IE 324 - Simulation
Spring 2017
Homework #3
Due: Mar 21, 2017, 5:30 pm

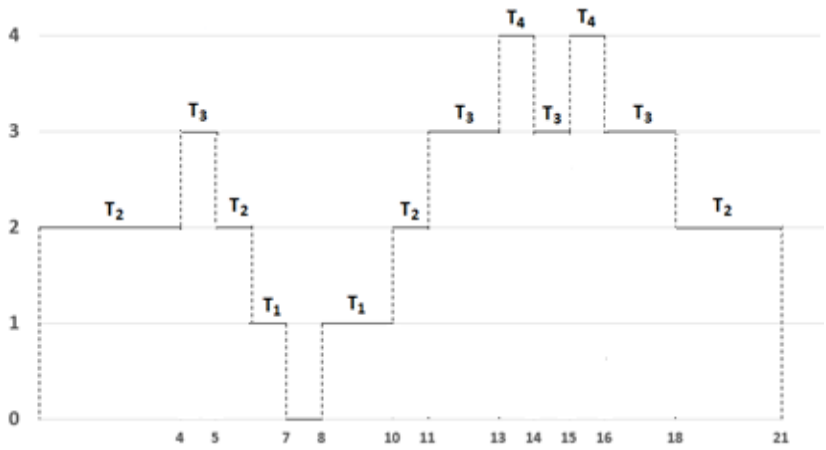
This homework must be done individually. No collaboration is allowed. Violation of academic integrity rules will not be tolerated and may result in serious consequences and disciplinary action.

1. Suppose that Figure 1 represents the number in system for a last-in-first-out (LIFO) single-server system. Customers are not preempted (not kicked out of service), but, upon service completion, the most recent arrival next begins service. For this LIFO system, apportion the total area under *L(t)* to each individual, i.e., determine the time each customer spends in the system during [0, 21].

Do equations (1) and (2) hold for the single-server LIFO system? If so, prove it. Otherwise, explain why these equations do not hold.

$$\sum_{i=1}^{N} W_i = \int_0^T L(t)dt \quad (1)$$

$$\hat{L} = \hat{\lambda}\hat{w} \quad (2)$$



2. Consider a **G/G/2 FIFO** queuing system in which customers take service from Server 1 when both servers are available. At $t = 2.5$, Server 1 leaves permanently and all customers start to take service from Server 2. If Server 1 is already serving a customer at $t = 2.5$, then Server 1 leaves immediately after the customer departs. The queuing rule changes to **LIFO** at $t = 2.5$ and remains as **LIFO** until the termination time. If there is an entity that leaves the queue at $t = 2.5$, the queuing rule changes immediately after the entity leaves.

Inter-arrival times: 0.2, 0.5, 0.1, 0.4, 0.2, 0.1, 0.2, 0.1, 0.6, 0.5, 0.2, 0.4, 0.2, 0.4, 0.6
Service times of Server 1: 0.3, 0.1, 0.1, 0.7, 0.6, 0.1, 0.4, 0.7, 0.5, 0.4, 0.8, 0.3, 0.3
Service times of Server 2: 0.3, 0.7, 0.5, 0.4, 0.8, 0.2, 0.5, 0.4, 0.6, 0.5, 0.7

Using the inter-arrival and service times given above (in minutes) in the given order, create a record of hand simulation using the event-scheduling algorithm until time *TE = 4* minutes and

compute the below stated performance measures. Assume that the first arrival occurs at time *t = 0*.

When using the service times, consider the rule given in the following example: If Entity 2 is the first entity that enters Server 2, it should use the **first** time given for Server 2, not the **second** time.

- **a.** Average number of jobs in the system
- **b.** Average number of jobs in the queue
- **c.** Utilization of Server 1
- **d.** Utilization of Server 2
- **e.** Average time in the system
- **f.** Average waiting time in the queue
- **g.** Maximum number of jobs in the queue
- **h.** Maximum waiting time in the queue
- **i.** Total number of jobs that arrived at the system
- **j.** Total number of jobs that started service
- **k.** Total number of jobs completed service

3. Approximate the integral

$$\int_2^6 \left(\sqrt[3]{3x} + \frac{2x-1}{x+1} + 5x^4\right) dx$$

- **a.** Write a Matlab code that calculates the approximate value of θ, by performing 7,500 replications.
- **b.** Write a Matlab code that calculates the approximate value of θ, by performing 500,000 replications.
- **c.** Calculate confidence interval of θ for part a and part b.
- **d.** Compare your findings of parts a,b and c. Discuss the reasons for differing results.

Do questions 4, 5, and 6 in Microsoft Excel and hand in the printouts of Excel sheets along with a detailed explanation of your solution.

4. Consider the assembly of two rectangle steel plates, each plate having a hole drilled in its center. The plates are to be joined by a pin. The plates are aligned for assembly relative to the bottom left corner (0, 0). The hole placement is centered at (2, 4) on each plate. The standard deviation in each direction is 0.003. The hole diameter is normally distributed, with a mean of 0.2 and a standard deviation of 0.0035. The pin diameter is also distributed normally, with a mean of 0.32 and a standard deviation of 0.0045. What fraction of pins will go through the assembled plates? Base your answer on a simulation of 150 observations.

Hint: $Clearance = min(h_1, h_2) - [(x_1 - x_2)^2 + (y_1 - y_2)^2]^{0.5} - p$, where

h_i = hole diameter, i = plate 1,2
p = pin diameter
x_i= distance to center of plate hole, horizontal direction, i = 1, 2
y_i= distance to center of plate hole, vertical direction, i = 1, 2

5. In the question above, the pin will wobble if it is too loose. Wobbling occurs if $\min(h_1,\ h_2) - p \geq 0.0055$. What fraction of the assemblies wobble? (Conditional probability, i.e, given that the pins go through).

6. Three points are chosen at random on the circumference of a circle. Estimate the probability that they all lie on the same semicircle, by Monte Carlo sampling methods. Perform 100 replications.