IE 375 Spring 2020

EXAM I

Duration: 110 minutes

March 12, 2020

Academic integrity is expected of all students of Bilkent University at all times, whether in the presence or absence of members of the faculty.

Understanding this, I declare that I shall not give, use or receive unauthorized aid in the examination.

Name: KEY

ID number:

Signature:

PLEASE READ ALL THE QUESTIONS CAREFULLY. YOUR ANSWERS SHOULD BE CONCISE AND NOT CONSISTENT. ANY ANSWER WITH INCONSISTENT REMARKS MAY NOT RECEIVE ANY PARTIAL GRADES:

Question	Grade	Your Grade
1	35	
2	45	
3	20	
Total	100	

Useful Formula:

$F_{t+1} = F_{t,t+1} = \alpha D_t + (1-\alpha) F_{t-1,t}$

$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$

$S_t = \alpha (D_t/c_{t-N}) + (1-\alpha)(S_{t-1} + G_{t-1})$

$d_t = (\mu + G_t) c_t + \varepsilon_t$

$e_t = F_{t-1,t} - D_t$

$N = \sum_{j=t-N+1}^{t} c_j$

$MAD_t = \alpha |e_t| + (1-\alpha) MAD_{t-1}$

$G_t = \beta (S_t - S_{t-1}) + (1-\beta) G_{t-1}$

$c_t = \gamma (D_t / S_t) + (1-\gamma) c_{t-N}$

$F_{t,t+\tau} = (S_t + \tau G_t) c_{t+\tau-N}$

$F_{t,t+1} = 1/N \sum_{j=t-N+1}^{t} D_j$

Question 1 (35 points)

A forecasting system with trend and seasonal factors has been installed in a company. Seasonality is specified with respect to quarters, and prior to the current quarter, say t=5, after observing period t-1's demand, most recent (updated) values of the forecast parameters are as follows:

- Seasonality factors: 0.9, 1, 0.8, 1.3 for quarters t-4 through t-1, (say, 1 through 4), respectively
- Slope (trend factor): 10.4
- Intercept (level factor): 100
- All smoothing parameters are 0.1

a) **(10 points)** What are the forecasts for quarter 5 and quarter 6?

+5 $F_{4,5} = (S_4 + 1 \cdot G_4)\, C_1 = (100 + 10.4)\, 0.9 = 99.36$

+5 $F_{4,6} = (S_4 + 2 \cdot G_4)\, C_2 = (100 + 2 \times 10.4)\, 1 = 120.8$

+2 each computation

b) **(20 points)** Period t's (quarter 5) demand is realized and is 80 units. Update all the factors.

$S_5 = \alpha (D_5 / C_1) + (1-\alpha)(S_4 + G_4)$

each +5

$= 0.1(80/0.9) + 0.9(100 + 10.4)$

$= 8.89 + 99.36 = 108.25$

each +2 computation

$G_5 = \beta (S_5 - S_4) + (1-\beta)\, G_4$

$= 0.1(108.25 - 100) + 0.9(10.4)$

$= 0.83 + 9.36 = 10.19$

$C_5 = \gamma (D_5 / S_5) + (1-\gamma)\, C_1$

$= 0.10 (80/108.25) + 0.90 \times 0.9$

$= 0.074 + 0.81 = 0.884 \Rightarrow$ take it as $\underline{\underline{0.884}}$

$\sum_{i=1}^{4} C_i = 4 \Rightarrow C_1 = 0.884 \times \frac{4}{3.984} = 0.888 \equiv C_5$

$C_2 = 1.004$

$C_3 = 0.803$

$C_4 = 1.305$

c) **(5 points)** What is the absolute value of the forecast error computed for quarter 5? If smoothing approach is used to update MAD values, what is the new MAD value if the most recent MAD value was 15? Note that all smoothing parameters are 0.1.

+2 $|e_5| = |F_{4,5} - D_5| = |99.36 - 80| = 19.36$

+3 $MAD_5 = 0.10\,|e_5| + 0.9\,(15)$

$= 0.10 \times 19.36 + 0.9\,(15)$

$= 1.936 + 13.5 \cong 15.44$

+1 for computation

Question 2 (45 points)

Consider the **Local** Airlines Case with additional information supplied. Use the following notation if needed:

T_t: the number of trainees hired in the beginning of period t, $t=t=1 \ldots T$.

J_t: the number of junior flight attendants available in the beginning of period t, $t=1 \ldots T$.

F_t: the number of experienced flight attendants available in the beginning of period t, $t=1 \ldots T$.

I_t: the number of instructors available in the beginning of period t, $t=t=1 \ldots T$.

Assume that you know the initial values for the numbers; i.e. T_0, T_{-1}, J_0, F_0, and I_0 are known.

a) **(33 points)** Consider the Figure presented. Assuming that the information stated in %'s represent point estimates, they can be used in computations to determine expected numbers. As an example, given that 20% of the trainees hired in any period t either quit or don't pass to the next stage will result in 0.80 T_t trainees to become junior flight attendants.

(i) **(3 points)** Find the number of trainees, junior flight attendants and flight attendants available in the beginning of July using the data supplied.

month problem ✓

add instructors ✓ $\Rightarrow$

$T_0 = 0$

$J_0 = 25 \times 0.8 = 20$

$F_0 = 195(0.92) + 15 \times 0.8 \times 0.95 = 190.8$

(ii) **(3 points)** Write the constraint which reflects Policy 3 for any period t.

add instructors who act like flight att.

$J_t \times 140 \leq 0.25\,(J_t \times 140 + F_t \times 125) \quad \forall t$

(iii) **(3 points)** How many instructors do we have in the beginning of period t, $t=1 \ldots T$ if we hire H_t new instructors in the beginning of period t? (Instructor balance equations)

$I_t = 0.95\, I_{t-1} + H_t$

(iv) **(6 points)** What are the decision variables you have in this problem if your objective is to minimize total cost for T months? Define any terms (decision variables) when needed.

I_t, J_t, F_t $\quad H_t^I$ - # of instr. hired

$T_t \equiv H_t^T$ - # of trainees hired $\quad I_t$ - # of instructors used as flight attendants.

(v) **(10 points)** Write the manpower balance equations to describe the number of experienced flight attendants for $t=1 \ldots T$ in terms of the relations given in the Figure. You may need to use the definitions given in the beginning of the problem, as well as defining any terms (decision variables) when needed. Make sure that your answer is clear.

No need to include instructors

$F_t = 0.92\, F_{t-1} + 0.95 J_{t-1}$

(vi) **(3 points)** What is the lowest value for T that the planner may select in order to make meaningful use of the planning model?

T should be at least 2 to see the affect of new hired people

(vii) **(5 points)** Write the objective function (minimize total cost) explicitly for the LP problem. Define any terms (parameters and/or decision variables) when needed.

2 if Σ is not present.

S_J, S_T, S_E, S_I — salaries of junior flight. ... etc.

$$\min \sum_{t=1}^{T} (S_J J_t + S_T T_t + S_E F_t + S_I I_t)$$ ⇒ no other cost that we know.

b) (12 points) Respond to each of the following questions, which are independent of each other.

(i) **(4 points)** Consider a change in Policy 1 by increasing the number per instructor to be maximum six rather than five. Will the total optimal cost increase or decrease? How can you find the exact cost change numerically? Explain.

total cost will decrease or stay the same - feasible region expands

Solve both LP's & take the difference

(ii) **(2 points)** While questioning the affect of Policy 4 you noticed that you may use the pool of experienced attendants as an alternative source for feeding instructor pool. Hence, you may decide to have experienced attendants as instructors in one period and of course they will remain as such for later periods. What is the effect of changing Policy 4 (i.e. allowing for experienced attendees become instructors) on the optimal costs? Specifically, will the total optimal cost increase or decrease?

total cost will decrease or stay the same.

feasible region expands

(iii) **(6 points)** You implement the planning problem on a rolling horizon basis. This implementation requires that you forecast (and hence update the previous forecasts) the requirements for the future T periods every period. Given a volatile market, the management has decided to lease new aircraft and add new flights. AS a result the following situation occurs: In period u, you forecast for period u+τ and notice that

$F_{u,u+\tau} >> F_{u-1,u+\tau}$, where $F_{t1,t2}$ is the forecast made at the end of period t1 for period t2.

With this change, is it possible to have the mathematical program infeasible? Explain your answer.

Looks like Yes; since there is a training period it may not be possible to train enough flight attendants.

+3/4

However, you can always hire instructors & use it as flight attendants.

So answer is NO as we have instructor pool available.

Question 3 (20 points)

A company wants a high level, aggregate production plan for the next 4 months. Projected orders for the company's product are listed in the table. Over the 4-month period, units may be produced in one month and stored in inventory to meet some later month's demand. Because of seasonal factors, the cost of production is not constant, as shown in the table. The cost of holding an item in inventory for 1 month is $\$h$/unit, charged for those items in the inventory at the end of the period. The maximum number of units that can be held in inventory is I_{max}. The initial inventory level at the beginning of the planning horizon is I_0 units; the desired final inventory level at the end of the planning horizon is I_f.

The problem is to determine the optimal amount to produce in each month so that demand is met while minimizing the total cost of production and inventory.

Aggregate planning data

Period	1	2	3	4
Demand (units)	D_1	D_2	D_3	D_4
Cost of production (\$/unit)	CP_1	CP_2	CP_3	CP_4

a) **(6 points)** Formulate the problem as an LP model. Assume that shortages are not permitted and you plan to meet all the demand on time. Note that you need to define decision variables you use.

Decision Variables

P_t - Quantity produced in period t

I_t - inventory at the end of period t

$$\min \sum_{t=1}^{4} (CP_t \cdot P_t + h I_t)$$

s.to

$$I_t + D_t = I_{t-1} + P_t \quad t = 1, \ldots, 4$$

$$I_0, \quad I_4 = I_f$$

$$I_t \le I_{max} \quad t = 1, \ldots, 4$$

$$I_t, P_t \ge 0$$

-3

b) **(6 points)** Now consider the situation where you are allowed to have backorders in periods 2 and 3, meaning that you can delay the satisfaction of demand in periods 2 and, but all backordered demand should be satisfied latest at the end of period 4. The following data summarizes the cost of backorder per unit per period for each period.

Period	1	2	3	4
Cost of backorder ($/unit/period)	Not allowed	CB_2	CB_3	Not allowed

How is your formulation going to change? The problem is to determine the optimal amount to produce in each month so that the total cost of production, inventory and backorders is minimized. You need to define new, as well as use some of the previous decision variables.

I_t^+ & I_t^- for $t=2,3$ (backorder at the end of period t)

$$\min \left[\sum_{i=1,4} (C_p P_t + h I_t) + \sum_{i=2,3} (C_p P_t + h I_t^+ + CB_t I_t^-) \right]$$

s to $\quad I_1 + D_1 = I_0 + P_1 \quad t=1,4$

$$I_2^+ - I_2^- + D_2 = I_1 + P_2$$

×3 $\quad I_3^+ - I_3^- + D_3 = I_2^+ - I_2^- + P_3$

$$I_4 + D_4 = I_3^+ - I_3^- + P_4$$

$$I_1, I_2^+, I_3^+, I_4 \le I_{max} \quad + \text{ non-negativity}$$

c) **This question has three independent parts:**

i. **(2 points)** When you solve the above problems, part a) and part b), let the optimal objective function value you have obtained is OF_a and OF_b, respectively. Write an inequality describing their relation.

$$OF_a \ge OF_b$$

ii. **(2 points)** What if you noticed that the inventory limitation described in the problem is actually $2I_{max}$ value rather than I_{max}, i.e. two times more than what you thought. When you solve the problem again according to this change (let us say part **a**), are you going to obtain a lower or higher objective function value?

Lower cost as feasible region expands

iii. **(4 points)** At optimality, you observe that the dual variable corresponding to the constraint regarding the inventory limitation I_{max} attains a value, V. How would you interpret the meaning if $V = 0$? If $V > 0$?

If $V=0$ then constraint is not binding, I_t (or I_t^+) $< I_{max}$

If $V>0$, then constraint is binding.