IE 375 Spring 2020

A Note on Multi-item Problem

Consider the multi-item problem presented

$$\sum_{i=1}^{n}\left(\frac{K_i\lambda_i}{Q_i}+h_i\frac{Q_i}{2}\right)$$

$$\sum_{i=1}^{n} c_i\frac{Q_i}{2}\leq B$$

This is slightly different version of the problem given in Appendix 4 of the text book.

Define $h_i = I\ c_i$ for all i. This means that holding cost is proportional to unit cost.

For the case when constraint is binding we defined the Lagrange multiplier for the equation to be θ. After the derivation, one can obtain the following for the optimal values of θ, θ*, and $Q_i\ for\ all\ i$:

$$\theta^* = \left\{\frac{\sum_{i=1}^{n}\sqrt{\frac{K_i\lambda_i c_i}{2}}}{B}\right\}^2 - I$$

$$Q_i^* = \sqrt{\frac{2K_i\lambda_i}{c_i(I+\theta^*)}}\ \text{for all i}$$

Using the result in the text, this solution for lot sizes turned out to be proportionately less than their corresponding EOQ for all i.

To interpret the Lagrange multiplier, notice the following:

If I started the original problem with $(I + \theta^*)$ instead of I as the inventory carrying cost rate per unit $ per unit time, I would find that the budget would be exactly used by the corresponding EOQ solution. (See why: The optimal lot sizes we found with the constrained problem are given above. Hence, that term would have been the EOQ of the problem with the corresponding carrying rate. If you are still unsure, you can resolve the problem with that I value!).

Then the interpretation comes: I identifies my *initial* estimate of opportunity cost. However, by using a budget and the constraint in the given form as above, I am implying that my opportunity cost may be even more than that as I have a budget limitation, i.e. short for money. Hence, the interpretation of θ*is simply the additional opportunity cost enforced by the budget constraint.

This interpretation is important as you can now solve the following problem to estimate the value of opportunity cost:

$$\sum_{i=1}^{n} \frac{K_i \lambda_i}{Q_i}$$

$$\sum_{i=1}^{n} c_i \frac{Q_i}{2} \leq B$$

In this case, the constraint will always be binding, and the optimal Lagrange multiplier is an estimate of the opportunity cost term.