IE 324 SIMULATION
SPRING 2020 LAB STUDY 2

Procedure:

- Download the excel file from the course web site.
- Feel free to ask the TA about any part you did not understand.
- This lab study will not be graded.

1) GAMBLER'S RUIN PROBLEM

Let's make gambling fun with a touch of simulation. Consider a two player game where in each step player 1 wins with probability $p_1 \in (0,1)$ i.e. player 2 wins with probability $1 - p_1$. Two players play the game until one of them is out of money. Let's say that each player begins playing the game with a sum of money $C_1, C_2 > 0$, respectively. First, we can analyze this game theoretically by using some mathematics.

Theoretical Approach: Let's play the game as player 1. As player 1, you can finish the game either at $C_1 + C_2$ or 0 (bankruptcy or ruin), at the end of the game. In each step you observe a change in your wealth which has the following distribution:

Let X_i's be the IID changes in the wealth of player 1 in the i^{th} step and S_i be the wealth of player 1 in i^{th} step. Given $S_0 = C_1$

$$P(X_i = +1) = p_1$$

$$P(X_i = -1) = 1 - p_1$$

$$S_i = \sum_{j=1}^{i} X_j + S_0$$

Let $T = min\{t \in \mathbb{N} : S_t = C_1 + C_2 \; or \; S_t = 0\}$. The game terminates at time T defined above i.e. the game ends whenever a player is out of money. Note that, $E[T] = C_1 C_2$ when $p_1 = 0.5$.

Simulation Task: Observe that, $E[T] = C_1 C_2$ with $C_1 = 5$, $C_2 = 5$ via simulating the game on spreadsheet with 10 replications. To have randomness you need to generate random numbers using Microsoft Excel's rand() function. Then, you need to update the current status of the game, that is, update the money of each player after the uncertainty is resolved.

Hint: rand() generates a random number (R) from the uniform distribution over $[0,1]$. You can use it to generate a Rademacher (R') i.e.

$$R' = \begin{cases} 1, & if \; R > p_1 \\ -1, & otherwise \end{cases}$$

Repeat the step simulation until the game is over i.e. repeat until one of the players run out of money. When one of the players is out of money one replication is done.

a) Calculate the average number of game length from 10 replications and check if it coincides with the theoretical results provided above.

b) Observe how average number of game length changes when, (i) p_1 changes and (ii) C_1, C_2 changes.

(Optional) House always wins (Casino vs. Player): You can also try simulating the game with C_1, C_2 such that $C_1 >> C_2$ i.e. C_1 is much larger than C_2. In this setting player 1 is the casino which has much more money than an ordinary player which is player 2. You can see that house always wins! Even when the game is "fair" ($p_1 = 0.5$).

2) NEWSVENDOR PROBLEM

Let's recall the good old newsvendor problem: Every morning, the newsvendor decides how many newspapers to buy before observing the demand for newspapers. Before the resolution of uncertainty the newsvendor only knows the cumulative distribution function the demand. He follows a fixed order quantity policy and purchases $\hat{Q}$ newspapers every morning. Random demand (D) is IID with cumulative distribution function $F_D(x)$. He purchases newspapers from unit cost u, sells the newspaper for price p and salvages excess newspapers from salvage value s. Also when the newsvendor is unable to meet the demand newsvendor occurs a shortage cost b for every unit of demand not met.

$$D \in \{110, 140, 160\}$$

$$F_D(x) = \begin{cases} 0 & x < 110 \\ 0.10 & 110 \leq x < 140 \\ 0.60 & 140 \leq x < 160 \\ 1 & 160 \leq x \end{cases}$$

Simulate the system using spreadsheets for 31 days with the following parameters:

$$\hat{Q} = 150, p = \$2, u = \$1, b = \$1, s = \$0.1$$

Hint: Use a method similar to the method used in Question 1 to virtually create the desired stochastic behavior.

a) Calculate average profit.

b) (Optional) Try different values of $\hat{Q}$. Comment on resulting average profit values.