IE 324 SIMULATION
SPRING 2020 LAB STUDY 4

Procedure:

- Download the excel file from the course web site.
- Feel free to ask the TA about any part you did not understand.
- This lab study will not be graded.

RANDOM NUMBER GENERATION

Random number generators should have full period and be generated dense enough in order to satisfy the uniformity property. The generated numbers should also look independent. However, the Linear Congruential Generators may fail to satisfy these requirements.

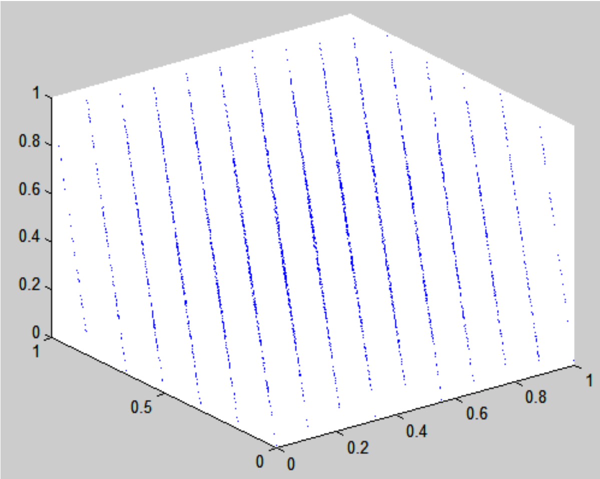
1. On the worksheet LCG, try to input the candidate parameters and examine visually if the generated numbers satisfy the requirements. (Candidate parameters are on the worksheet)

2. The following theorem guides in selecting the parameters properly in order to satisfy the maximum possible period. Try to find <u>three parameter combinations</u> and check if the above-mentioned requirements are satisfied visually.

- For m a power of 2, say $m = 2^b$, and $c \neq 0$, the longest possible period is $P = m = 2^b$, which is achieved whenever c is relatively prime to m (that is, the greatest common factor of c and m is 1) and $a = 1 + 4k$, where k is an integer.
- For m a power of 2, say $m = 2^b$, and $c = 0$, the longest possible period is $P = \frac{m}{4} = 2^{b-2}$, which is achieved if the seed Z_0 is odd and if the multiplier a is given by $a = 3 + 8k$ or $a = 5 + 8k$, for some $k = 0,1, \ldots$
- For m a prime number and $c = 0$, the longest possible period is $P = m - 1$, which is achieved whenever the multiplier a has the property that the smallest integer k such that a^{k-1} is divisible by m is $k = m - 1$.

3. In 1960s IBM used a multiplicative linear congruential algorithm, called RANDU, with the following parameters: $a = 65539, c = 0, m = 2 \times 10^{31}$ (In Excel 2E+31).

- Use these parameters and check the uniformity and independence visually.
- George Marsaglia shows that the numbers generated by the multiplicative linear congruential algorithms mainly fall in parallel hyperplanes in an n-dimensional space[1]. This may not be obvious when the tuples (R_i, R_{i+1}) of random numbers are plotted. RANDU seems to satisfy the uniformity and independence when the tuples of (R_i, R_{i+1}) are plotted. However, if a 3-tuple (R_i, R_{i+1}, R_{i+2}) are plotted and viewed from a specific angle, one can see that the points fall on planes, and this shows a high correlation[2].



4. For the dataset given on worksheet "Dataset" apply, Kolmogorov-Smirnov, Chi Square, Runs and Autocorrelation Tests to find out if there is a significant problem related to uniformity and independence of the generated numbers.

---

[1] Random Numbers Fall Mainly in the Planes, George Marsaglia, Proceedings of the National Academy of Sciences Sep 1968, 61 (1) 25-28; DOI: 10.1073/pnas.61.1.25 (https://www.pnas.org/content/pnas/61/1/25.full.pdf)

[2] See https://demonstrations.wolfram.com/PoorStatisticalQualitiesForTheRANDURandomNumberGenerator/ for an interactive plot.