

Chapter 4 Notes

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IE375 On-Line

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Remaining Problems of Interest

- We are done with the following:
 - Basic EOQ
 - Lead-time
 - Production Rate
 - Multi-item
- Quantity Discounts
- EOQ with backorders
- Practicality of the EOQ Solution and powers-of-two restriction
- Periodic Review – Lot Sizing Problem

Quantity Discount Models

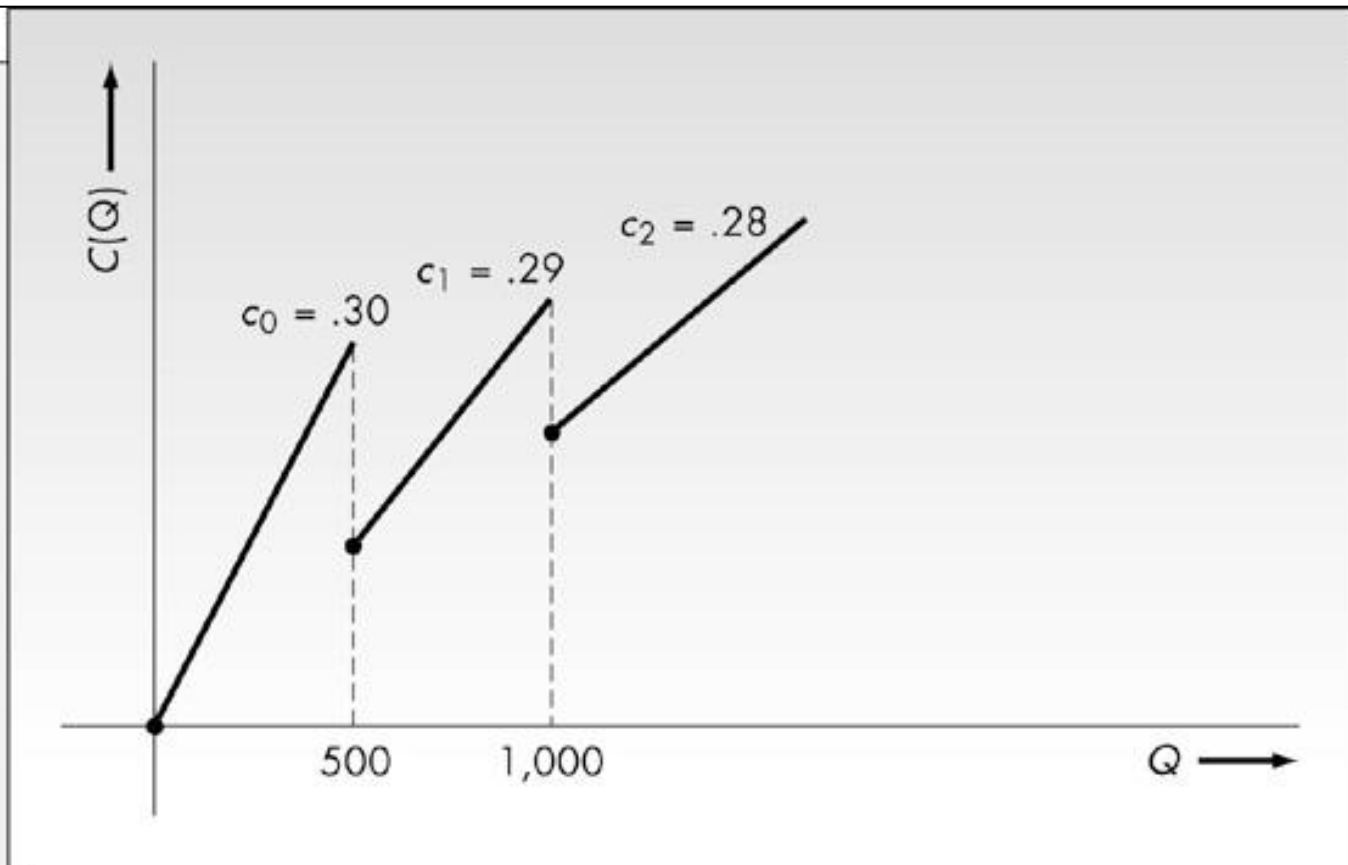
- In EOQ models, the unit cost c is independent of the size of the order
- However, usually suppliers are willing to charge less per unit for larger orders
- Two type of discount schedules
- All-units discount
 - The discount is applied to all units
- Incremental quantity discounts
 - The discount is applied to additional units beyond the breakpoint

All units quantity discounts

- Example
 - orders that are less than 500 units are charged 30 cents per unit
 - orders of 500 or more, but less than 1000 units are charged 29 cents per unit
 - order of 1000 or more are charged 28 cents per unit
- The purchase cost function $C(Q)$ can be written as

$$C(Q) = \begin{cases} 0.30Q & \text{for } 0 \leq Q < 500 \\ 0.29Q & \text{for } 500 \leq Q < 1000 \\ 0.28Q & \text{for } 1000 \leq Q \end{cases}$$

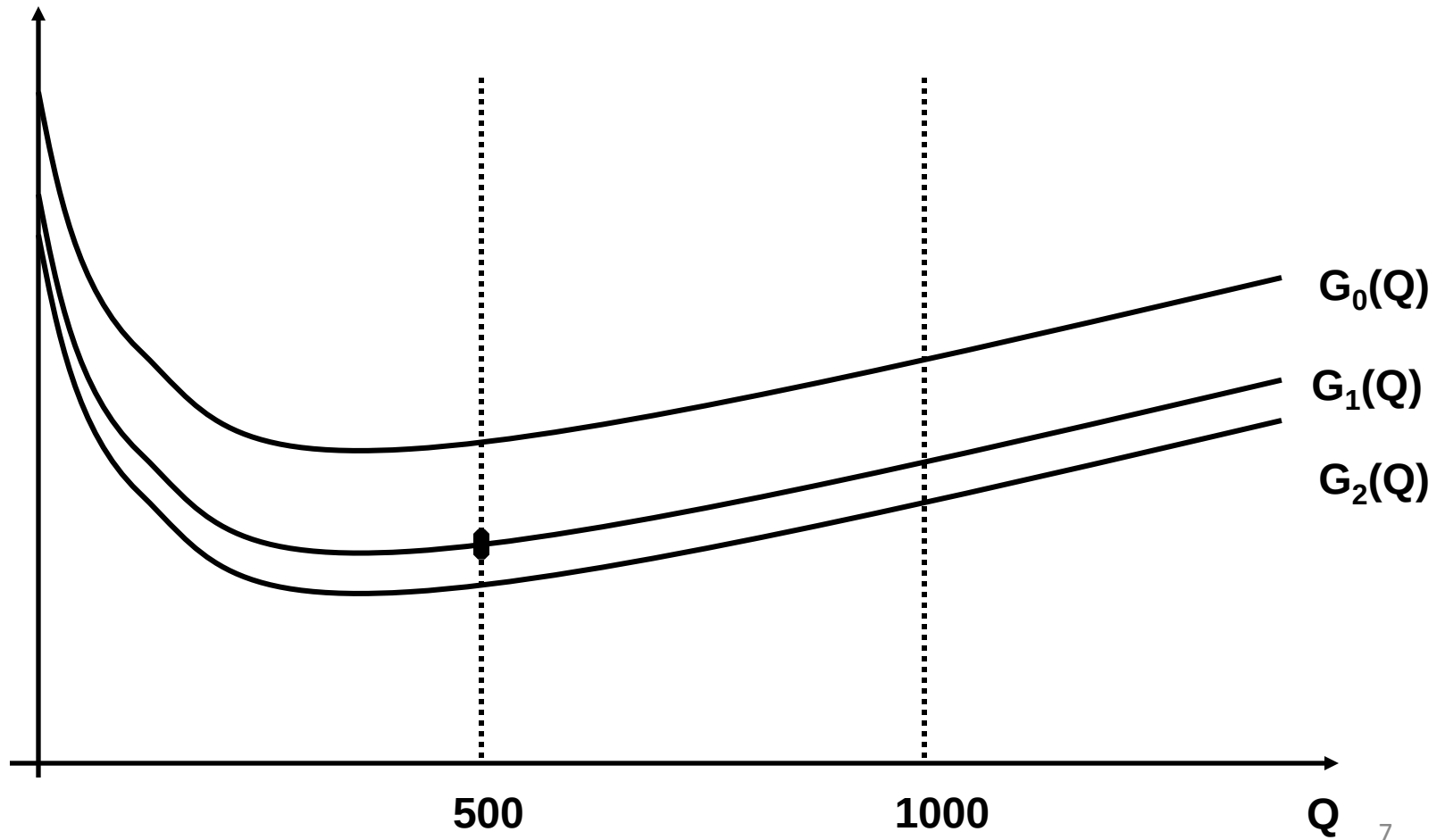
All-Units Discount Order Cost Function



All units quantity discounts – finding optimal Q

- Determine the largest realizable EOQ value. Start from the lowest price first, continue with the next higher price. Stop when the first EOQ value is realizable.
- Compare the value of the average annual cost at the largest realizable EOQ and all of the price breakpoints that are greater than the largest realizable EOQ. The optimal Q is the point at which the average annual cost is a minimum
- Example continued
 - **Annual demand of 600 units. If $K=\$8$ per order and a 20% inventory holding cost is applied on cost, find the optimal order quantity for all units quantity discounts scheme**

Average annual cost for all-units discount

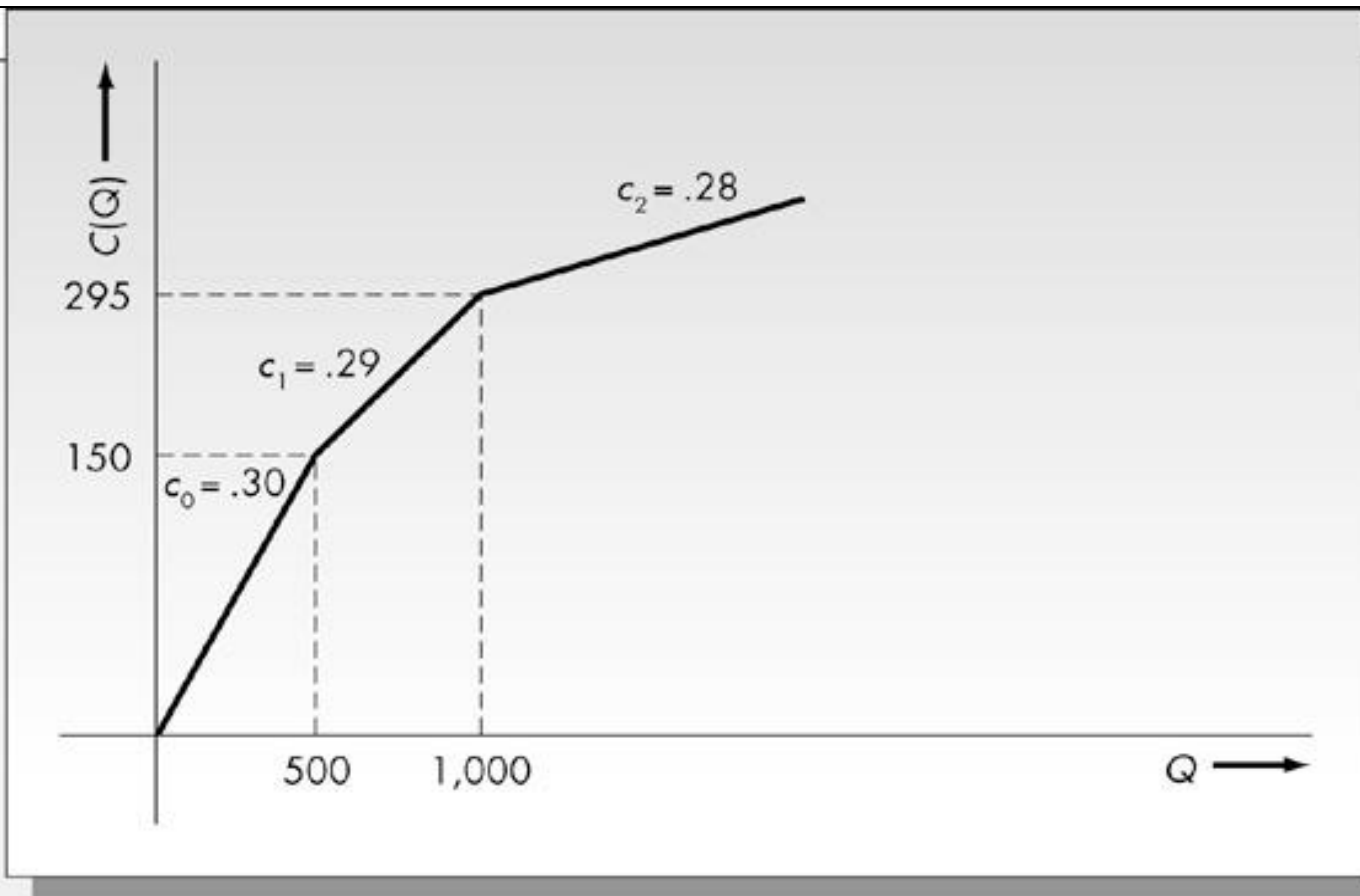


Incremental quantity discounts

- Example
 - For orders that are less than 500 units are charged 30 cents per unit
 - For orders of 500 or more, but less than 1000 units, 499 units are charged 30 cents per unit, remaining units are charged 29 units
 - For order of 1000 or more, 500 units are charged 30 cents per unit, 500 units are charged 29 cents per unit, remaining units are charged 28 cents per unit
- The purchase cost function $C(Q)$ can be written as

$$C(Q) = \begin{cases} 0.30Q & \text{for } 0 \leq Q < 500 \\ 150 + 0.29(Q - 500) & \text{for } 500 \leq Q < 1000 \\ 295 + 0.28(Q - 1000) & \text{for } 1000 \leq Q \end{cases}$$

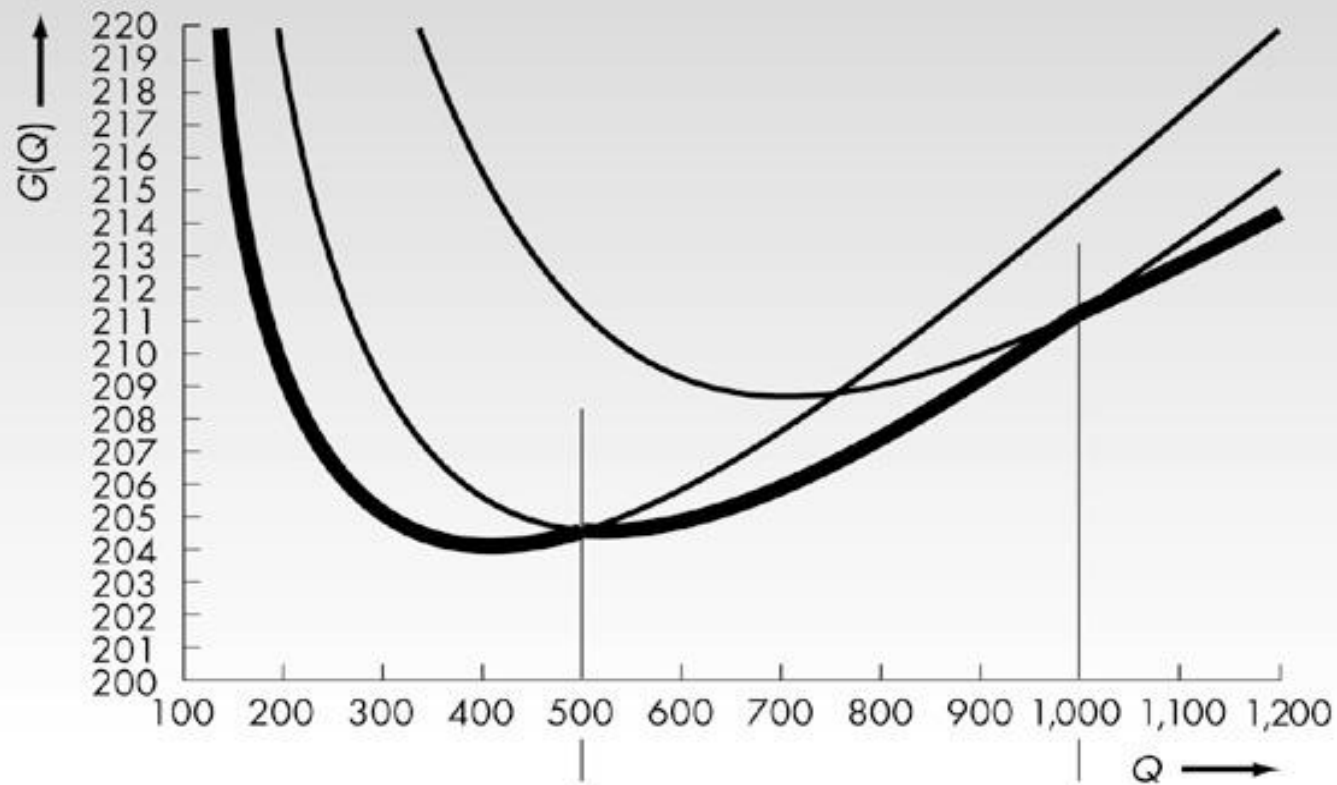
Incremental Discount Order Cost Function



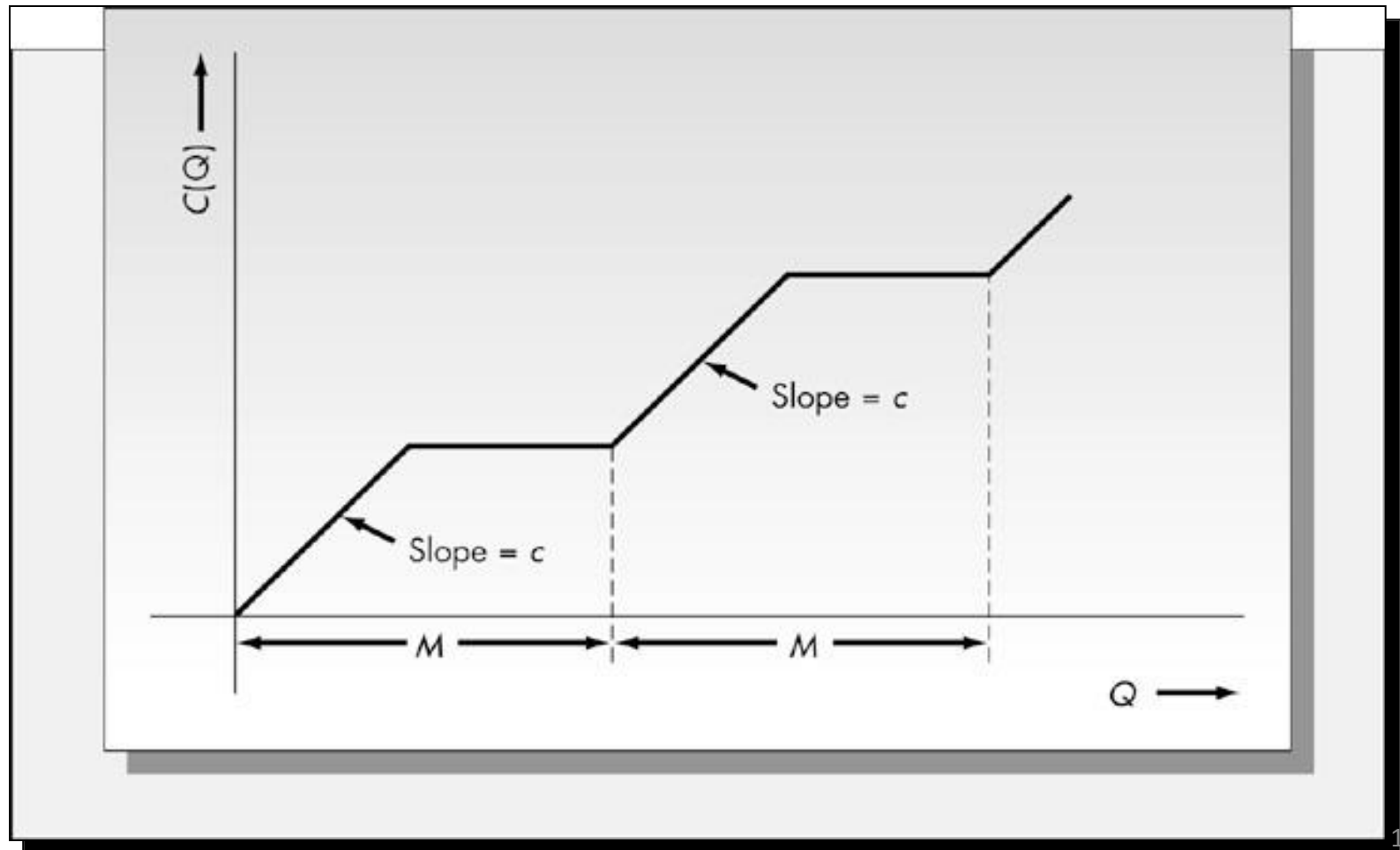
Incremental quantity discounts – finding optimal Q

- Determine the algebraic expression for $C(Q)$ corresponding to each price interval. Use this to find $C(Q)/Q$
- Use $C(Q)/Q$ to find $G(Q)$ for each price interval. Find the optimal value of Q for each price interval.
- Determine which of the minima computed are realizable. Compare the values of the average annual cost at the realizable EOQ values and pick the lowest.
- Example continued
 - **Annual demand of 600 units. If $K=\$8$ per order and a 20% inventory holding cost is applied on cost, find the optimal order quantity for all units quantity discounts scheme**

Average Annual Cost Function for Incremental Discount Schedule



Other discount schedules: carload discount schedule

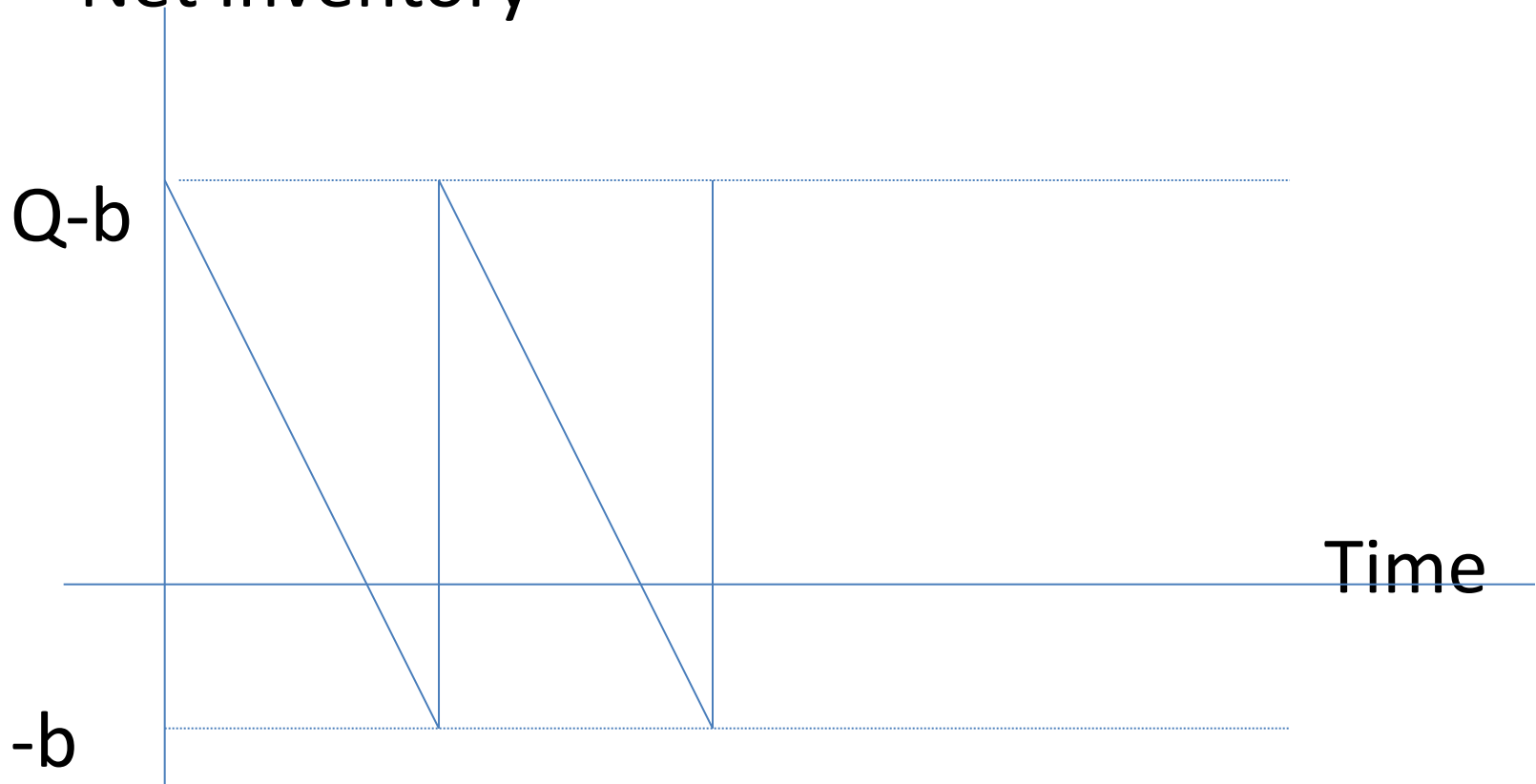


EOQ with Backorders

- Consider the extension of EOQ which allows for backorders.
- Cyclic structure holds: We order, Satisfy previous backorders, satisfy demand from inventory and backorder at the end – true for every cycle.
- As additional parameters, we solve for b , where b is defined as the maximum number of backorders allowed. We also define p as the cost of backorders/unit/unit time.

EOQ with Backorders

- Net Inventory



EOQ with Backorders

- Formulation – similar to EOQ - Cycle Time = $T = \lambda^* Q$
- Write average fixed ordering cost , average inventory carrying cost (area/ T) and average backorder cost (area/ T)
- One can obtain the optimal solution to this problem as follows:
 - $b^* = \{h/(p+h)\}Q^*$
and $Q^* = \{(2KI/h)^*[(p+h)/h]\}^{1/2}$
- Using these results, write the average total cost function and show that it is always less than the average total cost obtained by EOQ with no backorders –
- Question 2) of the Mini-HW2.

Powers-of-two Policies

- Practicality of the EOQ Solution.
- Multiple items
- Correction for realistic times
- T_L : base-time (1 hour, 1 week)
- Limit your cycle times to $2^k T_L$ for $k=0, 1, 2, \dots$
- It is shown that with this restriction at most an additional 6% of the optimal EOQ average cost will be paid.

Statement of the Lot Sizing Problem

- Known set of requirements $(d_1, d_2, \dots, d_T)$ over an T period planning horizon.
- Costs: set up cost, K , and the holding cost, h , per unit in the ending inventory per period
- The objective is to determine production quantities $(X_1, X_2, \dots, X_T)$ to meet the requirements at minimum cost.
- The feasibility condition to assure there are no stockouts in any period is:

$$I_{t-1} + X_t - d_t = I_t \text{ for all } t$$

Lot Sizing Problem

- Periodic Review where periods are given
- Formulation (we already did in class)
 - minimize total cost (excluding sum of unit purchasing costs which is constant)

$$\text{Min } \sum_t \{ \delta(X_t)K + hI_t \}$$

$$I_{t-1} + X_t - d_t = I_t \text{ for all } t$$

$$\delta(X_t) = 1 \text{ if } X_t > 0; 0 \text{ o.w.}$$

$$X_t \geq 0$$

- Solution: Dynamic Programming in general

Lot Sizing Problem

- Special Case – Observations
- (1) Do we order when there is inventory? NO
- (2) So to ensure (1) how much do we order, if we order in a period t ?

d_t , or $d_t + d_{t+1}$, or $d_t + d_{t+1} + d_{t+2}$, ...

- Solution: DP + shortest path problem:
 - *Node 1 – corresponds to time 1; *node $T+1$ corresponds to time $T+1$ (end of horizon)
 - *Arc(i,j) – corresponds to the cost of ordering at i to satisfy all the demand requirements from i to $j-1$, $j > i$