IE 324 – Simulation
Fall 2017
Study Set 2

1. How could random numbers that are uniform on the interval $[0, 1]$ be transformed into random numbers that are uniform on the interval $[-21,12]$?

2. Use the linear congruential method to generate a sequence of four two-digit random integers. Let $X_0 = 9,\ a = 14,\ c = 23$, and $m = 2^6$.

3. Given the cdf $F(x) = x^3/125$ on $0 \le x \le 5$, develop a generator for this distribution.

4. Develop a generation scheme for the triangular distribution with pdf

$$f(x) = \begin{cases} \frac{1}{15}(x-2), & 2 \le x \le 5 \\ \frac{12-x}{35}, & 5 < x \le 12 \\ 0, & otherwise \end{cases}$$

Generate 1000 random variates, compute the sample mean, and compare it to the true mean of the distribution.

5. Develop a generator for a random variable whose pdf is given by

$$f(x) = \begin{cases} \frac{\beta}{\alpha^\beta} x^{\beta-1} e^{-(\frac{x}{\alpha})^\beta}, & x \ge 0 \\ 0, & otherwise \end{cases}$$

where $\alpha > 0$ and $\beta > 0$.

6. The cdf of a discrete random variable X is given by

$$F(x) = \frac{x(x+1)(2x+1)}{n(n+1)(2n+1)}, \quad x = 1,2,\dots,n$$

When n=6, generate seven values of X using $R_1 = 0.04$, $R_2 = 0.21$, $R_3 = 0.33$, $R_4 = 0.09$, $R_5 = 0.60$, $R_6 = 0.95$, and $R_7 = 0.01$.

7. Develop a technique for generating a Binomial random variable X via convolution. Then, write a Matlab code and generate a Binomial random variable X, using the technique you developed (take $n = 20$ and $p = 0.6$). (Hint: X can be represented as the number of successes in n independent Bernoulli trials, each success having probability p. Thus, $X = \sum_{i=1}^{n} X_i$, where $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p$)

8. Develop an acceptance-rejection technique for generating a random variable X with pdf below, then write a Matlab code and generate 1000 samples.

$$f(x) = \begin{cases} 18x^3(1-x)^2(3-x^3), & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$