IE 324 - Simulation
Spring 2017
Homework #2
Due: Mar 14, 2017, 5:30 pm

This homework must be done individually. No collaboration is allowed. Violation of academic integrity rules will not be tolerated and may result in serious consequences and disciplinary action.

You need to perform relevant mathematical calculations for all questions, then you need to write a Matlab code if it is required.

Note: Please notice the update on **Q8a** and **Q9b.**

1. **(3 points)** How could random numbers that are uniform on the interval $[0, 1]$ be transformed into random numbers that are uniform on the interval $[-24, 16]$?

2. **(3 points)** Use the linear congruential method to generate a sequence of four two-digit random integers. Let $X_0 = 26$, $a = 13$, $c = 27$, and $m = 2^6$.

3. **(4 points)** Given the cumulative distribution function $F(x) = x^5 /7776$ on $0 \leq x \leq 6$, develop a generator for this distribution.

4. **(10 points)** Develop a generator for a random variable whose probability density function is given below,

$$f(x) = \begin{cases} \frac{\beta}{\alpha^\beta} x^{\beta-1}\, e^{-\left(\frac{x}{\alpha}\right)^\beta}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

where $\alpha > 0$ and $\beta > 0$.

5. **(10 points)** The cumulative distribution function of a discrete random variable X is given by

$$F(x) = \frac{x(x+1)(2x+1)(2x+2)(4x+3)}{n(n+1)(2n+1)(2n+2)(4n+3)}, \quad x = 1,2,\dots,n$$

For $n = 5$, generate ten values of X using $R_1 = 0.01$ $R_2 = 0.02$, $R_3 = 0.09$, $R_4 = 0.26$, $R_5 = 0.54$, $R_6 = 0.63$, $R_7 = 0.74$, $R_8 = 0.81$, $R_9 = 0.89$, and $R_{10} = 0.97$.

6. **(15 points)** Develop a generation scheme for the distribution with pdf

$$f(x) = \begin{cases} \frac{1}{96}(x-8), & 8 \leq x \leq 16 \\ \frac{1}{24}\left(4 - \frac{x}{8}\right), & 16 < x \leq 32 \\ 0, & otherwise \end{cases}$$

Using inverse transform technique, write a Matlab code that generates a sample 200 random variates, compute the sample mean, and compare it to the theoretical mean.

7. **(15 points)** Develop a technique for generating a Binomial random variable X via convolution. Then, write a Matlab code and generate a Binomial random variable X, using the technique you developed (take $n = 18$ and $p = 0.7$).

(**Hint:** X can be represented as the number of successes in n independent Bernoulli trials, each success having probability p. Thus, $X = \sum_{i=1}^{n} X_i$, where $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p$).

8. **(20 points)**

   a) Show that inverse transform technique does not work well for the pdf given below.

$$f(x) = \begin{cases} 168x^4(1-x)^3(2-x^2), & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

   b) Develop an acceptance-rejection technique for generating a random variable X with pdf given in the previous part, then write a Matlab code and generate 1000 samples.

9. **(20 points)** Prove the following, show all your work:

   a) If $R \sim U(0,1)$, then the random variable $X = F^{-1}(R)$ has the distribution function F.

   b) Let Z be a random variable with pdf given below.

$$f(z) = \begin{cases} (z-2)/16, & 2 \le z \le 6 \\ (10-z)/16, & 6 < z \le 10 \\ 0, & otherwise \end{cases}$$

   Z is the sum of two independent random variables X and Y, such that $X, Y \sim U(1,5)$.