Chapter 4 –
Presentation Slides for
March 26 and March 31st

IE 375
On-Line Education – Spring 2020
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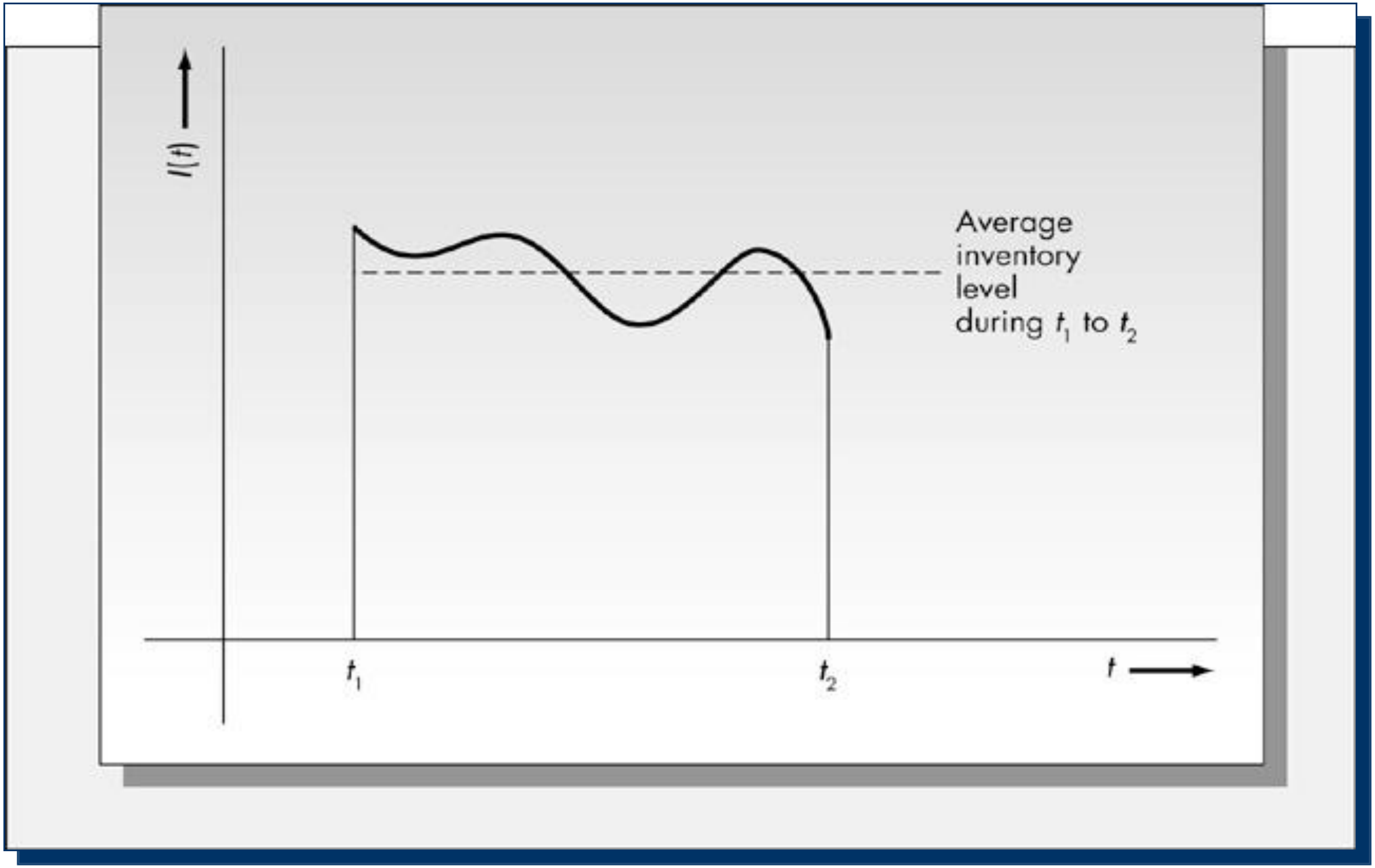
Inventory in industry

- Arçelik in 2002
  - Cost of goods sold = 1,232,496 billion TL
  - Average inventory level = 39,530 billion TL
  - Inventory turnover rate = 31.2
- Migros in 2002
  - Cost of goods sold = 657,065 billion TL
  - Ending inventory level = 63,323 billion TL
  - Inventory turnover rate = 10.4
- Inventories in the US in 1999
  - 1,370 billion USD

Relevant costs

- Holding costs (h)
  - Cost of physical storage
  - Taxes and insurance
  - Breakage, spoilage, deterioration, obsolescence
  - Opportunity cost of alternative investment
- Order cost (K)
- Penalty costs (p)

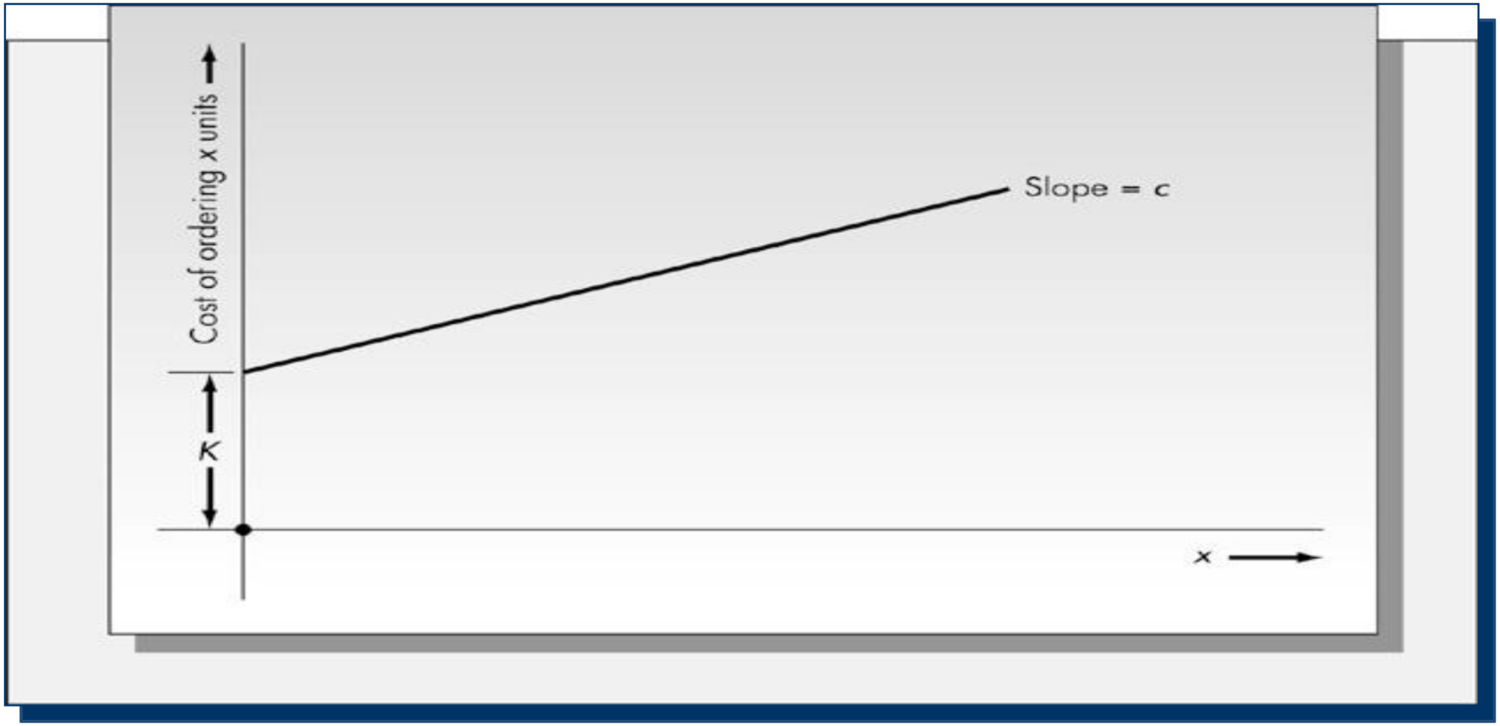
Inventory as a Function of Time



The basic EOQ model

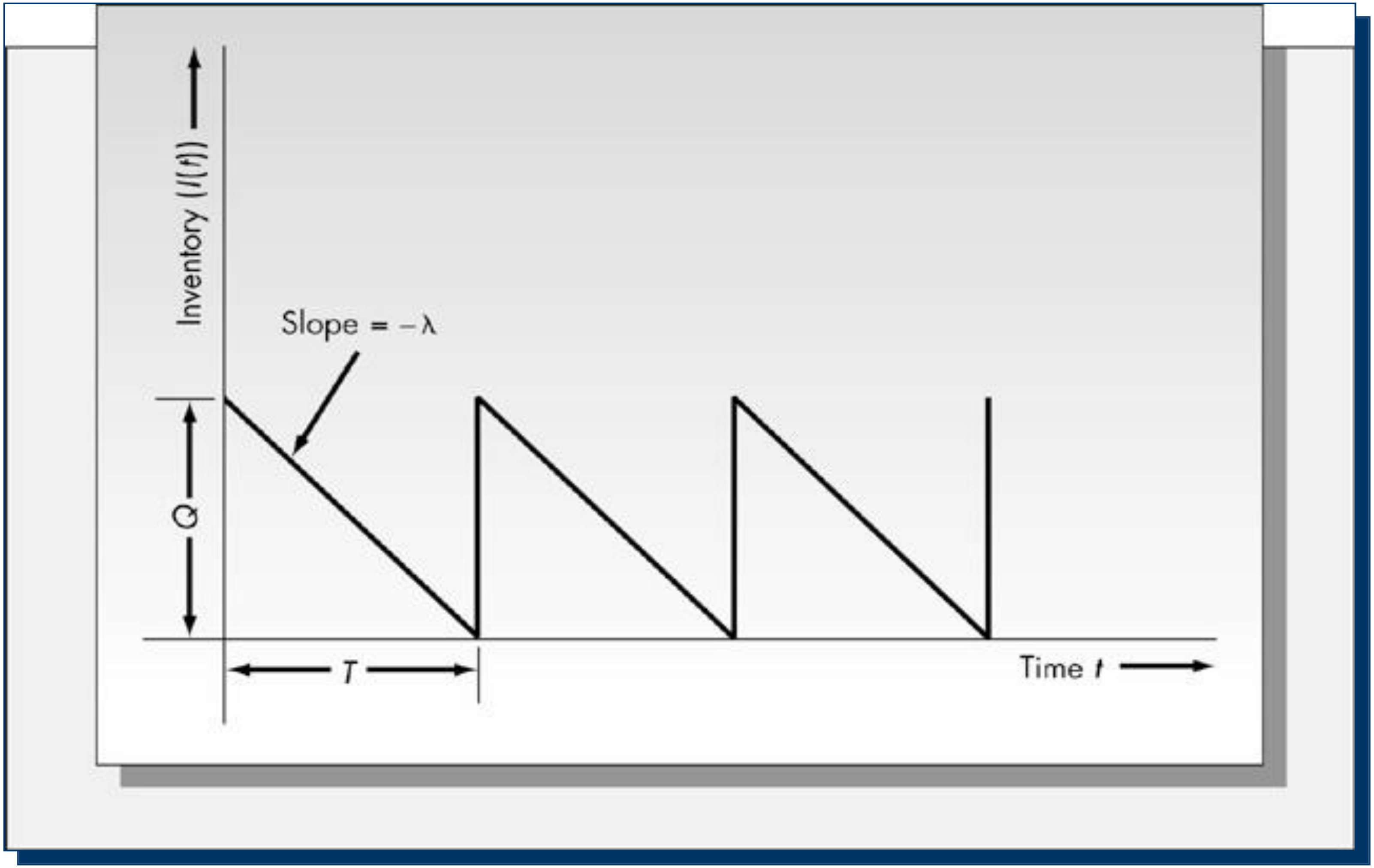
- Assumptions
  - The demand rate is known and constant λ units per unit time
  - Shortages are not allowed
  - There is no order lead time
  - The costs include
    - Setup cost at K per positive order placed
    - Proportional order cost at c per unit ordered
    - Holding cost at h per unit held per unit time

Order Cost Function



$$C(x) = \begin{cases} 0, & \text{if} \quad x = 0 \\ K + cx, & \text{if} \quad x > 0 \end{cases}$$

Inventory Levels for the EOQ Model



Cost function

$$c(Q) = K + cQ$$

$$G(Q) = \frac{K + cQ}{T} + \frac{hQ}{2}$$

$$T = Q / \lambda$$

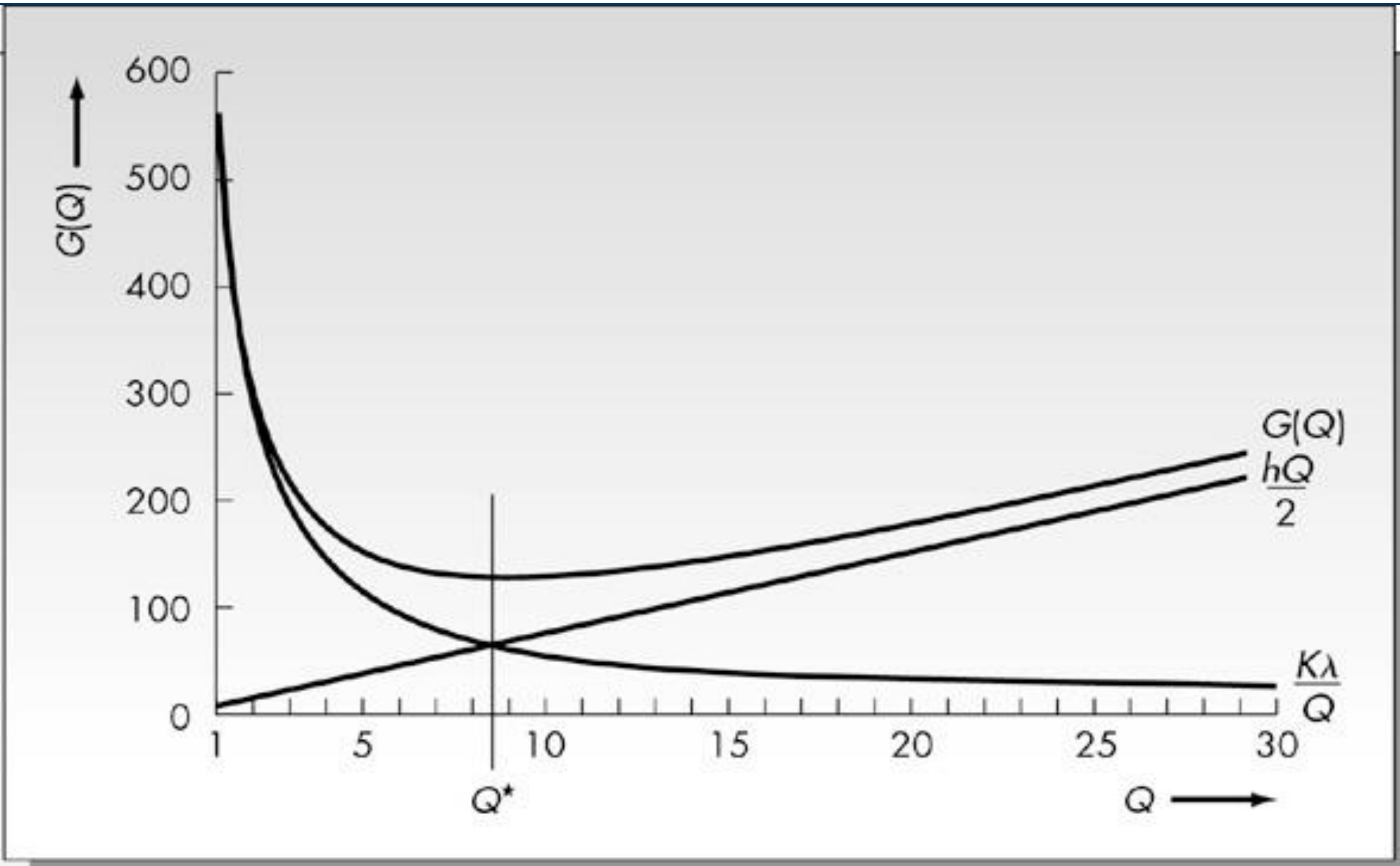
$$G(Q) = \frac{K\lambda}{Q} + \lambda c + \frac{hQ}{2}$$

$$G'(Q) = -K\lambda / Q^2 + h / 2$$

$$G''(Q) = 2K\lambda / Q^3 > 0, \text{ for } Q > 0$$

$$Q^* = \sqrt{\frac{2K\lambda}{h}}$$

Average Annual Cost Function G(Q)



Example

- Consider the photocopy shop at Bilkent library. The shop uses 50 reams (50*500 sheets) of A4 paper every day. The shop orders its papers from Xerox distributor in Istanbul. The distributor charges 20 TL for delivery (in addition to the cost of papers) regardless of the number of reams ordered. Photocopy shop is estimating the inventory holding costs at 22% of the purchase costs. Papers cost 3 TL per ream.
  - How many reams of paper should the shop order each time?
  - How frequently should the shop order?

Example

$$\lambda = 50 \times 365 \quad \text{reams per year}$$

$$K = 20 \text{ TL}$$

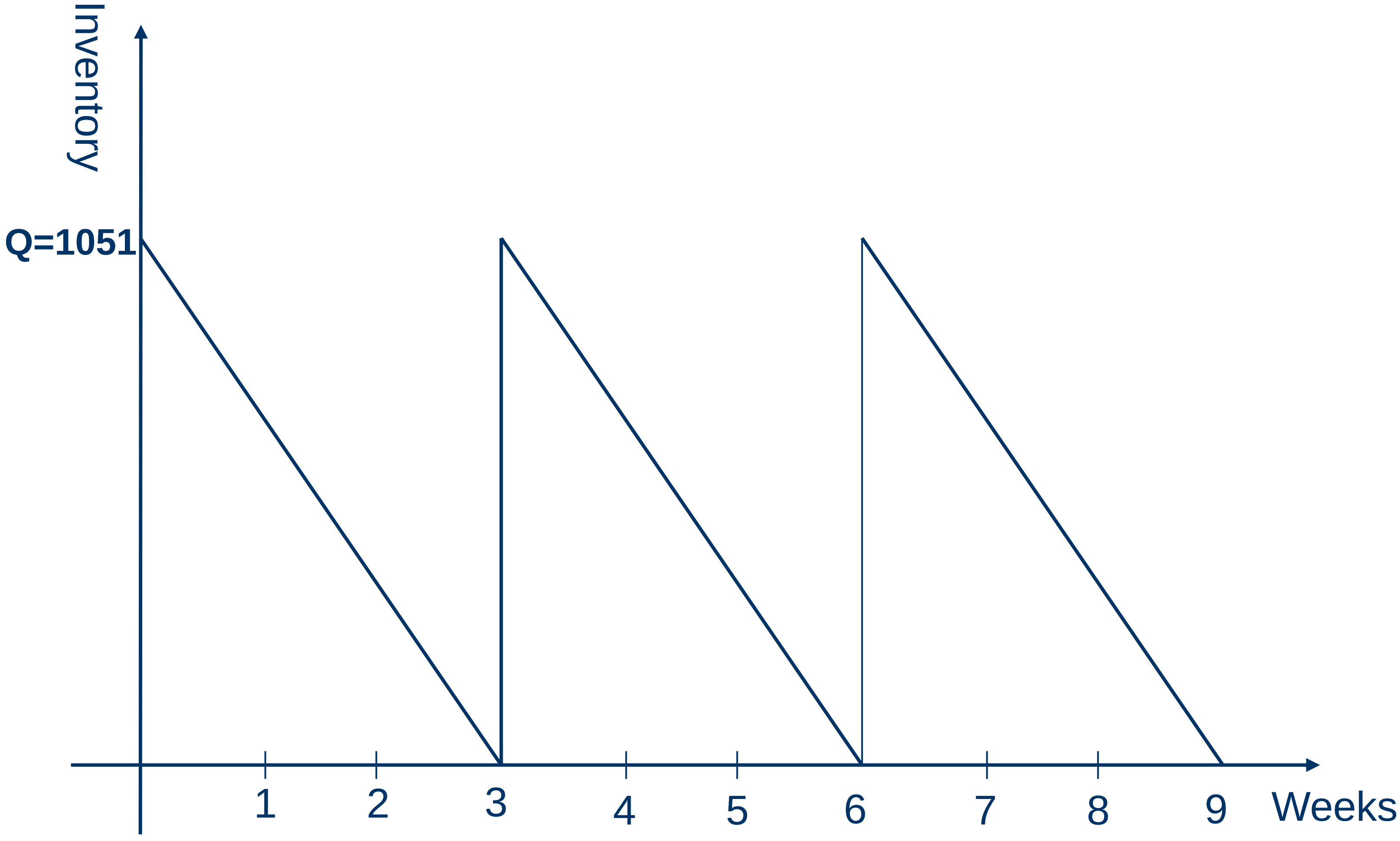
$$h = 3 \times 0.22 = 0.66 TL \text{ per year per ream}$$

$$Q^* = \sqrt{\frac{2K\lambda}{h}} = 1051$$

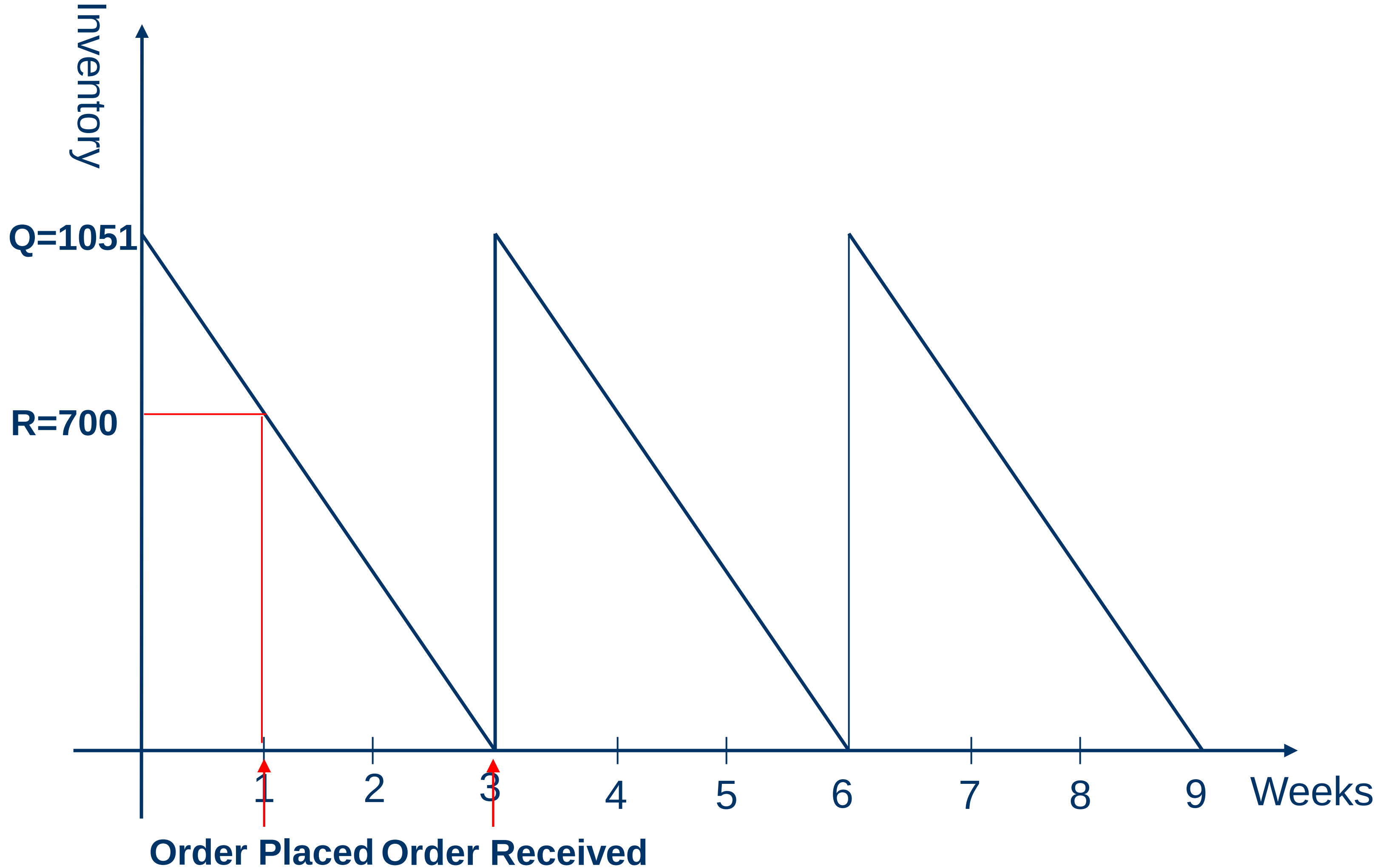
$$T^* = Q^* / \lambda = 1051/(50 \times 365) = 0.0576 \text{ years} = 3 \text{ weeks}$$

$$\text{or } 52/3 = 17.3 \text{ orders per year}$$

Example - continued



Example – delivery lead time = 2 weeks



Example – delivery lead time = 7 weeks

Sensitivity of annual cost to Q

$$G(Q^*) = K\lambda / Q^* + hQ^* / 2$$

$$Q^* = \sqrt{\frac{2K\lambda}{h}}$$

$$G(Q^*) = \frac{K\lambda}{\sqrt{2K\lambda / h}} + \frac{h}{2}\sqrt{2K\lambda / h} = \sqrt{2K\lambda h}$$

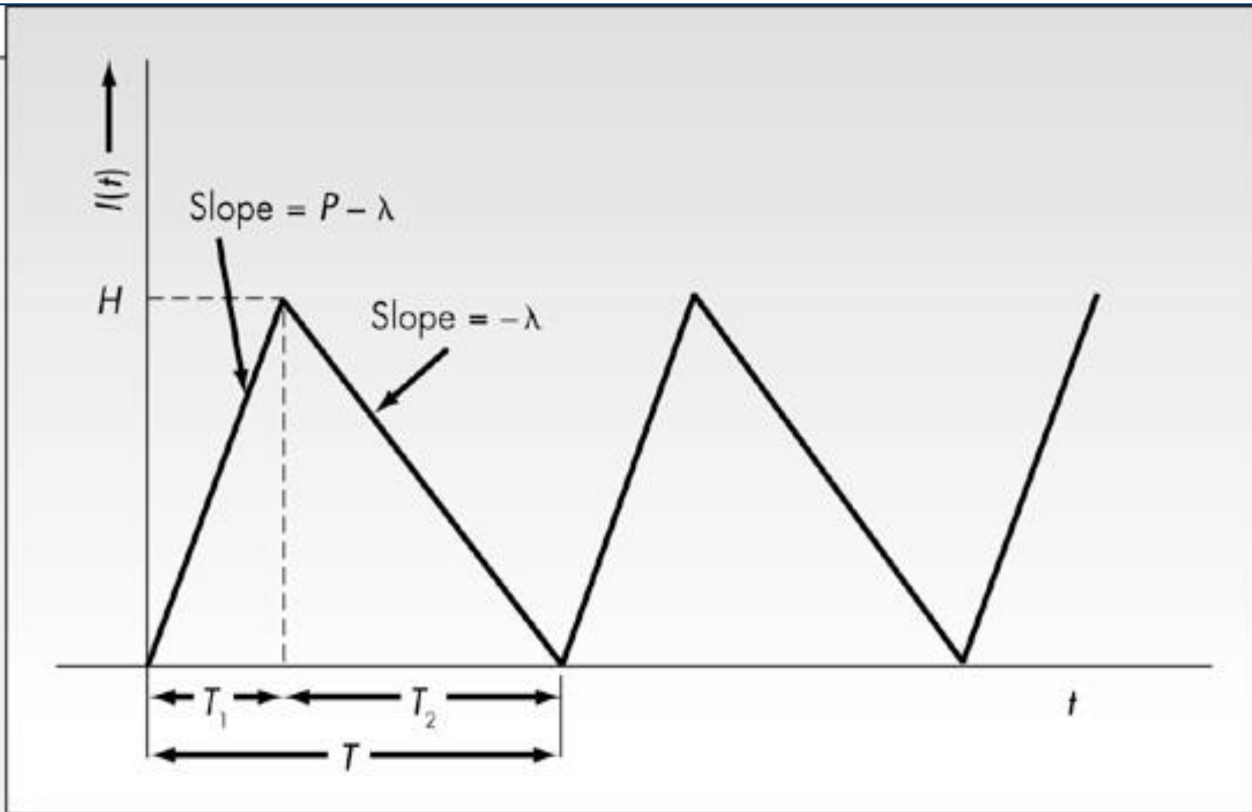
$$\frac{G(Q)}{G(Q^*)} = \frac{K\lambda / Q + hQ / 2}{\sqrt{2K\lambda h}} = \frac{1}{2Q}\sqrt{\frac{2K\lambda}{h}} + \frac{Q}{2}\sqrt{\frac{h}{2K\lambda}}$$

$$\frac{G(Q)}{G(Q^*)} = \frac{Q^*}{2Q} + \frac{Q}{2Q^*} = \frac{1}{2}\left[\frac{Q^*}{Q} + \frac{Q}{Q^*}\right]$$

Inventory Levels for Finite Production Rate Model

- In basic EOQ model, items are procured from an outside supplier; the entire replenishment is delivered at the same time
- What happens if you are actually producing the items in house with some finite production rate?
- Assume that items are produced with a rate of P which satisfies $P>\lambda$
- If there is a setup cost of K each time the production starts, what is the size of each production run (Q)?

Inventory Levels for Finite Production Rate Model



Inventory Levels for Finite Production Rate Model

$$T = T_1 + T_2 \qquad T = Q / \lambda$$

$$T_1 = H /(P - \lambda) \qquad T_2 = H / \lambda$$

$$\frac{Q}{\lambda} = \frac{H}{P - \lambda} + \frac{H}{\lambda} = H\left(\frac{1}{P - \lambda} + \frac{1}{\lambda}\right)$$

$$H = Q\left(1 - \frac{\lambda}{P}\right)$$

$$G(Q) = \frac{K}{T} + \frac{hH}{2} = \frac{K}{T} + h\left(1 - \frac{\lambda}{P}\right)\frac{Q}{2}$$

$$h' = h\left(1 - \frac{\lambda}{P}\right) \qquad Q^* = \sqrt{\frac{2K\lambda}{h'}}$$

Example

- Consider the Renault car factory. The assembly line at the Bursa plant can produce 300 Megane models per day. With the current demand, Renault is shipping out 100 Megane models per day. Each time the assembly line switches production to Megane (from another model like Clio), the assembly line needs to be set up, which costs 30 000 TL. Each Megane costs Renault 15 000 TL and inventory holding costs are 25% of the cost per year. How many Meganes should Renault assemble at each run?

EOQ models for production planning

- Consider a production planning problem where *n* items are produced on single resource
- Each item has a finite production rate
- How do you schedule production of these n items on the single resource so that total of inventory holding costs and ordering costs is minimized
- Assumptions
  - Enough capacity to meet demand
  - Each item requires the same setup time (or cost) regardless of what was produced before in the single resource

EOQ models for production planning

- Assumption: Rotation cycle policy. In each cycle there is exactly one setup for each product, and products are produced in the same sequence in each cycle

λ_j : Demand rate for product j

Pj : Production rate for product j

h_j : Holding cost per unit per unit time for product j

K_j : Cost of setting up the production facility to produce j

$$\sum_{j=1}^{n} \lambda_j / P_j \leq 1$$

Can we use individual EOQ for each part?

$$\mathrm{Q_j} = \sqrt{\frac{2K_j\lambda_j}{h'_j}}$$

EOQ models for production planning

- Let T be the cycle time. During T exactly one lot of each product is produced

$$Q_j = \lambda_j T$$

$$G(Qj) = K_j \lambda_j / Q_j + h'_j Q_j / 2$$

$$\sum_{j=1}^{n} G(Qj) = \sum_{j=1}^{n} \left(K_j \lambda_j / Q_j + h'_j Q_j / 2 \right)$$

$$G(T) = \sum_{j=1}^{n} \left(K_j / T + h'_j \lambda_j T / 2 \right)$$

$$\frac{dG(T)}{dT} = \sum_{j=1}^{n} \left(- K_j / T^2 + h'_j \lambda_j / 2 \right) = 0$$

$$T^* = \sqrt{\frac{2 \sum_{j=1}^{n} K_j}{\sum_{j=1}^{n} h'_j \lambda_j}}$$

EOQ models for production planning

- Consider the case where each production run requires a setup time of s_i in addition to the setup cost. What should be our cycle time?

$$\sum_{j=1}^{n}\left(s_j + Q_j / P_j\right) \le T$$

$$\sum_{j=1}^{n}\left(s_j + \lambda_j T / P_j\right) \le T$$

$$T \ge \frac{\sum_{j=1}^{n} s_j}{1 - \sum_{j=1}^{n}(\lambda_j / P_j)} = T_{\min}$$

$$\text{Choose } \max(T^*, T_{\min})$$

Example

- Bali produces several styles of men’s and women’s shoes at a single facility near Bergamo, Italy. Bali would like to schedule a rotation policy for production that meets all demand and minimizes setup and holding costs. Setup costs are proportional to setup times. The firm estimates that setup costs amount to an average of $110 per hour, based on the cost of worker time and the cost of forced machine idle time during setups. Inventory holding costs are 22% of the unit cost and the company has 250 working days in a year and 8 working hours per day and no overtime is allowed.

Style	Annual demand (units/year)	Production rate (units/year)	Setup time (hrs)	Variable cost ($/unit)
Women’s pump	4520	35800	3.2	40
Women’s loafer	6600	62600	2.5	26
Women’s boot	2340	41000	4.4	52
Women’s sandal	2600	71000	1.8	18
Men’s wingtip	8800	46800	5.1	38
Men’s loafer	6200	71200	3.1	28
Men’s oxford	5200	56000	4.4	31