IE375 Spring 2020 Homework II - Due: March 6, Friday 17:00

- **Homework II is 2%. Make sure that you review course material before you start. Cite sources used if any.**
- **Grading Policy:** The homework grades will not be only based on the correctness of the answers provided, but on the perceived degree of effort on the returned assignment. This means that even without obtaining the correct answers one can obtain full credit. The grade scale will be "2" (full effort, all questions); "1" (some effort or full effort, some questions); "0" (no effort or no submission). The submissions that exceed expectations may get a bonus – grade will be "3" (extra effort). Note that "2" corresponds to 100% - 2% of the overall grade in this case. Late submissions will not be accepted.
- The grading scale for each question is 2 points (full effort), 1 point (some effort), 0 (no effort or no submission). Total points that would lead to your overall HW grade are given below:

Total Points	HW Grade
6+	3
5-6	2
2-4	1
0	0

- **Academic Dishonesty (taken from IE375 Course Information):** Academic misconduct is not tolerated, and violations will be treated in accordance with the University policy. For assignments: Discussion of the course material and working in groups are effective ways of learning. However, this does not mean that you can copy the assignment from one of your friends. You can discuss with your friends, but you should be alone, without any written solution in front of you, when you actually prepare your assignment. It is extremely important to understand the value of this exercise in the process of learning. Any violation of the above understanding will result measures outlined in the student rules and regulations.
- **Submission Information:** An assignment submission box is to be placed in front of the office EA325 on Thursday, one day prior the submission date and is to be removed at the time of the deadline. You need to put your assignment in the designated box before the submission deadline. If you want to submit before Thursday, please contact the TA. **Submission via e-mail is NOT accepted. In case you may want to submit your homework earlier, please consult your TA's.**
- **More Information on Questions: What is expected? How to study?**
- To answer the questions, make sure that you understand the material presented during the course, as well as the reading material from the textbook. Homework II is made up of computational type questions, as well as modeling questions. For computational questions the key is to understand how standard methods are applied, and interpret the results. Question 1, for example both ask you to evaluate certain production planning approaches. If you formulate these problems using an optimization model and solve, you can see how the objective function value obtained by applying a given rule can be improved by use of an optimizing algorithm. You are asked to formulate such a problem as part c) of Question 1. We do not have any homework problem where we ask you to solve the obtained model by using software. However, we will have such questions in the exercise set and we are hoping that you will appreciate those exercises. Questions 2 is related to modeling, whereas, Question 3 asks you to interpret the optimal solution of an LP. Both questions may take shorter time to solve, assuming that YOU HAVE STUDIED AND KNOW THE MATERIAL, as they are examples of problems which were asked in the previous exams of this course. Make sure that you understand the problem, and only present your answer accordingly.
- **One final remark:** In the exams, please get used to receiving question types that you have not seen before. One useful purpose of homework and exercises is to cover the "routine" so that you can go beyond that during the exams, and elsewhere.

PLEASE READ QUESTIONS CAREFULLY. FOR THE QUESTIONS FROM THE TEXT-BOOK THERE MIGHT BE CHANGES IN THE GIVEN VALUES PRESENTED BEFORE THE PROBLEM STATEMENT.

1) **Question 21, Section 3.4, p.144 for 7th Edition (**Page 133, Problem 8 for 6th Edition)
*********NUMBER OF NURSES REQUIRED PER PATIENT FOR CRITICAL CARE IS NOW ESTIMATED TO BE 0.55*********

8. St. Clair County Hospital is attempting to assess its needs for nurses over the coming four months (January to April). The need for nurses depends on both the numbers and the types of patients in the hospital. Based on a study conducted by consultants, the hospital has determined that the following ratios of nurses to patients are required:

Patient Type	Numbers of Nurses Required per Patient	Patient Forecasts Jan.	Feb.	Mar.	Apr.
Major surgery	0.4	28	21	16	18
Minor surgery	0.1	12	25	45	32
Maternity	0.5	22	43	90	26
Critical care	0.6	75	45	60	30
Other	0.3	80	94	73	77

a. How many nurses should be working each month to most closely match patient forecasts?

b. Suppose the hospital does not want to change its policy of not increasing the nursing staff size by more than 10 percent in any month. Suggest a schedule of nurse staffing over the four months that meets this requirement and also meets the need for nurses each month.

c) Knowing LP and understanding needs of a hospital, you decide to build an LP model to approach the above problem. You want to make sure instead of closely matching patient forecasts you want to create a buffer in the number of nurses available. Let us say the nurse buffer is defined as a certain % (say β%) of the number found in Part a) - note that we allow for non-integer values in this part as we allow for part-time nurses, as well. Training each nurse takes one month, hence if you hire new nurses, the salaries for the nurses are to be paid, but those nurses will not be used in operations. Moreover, the nurses are specialized for patient types, meaning that a nurse who takes care of a patient type cannot help another patient type. Assume that you have monthly salaries, hiring costs, lay-off cost associated with the nurses. Formulate this problem as an LP, assuming that you have N_0 nurses in the beginning of January, assuming that N_0 will be sufficient for January requirements.

2) A textile firm is developing its aggregate production plan for the next T months. The firm owns two plants that are closely located: a yarn manufacturing plant that converts cotton and other synthetic fibers into yarn, and a fabric manufacturing plant that converts yarn into fabric. An aggregate item is defined for the fabric. The demand for fabric (in meter squares) in month t is given by D_t .1 ton of yarn can be used to produce e^{YF} meter squares of fabric. The firm can keep inventory either as yarn or as fabric. Monthly inventory holding costs for one ton of yarn is h^Y and one meter square of fabric is h^F. Production costs in month t for one ton of yarn are c_t^Y and one meter square of fabric is c_t^F. Monthly physical (technological) capacity of the yarn plant is B^Y tons and the fabric plant is B^F meter squares. The two plants can share the workers as they are located closely. Producing a ton of yarn requires a^Y man-months and a meter square of fabric requires a^F man-months. The firm has a total permanent workforce of W^p which is paid c^p per employee per month. Additional seasonal workers can be employed in any given month for a wage of c^S per employee per

month. It is not possible to change the size of the permanent workforce, but there is no additional hiring and laying off costs for the seasonal employees.

Define additional parameters to the above given, if needed, and define decision variables for the model. Formulate the problem as a linear programming problem, assuming continuity of the decision variables.

3) A plant manager employed by a national producer of kitchen products is planning workforce levels for the next three months. An aggregate unit of production requires an average of four labor hours. Forecast demands for the three-month horizon are as follows.

Month	Workdays	Forecasted Demand (in aggregate units)
July	23	2,400
August	16	3,000
September	20	800

Assume that a normal workday is eight hours. Hiring costs are $500 per worker and layoff costs are $1,000 per worker. Holding costs are $40.00 per aggregate unit held per month. Shortages are not permitted. Assume that the ending inventory was 600 and the manager wishes to have at least 800 units on hand at the end of September. The June work force level is 30 workers.

To find the optimal aggregate production plan the following LP is used:

Minimize $\left(500\sum_{t=1}^{3} H_t + 1{,}000\sum_{t=1}^{3} F_t + 40\sum_{t=1}^{3} I_t \right)$

s.to:

(c1) $W_1 - W_0 - H_1 + F_1 = 0$

(c2) $W_2 - W_1 - H_2 + F_2 = 0$

(c3) $W_3 - W_2 - H_3 + F_3 = 0$

(c4) $P_1 - I_1 + I_0 = 2400$

(c5) $P_2 - I_2 + I_1 = 3000$

(c6) $P_3 - I_3 + I_2 = 800$

(c7) $P_1 - 46W_1 = 0$

(c8) $P_2 - 32W_2 = 0$

(c9) $P_3 - 40W_3 = 0$

(c10) $W_0 = 30$

(c11) $I_0 = 600$

(c12) $I_3 \geq 800$

All variables ≥ 0

The optimal solution to this problem is (Subscripts in the variables are written as a part of the "Variable Name"):

Variable Name	Solution Value
H1	31.538462
F3	28.885400
I1	1030.769231
I3	800.000000
W1	61.538462
W0	30.000000
W2	61.538462
W3	32.653061
P1	2830.769231
I0	600.000000
P2	1969.230769
P3	1600.000000

All other variables in the range 1-17 are 0.

The dual variables are (Constraints are numbered consecutively in the order written above; "Constraint Name" contains "c" followed by the constraint number):

Constraint Name	Dual Price
c1	-500.000000
c2	-370.256410
c3	1000.000000
c4	2.820513
c5	42.820513
c6	-20.408163
c7	-2.820513
c8	-42.820513
c9	20.408163
c10	-500.000000
c11	-2.820513
c12	19.591837

a) c4, c5 and c6 are constraints on the forecasted demand. If you were to revise your demand forecasts (i.e. the numbers 2400, 3000, 800), which will be preferable? Why? (Consider the effect of changing forecasted demand - up/down.)

b) Compute the optimal value of the objective function twice; first using primal, and then dual solution data.

c) Which forecasted demand value you would increase by 5 units to obtain the highest reduction in cost? What is the upper bound for this reduction?