IE 324 – Simulation
Fall 2016
Homework #2
Due Date: Oct 27, 2016, 1:30 pm

1. **(3 points)** How could random numbers that are uniform on the interval $[0, 1]$ be transformed into random numbers that are uniform on the interval $[-32, 18]$?

2. **(3 points)** Use the linear congruential method to generate a sequence of four two-digit random integers. Let $X_0 = 52$, $a = 12$, $c = 28$, and $m = 2^7$.

3. **(4 points)** Given the cdf $F(x) = x^5 / 16807$ on $0 \leq x \leq 7$, develop a generator for this distribution.

4. **(15 points)** Develop a generation scheme for the distribution with pdf

$$f(x) = \begin{cases} \frac{1}{54}(x-6), & 6 \leq x \leq 12 \\ \frac{1}{18}\left(4 - \frac{x}{6}\right), & 12 < x \leq 24 \\ 0, & otherwise \end{cases}$$

Using inverse transform technique, generate 200 random variates, compute the sample mean, and compare it to the theoretical mean.

5. **(10 points)** Develop a generator for a random variable whose pdf is given by

$$f(x) = \begin{cases} \frac{\beta}{\alpha^\beta} x^{\beta-1} \, e^{-(\frac{x}{\alpha})^\beta}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

where $\alpha > 0$ and $\beta > 0$.

6. **(10 points)** The cdf of a discrete random variable X is given by

$$F(x) = \frac{x(x+1)(2x+1)}{n(n+1)(2n+1)}, \quad x = 1, 2, \ldots, n$$

For $n = 5$, generate five values of X using $R_1 = 0.01$ $R_2 = 0.02$, $R_3 = 0.09$, $R_4 = 0.26$, $R_5 = 0.54$.

7. **(20 points)** Prove the following:

a) If $R \sim U(0,1)$, then the random variable $X = F^{-1}(R)$ has the distribution function F.

b) Let Z be a random variable with pdf given below.

$$f(z) = \begin{cases} z/4, & 0 \leq z \leq 2 \\ (4-z)/4, & 2 < z \leq 4 \\ 0, & otherwise \end{cases}$$

Z is the sum of two independent random variables X and Y, such that $X, Y \sim U(0,2)$.

8. **(15 points)** Develop a technique for generating a Binomial random variable X via convolution. Then, write a Matlab code and generate a Binomial random variable X, using the technique you developed (take $n = 20$ and $p = 0.6$).

   (Hint: X can be represented as the number of successes in n independent Bernoulli trials, each success having probability p. Thus, $X = \sum_{i=1}^{n} X_i$, where $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p$)

9. **(20 points)**

   a) Show that inverse transform technique does not work well for the pdf given below.

   b) Develop an acceptance-rejection technique for generating a random variable X with pdf below, then write a Matlab code and generate 1000 samples.

$$f(x) = \begin{cases} 105x^4(1-x)^3(3-x^2), & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$