Mini-Homework 2 Key

Question 1

$\boxed{a}$

For Single Core

$\lambda = 500 \times 365$

$= 182\,500$ item/year

$K = \$10{,}000$

$h = 0.2 \times 80$

$= \$16$ /item-year

(i) $Q = \sqrt{\dfrac{2 \times 10000 \times 182500}{16}}$

$= 15103.81$ item

Common Formulas

$h = Ic$

$$Q = \sqrt{\frac{2K\lambda}{h}}$$

For Dual Core

$\lambda = 1000 \times 365$

$= 365{,}000$ item/year

$K = \$12{,}000$

$h = 0.2 \times 100$

$= \$20$ / item-year

$Q = \sqrt{\dfrac{2 \times 12000 \times 365000}{20}}$

$= 20928.45$ item

$$TC(Q) = \underbrace{\frac{K\lambda}{Q}}_{\text{setup cost}} + \underbrace{c\lambda}_{\text{purchase cost}} + \underbrace{\frac{hQ}{2}}_{\text{holding cost}}$$

(ii) $TC = \$14{,}841{,}660$ /year

$TC = \$36{,}918{,}568$ /year

$\boxed{b}$ Rotation cycle policy can solve this problem by p. 232 of textbook (7th ed.), the sentence starting with "We also will assume that ..."

We also will assume that the policy used is a *rotation cycle policy*. That means that in each cycle there is exactly one setup for each product, and products are produced in the same sequence in each production cycle. The importance of this assumption will be discussed further at the end of the section.

$$T^* = \sqrt{\frac{2 \times \sum_{i=1}^{n} K_i}{\sum_{J=1}^{n} h'_J \lambda_J}}$$ and for this problem, $h'_J = h_J\left(1 - \frac{\lambda_J}{P_J}\right) = h_J$

since $P_J = +\infty$ (orders are received instantly)

$$T^* = \sqrt{\frac{2 \times 12{,}000}{(81 \times 0.2 \times 182500) + (100 \times 0.2 \times 365000)}}$$

$= 0.0483$ year

(i) $Q^* = \lambda T^*$

Single Core: $Q^* = 182500 \times T^* = 8814.75$ item

Dual Core: $Q^* = 365000 \times T^* = 17629.5$ item

(ii) $$TC = \frac{K}{T^*} + \sum_{i=1}^{2}\left[\lambda_i c_i + \frac{h_i Q_i^*}{2}\right]$$

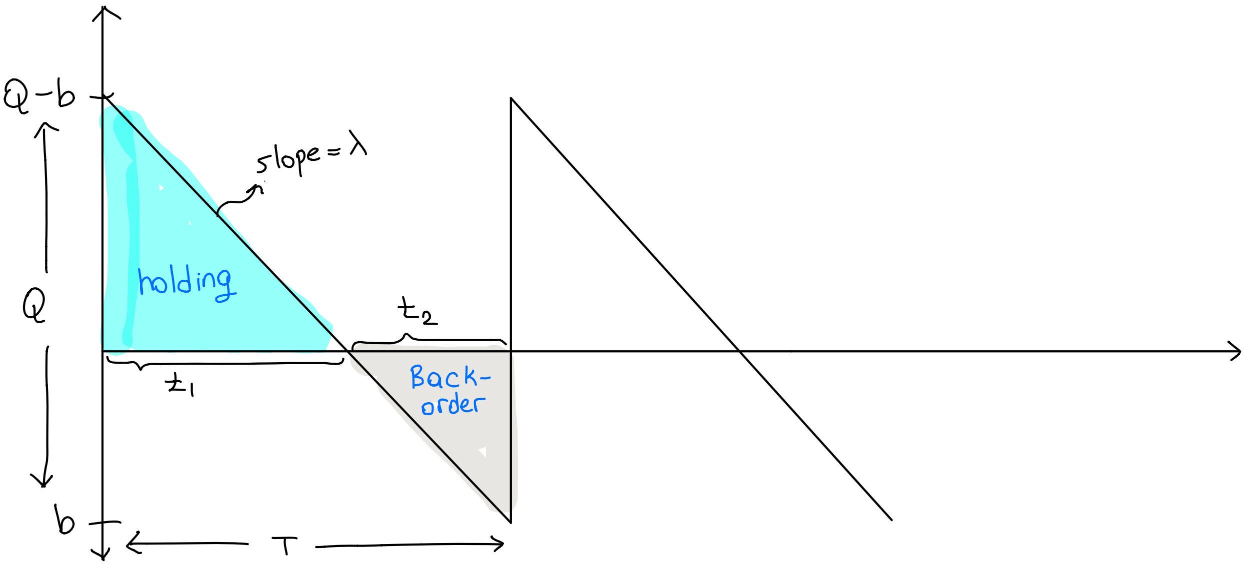
$$= \underbrace{\frac{12{,}000}{0.0483}}_{\text{common setup cost}} + \underbrace{182500 \times 81 + 365000 \times 100}_{\text{purchase cost}} + \underbrace{\frac{16.2 \times 8814.75}{2} + \frac{20 \times 17629.5}{2}}_{\text{holding cost}}$$

$= \$\,51778641/\text{year}$

(iii) Since the sum of total costs in part (a) ($\$51760228$) is smaller than the total cost in part (b) ($\$51778641$) Lenovo should <u>not accept</u> offer

Question 2

First, lets understand the effect of backorder intuitively by drawing a picture similar to Figure 4-4 on p. 211 of our textbook.



(a) Average costs:

Setup : $\dfrac{K}{T} = \dfrac{K\lambda}{Q}$ (same with EOQ)

Holding : $h \times \dfrac{(Q-b)\,t_1}{2\,T}$ (The region with color)

Backorder : $p \times \dfrac{b\,t_2}{2T}$ (The region with color)

Moreover, $\lambda . t_1 = Q-b \Rightarrow t_1 = \dfrac{Q-b}{\lambda}$, $\lambda . t_2 = b \Rightarrow t_2 = \dfrac{b}{\lambda}$, $T = Q/\lambda$

Hence,

$$G(Q,b) = \frac{K\lambda}{Q} + h\,\frac{(Q-b)^2}{2Q} + p\,\frac{b^2}{2Q}$$

Now, lets plug in Q^* and b^* in the objective function

$$G(Q^*, b^*) = \frac{K\lambda}{Q} + h\frac{(Q-b)^2}{2Q} + p\frac{b^2}{2Q}$$

$$= \frac{K\lambda}{Q} + \frac{h\left(\frac{pQ}{p+h}\right)^2}{2Q} + \frac{p\left(\frac{hQ}{p+h}\right)^2}{2Q} \quad \text{since } Q-b = Q - \frac{hQ}{p+h} = \frac{pQ}{p+h}$$

$$= \frac{K\lambda}{Q} + \frac{h.p^2.Q}{2.(p+h)^2} + \frac{p.h^2.Q}{2.(p+h)^2}$$

$$= \frac{K\lambda}{Q} + \frac{ph.(p+h)Q}{2(p+h)^2}$$

$$= \frac{K\lambda}{Q} + \frac{phQ}{2(p+h)}$$

- $$\frac{K\lambda}{Q} = K\lambda.\sqrt{\frac{h}{2K\lambda}\left(\frac{p}{p+h}\right)} = \sqrt{\frac{K\lambda h}{2}}.\sqrt{\frac{p}{p+h}}$$

- $$\frac{ph}{2(p+h)}Q = \frac{ph}{2(p+h)}\sqrt{\frac{2K\lambda}{h}.\left(\frac{p+h}{p}\right)} = \sqrt{\frac{K\lambda h}{2}}\sqrt{\frac{p}{p+h}}$$

<u>Result:</u> $$G(Q^*, b^*) = \sqrt{2K\lambda h}\sqrt{\frac{p}{p+h}}$$

Since $p/(p+h) < 1$, $G(Q^*, b^*)$ is always less than the average total cost obtained by EOQ with no backorders.

Derivation of Q^* and b^* (Not required in the homework)

$$\frac{\partial G(Q,b)}{\partial b} = -\cancel{2}.\frac{h.(Q-b)}{\cancel{2}Q} + \cancel{2}.\frac{p.b}{\cancel{2}Q} = 0 \quad (1)$$

$$\frac{\partial G(Q,b)}{\partial Q} = \frac{-K\lambda}{Q^2} + \frac{h}{2} - \frac{b^2h}{2Q^2} - \frac{pb^2}{2Q^2} = 0 \quad (2)$$

$$(1) \Rightarrow \frac{pb}{Q} = \frac{h(Q-b)}{Q} \Rightarrow (p+h)b = hQ \Rightarrow \underbrace{b = \frac{hQ}{p+h};}_{(3)}$$

$$(2) \Rightarrow \frac{-2K\lambda + hQ^2 - b^2(p+h)}{2Q^2} = 0$$

$$\Rightarrow hQ^2 = 2K\lambda + b^2(h+p) = 2K\lambda + \frac{h^2}{p+h}Q^2 \quad (\text{by } (3))$$

$$\Rightarrow Q^2\left(h - \frac{h^2}{p+h}\right) = 2K\lambda$$

$$\Rightarrow Q^2 = 2K\lambda\frac{(p+h)}{hp}$$

$$\Rightarrow Q = \sqrt{\frac{2K\lambda}{h}\cdot\left(\frac{p+h}{p}\right)} \quad (4)$$

Moreover, the objective function is "nice" so that taking derivative and equating to zero yields the optimal solution.

Now, we can replace Q and b with Q^* and b^* in equations (3) and (4), respectively.