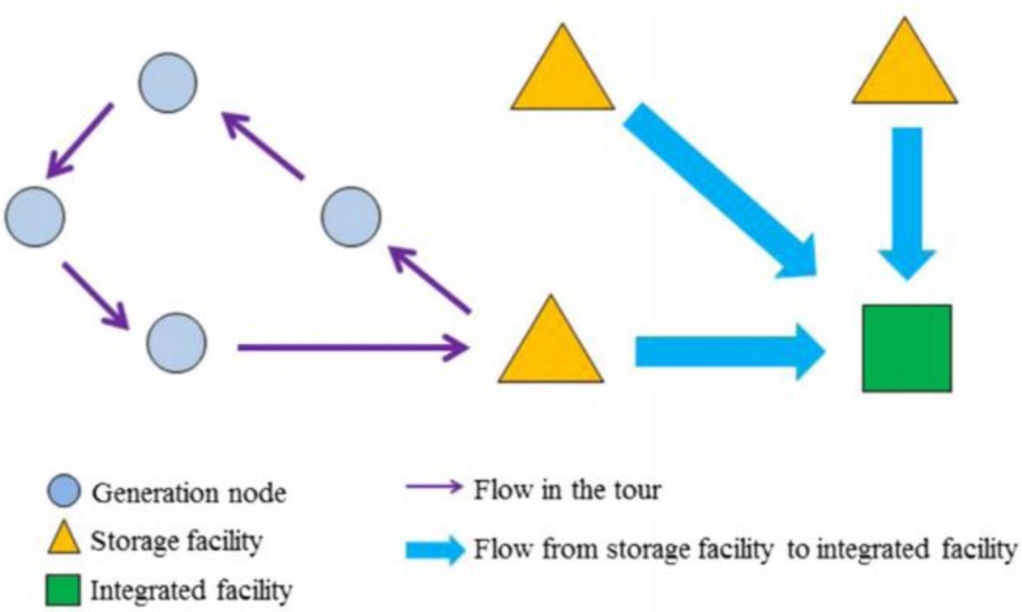
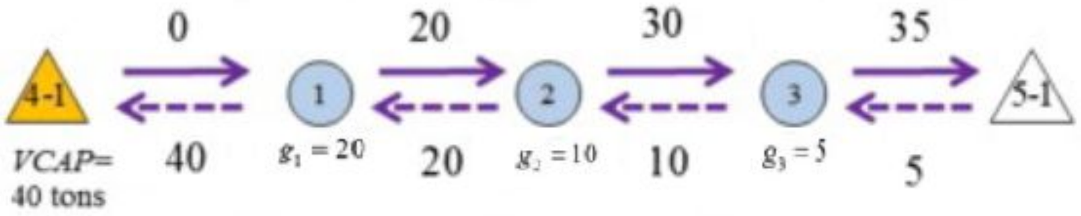
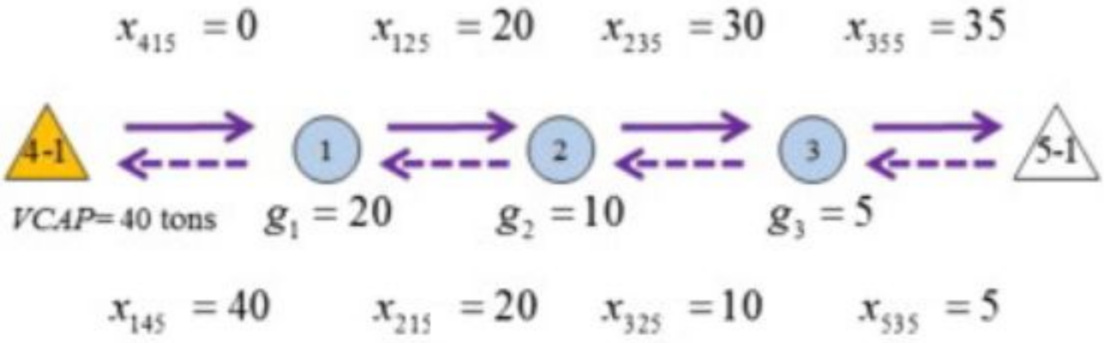



**Fig. 1.** The used oil management framework.

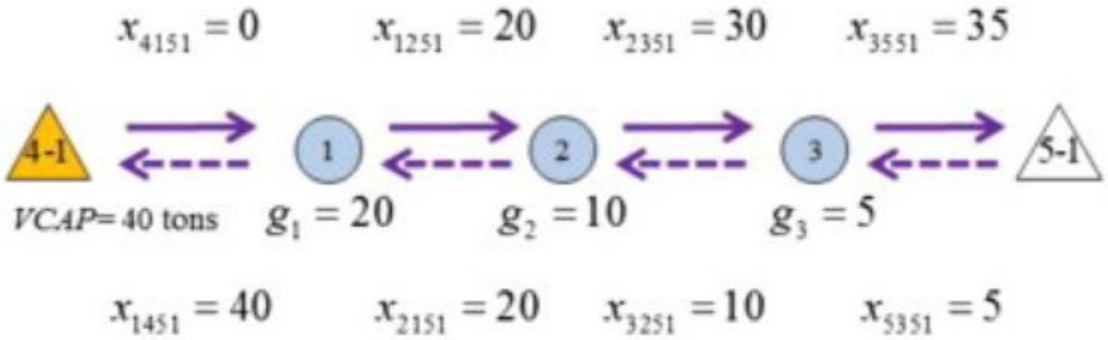
# Two-Commodity Flow



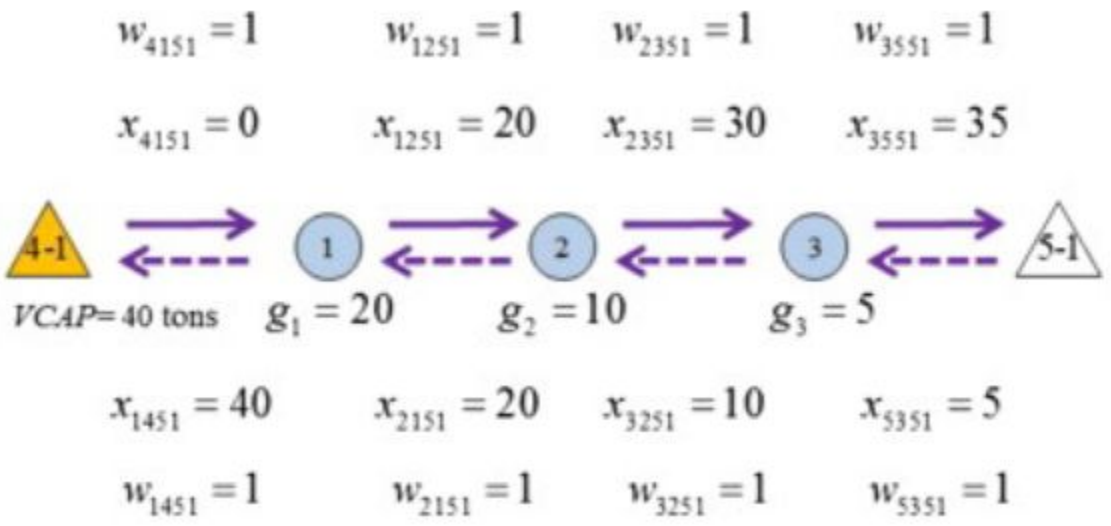
# Two-Commodity Flow



# Two-Commodity Flow



# Two-Commodity Flow



# Standard VRP Constraints

$$\sum_{\substack{j \in G \cup S \cup \bar{S} \\ j \neq i}} (x_{ijml} - x_{jiml}) = 2 \times g_i \times z_{iml}, \quad \forall i \in G, \forall m \in \bar{S}, \forall l \in L$$

Balancing of in- and outflow at generators

$$\sum_{i \in G} \sum_{m \in \bar{S}} \sum_{l \in L} x_{miml} = K \times VCAP - \sum_{i \in G} g_i$$

Total outflow of $\bar{S}$ = Remaining Capacity

$$\sum_{i \in G} x_{imml} = \sum_{i \in G} g_i \times z_{iml}, \quad \forall m \in \bar{S}, \forall l \in L$$

Total inflow to $\bar{S}$ = Total Amount Collected

$$\sum_{i \in G} \sum_{j \in S} \sum_{m \in \bar{S}} \sum_{l \in L} x_{ijml} \leq K \times VCAP$$

Total inflow is limited to vehicle capacity

# Additional Constraints

$$x_{ijml} + x_{jiml} = VCAP \times w_{ijml}, \quad \forall i,j \in G \cup S \cup \bar{S}, i \neq j, \forall m \in \bar{S}, \forall l \in L$$

Space + Storage = Capacity

$$\sum_{\substack{i \in G \cup S \cup \bar{S} \\ i \neq j}} w_{ijml} = 2 \times z_{jml}, \quad \forall j \in G, \forall m \in \bar{S}, \forall l \in L$$

Each generation node has two incident edges.

$$\sum_{\substack{i \in G \cup S \cup \bar{S} \\ i \neq m}} \sum_{l \in L} (w_{ijml} + w_{jiml}) = 0, \quad \forall j \in S, \forall m \in \bar{S}, m - j \neq |S|$$

One vehicle cannot visit more than one copy

$$\sum_{\substack{i \in G \cup S \cup \bar{S} \\ i \neq m}} \sum_{l \in L} (w_{ijml} + w_{jiml}) = 0, \quad \forall j, m \in \bar{S}, j \neq m$$

No links between copy storages

# Additional Constraints

$$\sum_{m \in \bar{S}} \sum_{l \in L} w_{ijml} = 0, \quad \forall i, j \in G \cup S \cup \bar{S}, i = j$$

Eliminate the trivial sub-tours

$$\sum_{m \in \bar{S}} \sum_{l \in L} z_{iml} = 1, \quad \forall i \in G$$

Transport oil to only one $\bar{S}$ with only one capacity level

$$w_{ijml} \leq o_{il}, \quad \forall i \in S, \forall j \in G \cup S \cup \bar{S}, \forall m \in \bar{S}, m - i = |S|, \forall l \in L$$

Guarantee that the vehicle can only start at an open storage facility

# Additional Constraints

$$\sum_{j\in F}\sum_{l\in L} y_{ijl} = \sum_{l\in L} s_{il}, \quad \forall i \in S$$

$$s_{il} \leq o_{il} CAP_{il}, \quad \forall i \in S, \forall l \in L$$

$$f_{il} \leq o_{il} CAP_{il}, \quad \forall i \in F, \forall l \in L$$

Capacity Constraints

$$\sum_{l\in L} o_{il} \leq 1, \quad \forall i \in S \cup F$$

Each facility can only take one capacity level

# Cost Function

$$h_1 = \sum_{i \in S \cup F} \sum_{l \in L} FC_{il} o_{il}$$

Fixed Cost of Opening Facility

$$+ \sum_{i \in S \cup F} \sum_{l \in L} VC_i \left( s_{il} + f_{il} \right)$$

Variable Operating Cost

$$+ \sum_{i \in S} \sum_{j \in F} \sum_{l \in L} TC^{treat} D_{ij} y_{ijl}$$

Storage to Integrated facility Transportation Cost

$$+ \sum_{i,j \in G \cup S \cup \bar{S}} \sum_{m \in \bar{S}} \sum_{l \in L} \frac{1}{2} TC^{store} \times D_{ij} w_{ijml}$$

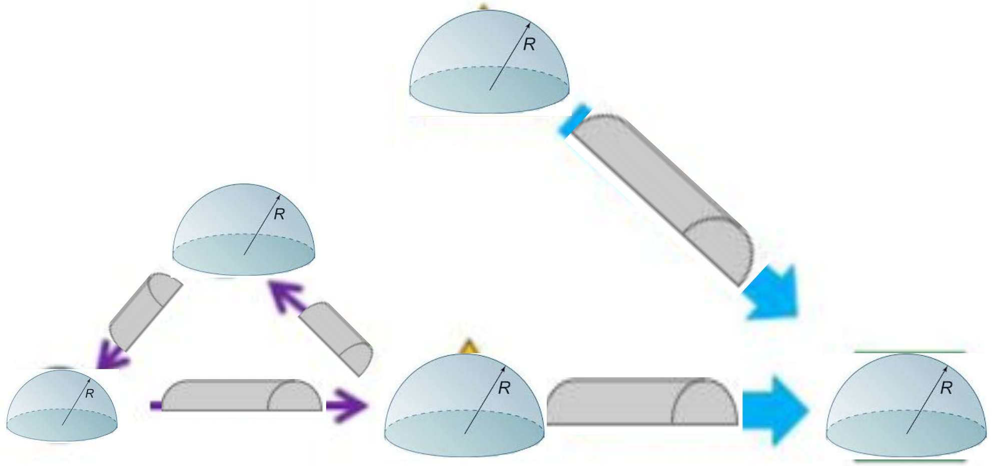
Generation to Storage Transportation Cost

$$+ K \times VFC$$

Fixed Vehicle Cost

# Environmental Risk Function

$$h_2 = \sum (\text{Volume of hemisphere around nodes}) + (\text{Volume of semi-cylinder around route}) + (\text{Risk of return trips})$$



# Bi-Objective Function

To construct the bi-objective function, we:

- ❑ Find the optimal value of each of $h_1$ and $h_2$ separately
- ❑ Multiply that with a normalization factor
- ❑ Assign each of $h_1$ and $h_2$ a weight $\lambda$

$$\min h_3 = \lambda \frac{100}{h_1^*}(h_1 - h_1^*) + (1-\lambda)\frac{100}{h_2^*}(h_2 - h_2^*)$$

# Chongqing Case

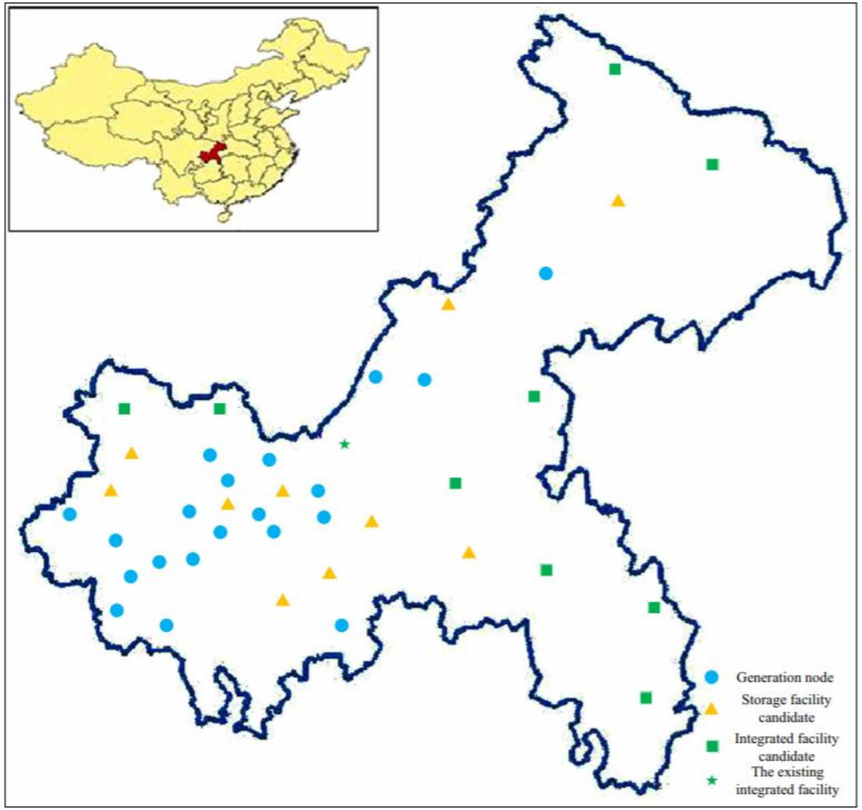



Fig. 3. Used oil generation nodes and alternative facility sites in Chongqing.

**Table 5**
Optimal values for $M(A)-1$ and $M(A)-2$.

| Model | Objective | Annual cost (yuan) | Environmental risk (km$^3$) | CPU time (s) |
|---|---|---|---|---|
| *M(A)-1* | Min cost | **433,360** | 84,222 | 1614.61 |
| *M(A)-2* | Min envir. risk | 1,213,400 | **22,351** | 2414.05 |

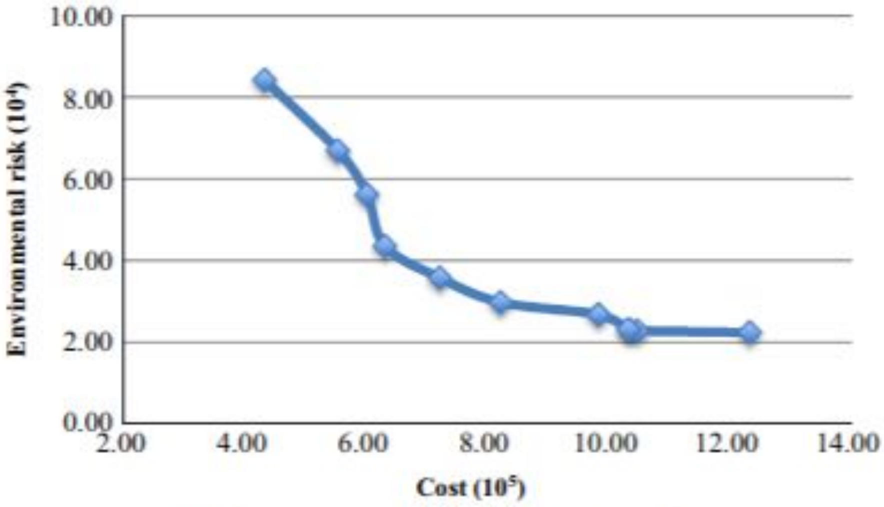



Fig. 4. Trade-off curve with $\lambda$ increment of 0.1.

# Chongqing Case

## Optimal Solution:

- ❑ 3 storage facilities
- ❑ 3 integrated facilities
- ❑ 6 vehicles
- ❑ 982510 yuan
- ❑ 26750 km^3

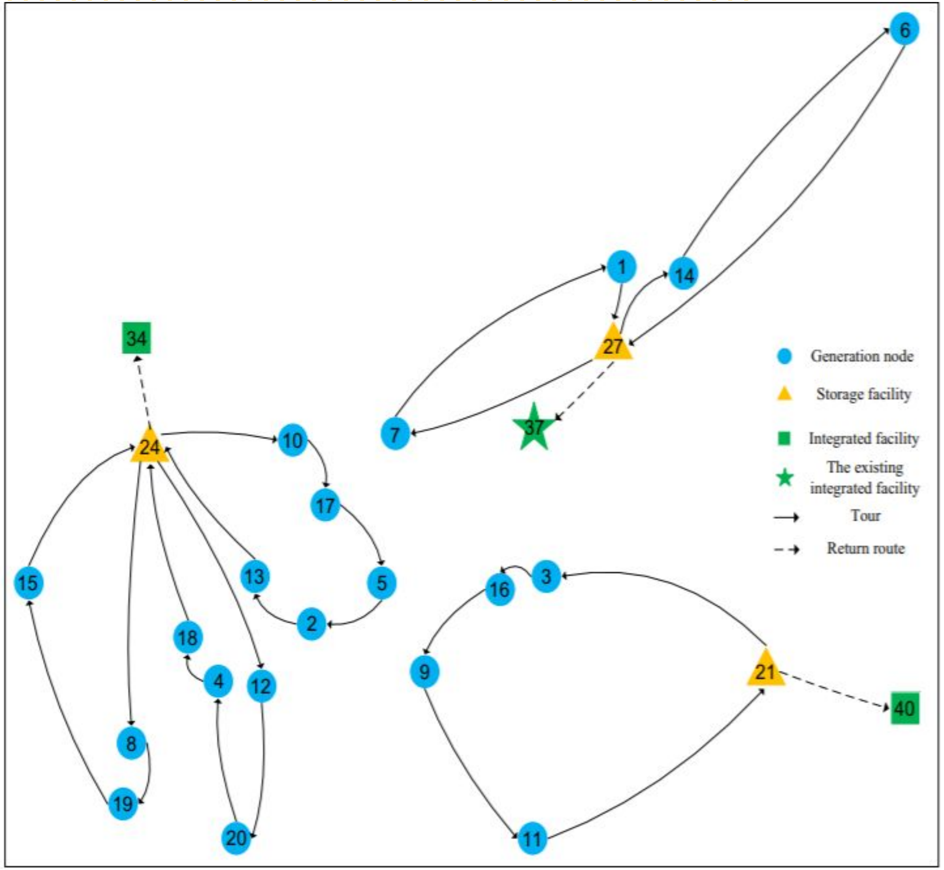
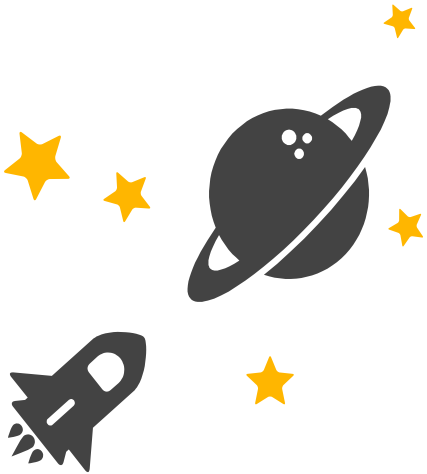



**Fig. 5.** The optimal solution to the used oil location-routing problem in Chongqing ($\lambda$=0.40).

# Big Questions



**The correctness and applicability of the paper is quite robust, however:**

1. Definition of environmental risk is not comprehensive
2. Too many indexes?

# “An Exact Algorithm for the Capacitated Vehicle Routing Problem Based on a Two-Commodity Network Flow Formulation ” - Baldacci R. et al.

$N(V, E)$

$V = G \cup S \cup \bar{S} \cup F$

$G(1,2,\ldots,g)$ used oil generation nodes.
$S(1,2,\ldots,s)$ storage facility candidates.
$\bar{S}(1,2,\ldots,\bar{s})$ the copies of storage facility candidate.
$F(1,2,\ldots,f)$ integrated facility candidates.

$\overline{G} = (\overline{V}, \overline{E})$

$\overline{V} = V \cup \{n+1\}$

$V' = \overline{V} \setminus \{0, n+1\}$

$\overline{E} = E \cup \{\{i, n+1\}, i \in V'\}$

$c_{in+1} = c_{0i} \ \forall \ i \in V'$

**Our Formulation:**

$$\sum_{\substack{j \in G \cup S \cup \bar{S} \\ j \neq i}} (x_{ijml} - x_{jiml}) = 2 \times g_i \times z_{iml}, \quad \forall i \in G,\ \forall m \in \bar{S},\ \forall l \in L \tag{13}$$

$$\sum_{i \in G} \sum_{m \in \bar{S}} \sum_{l \in L} x_{miml} = K \times VCAP - \sum_{i \in G} g_i \tag{14}$$

$$\sum_{i \in G} x_{imml} = \sum_{i \in G} g_i \times z_{iml}, \quad \forall m \in \bar{S}, \forall l \in L \tag{15}$$

$$\sum_{i \in G} \sum_{j \in S} \sum_{m \in \bar{S}} \sum_{l \in L} x_{ijml} \leq K \times VCAP \tag{16}$$

$$x_{ijml} + x_{jiml} = VCAP \times w_{ijml}, \quad \forall i,j \in G \cup S \cup \bar{S}, i \neq j, \forall m \in \bar{S}, \forall l \in L \tag{17}$$

$$\sum_{\substack{i \in G \cup S \cup \bar{S} \\ i \neq j}} w_{ijml} = 2 \times z_{jml}, \quad \forall j \in G, \forall m \in \bar{S}, \forall l \in L \tag{18}$$

**Formulation of Baldacci et al.:**

$$\sum_{j \in \bar{V}} (x_{ji} - x_{ij}) = 2q_i \quad \forall i \in V',$$

$$\sum_{i \in V'} x_{j0} = MQ - q(V'),$$

$$\sum_{j \in V'} x_{0j} = q(V'),$$

$$\sum_{j \in V'} x_{n+1j} = MQ,$$

$$x_{ij} + x_{ji} = Q\xi_{ij} \quad \forall \{i, j\} \in \bar{E},$$

$$\sum_{\substack{j \in \bar{V} \\ i<j}} \xi_{ij} + \sum_{\substack{j \in \bar{V} \\ i>j}} \xi_{ji} = 2 \quad \forall i \in V',$$

# Local Cases

# Central Anatolia Case

"A new model for the hazardous waste location-routing problem" -B. Kara, S. Alumur (2005)

**Locating treatment centers, disposal centers, allocating technology, all routing issues**

$$(\lambda \times \text{ Cost of link/Max link Cost}) + (1 - \lambda) \times (\text{Risk of link/Max link Risk})$$

# Central Anatolia Case

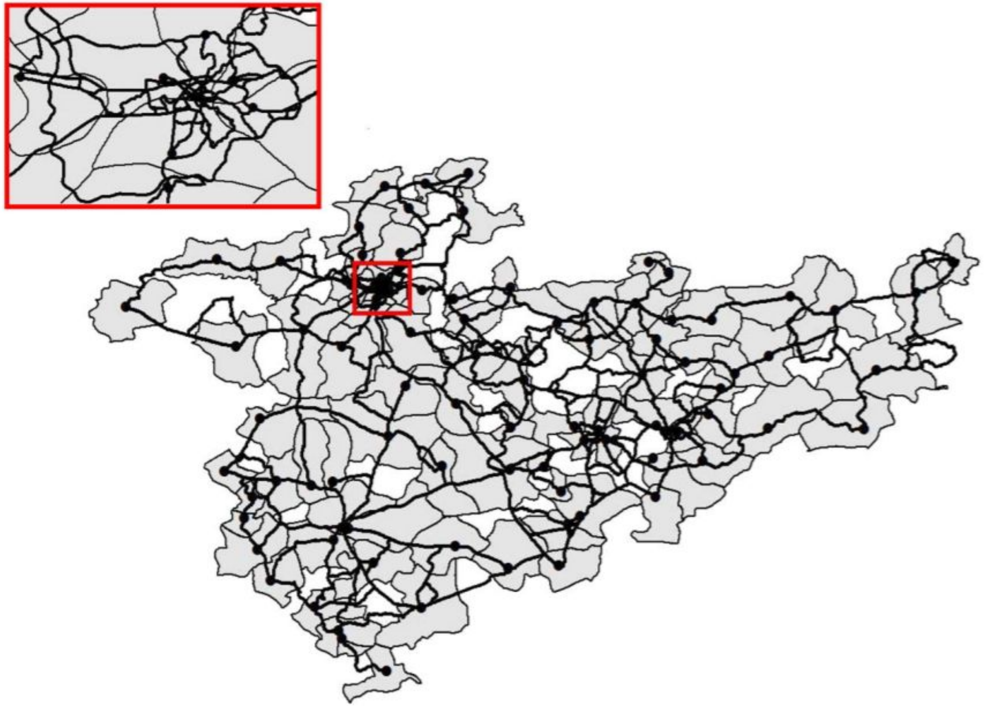


Fig. 3. Ninety-two administrative districts and the highway network in the region. The inset displays an enlarged view of the network around Ankara.

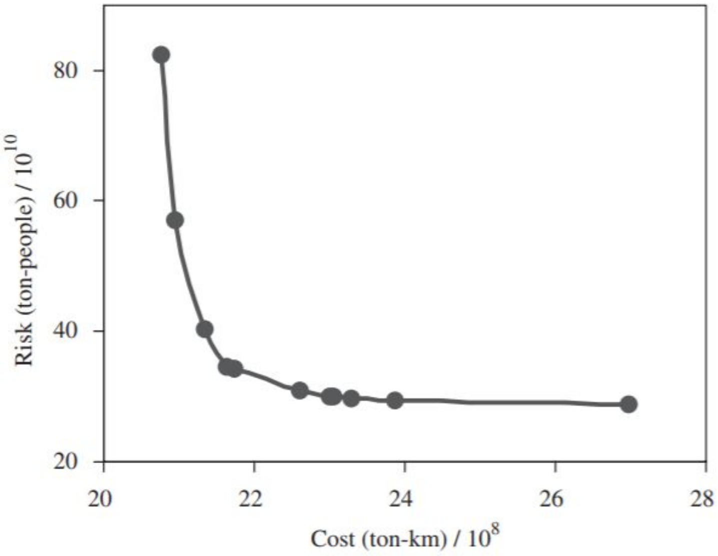



Fig. 4. Trade-off curve with 0.1 increments of $\lambda$.

# Marmara Region Case

"A multi-objective mathematical model for the industrial hazardous waste location-routing problem" -Samanlioglu, F. (2012)

**Locating treatment centers, disposal centers, recycling centers allocating technology, routing hazardous material and waste residues**

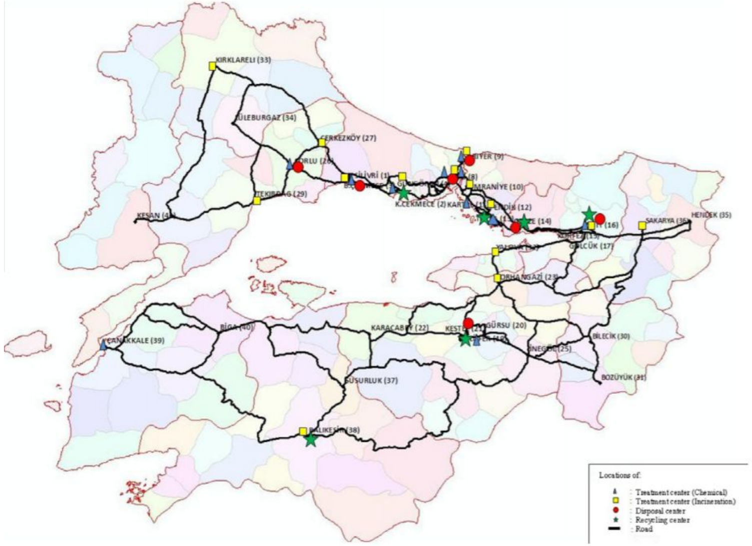



Fig. 3. Efficient solution obtained with equal weights (solution #13).

# References

[1] Jiahong Zhao, Vedat Verter, A bi-objective model for the used oil location-routing problem, Computers & Operations Research, Volume 62, 2015, Pages 157-168, ISSN 0305-0548, https://doi.org/10.1016/j.cor.2014.10.016.

[2] Baldacci, R.; Hadjiconstantinou, E. & Mingozzi, A. 2004. "An Exact Algorithm for the Capacitated Vehicle Routing Problem Based on a Two-Commodity Network Flow Formulation". *Operations Research* 52 (5): 723-738. Institute for Operations Research and the Management Sciences (INFORMS). doi:10.1287/opre.1040.0111.

[3] Sibel Alumur, Bahar Y. Kara, A new model for the hazardous waste location-routing problem, Computers & Operations Research, Volume 34, Issue 5, 2007, Pages 1406-1423, ISSN 0305-0548, https://doi.org/10.1016/j.cor.2005.06.012.

[4] Funda Samanlioglu, A multi-objective mathematical model for the industrial hazardous waste location-routing problem, European Journal of Operational Research, Volume 226, Issue 2, 2013, Pages 332-340, ISSN 0377-2217, https://doi.org/10.1016/j.ejor.2012.11.019.

# Thank you for listening!

Any questions?

# Vehicle-Routing Aspect of the Model

- Each vehicles leaves from and returns to the same storage
- Each vehicle visits at least one generation node
- Each generation node is visited exactly once
- Capacity of each vehicle should not be exceeded
- Two-commodity flow formulation

$$\min \sum_{i\in V}\sum_{j\in V} c_{ij}x_{ij}$$

subject to

$$\sum_{i\in V} x_{ij} = 1 \quad \forall j \in V\backslash\{0\}$$

$$\sum_{j\in V} x_{ij} = 1 \quad \forall i \in V\backslash\{0\}$$

$$\sum_{i\in V} x_{i0} = K$$

$$\sum_{j\in V} x_{0j} = K$$

$$\sum_{i\notin S}\sum_{j\in S} x_{ij} \geq r(S), \;\; \forall S \subseteq V \setminus \{0\}, S \neq \emptyset$$

$$x_{ij} \in \{0,1\} \quad \forall i,j \in V$$

SlidesCarnival icons are editable shapes.

This means that you can:

- Resize them without losing quality.
- Change fill color and opacity.
- Change line color, width and style.

Isn't that nice? :)

Examples: