# Mini-Homework 3 Key

## Question 1

(a) This problem can be solved by using the Newsvendor Model on p. 258 of our textbook (7th ed.)

$c_u = p - c = 35 - 8 = 27$ cents

$c_o = c - s = 8 - 3 = 5$ cents

critical ratio $= \dfrac{c_u}{c_u + c_o} = \dfrac{27}{32} = 0.84375$

By p. 262 of textbook (7th edition) $Q^*$ is the smallest number satisfying

$$F(Q^*) \geq \frac{c_u}{c_u + c_o}$$

where $F$ is the distribution function defined by the table in the question.

By observing the table, we can see that

$$Q = 20 \Rightarrow F(Q) = 0.70 < c_u / c_u + c_o = 0.84375$$

$$Q = 25 \Rightarrow F(Q) = 0.85 \geq c_u / c_u + c_o = 0.84375$$

Hence $Q^* = 25$.

(b) Introduce the following notation

Let the random variable $D$ denote the daily demand for bagels.

Let $F$ denote the cumulative distribution function of $D$ (given in the table.)

Let $\mu$ denote the mean of $D$, i.e., $\mu = \mathbb{E}[D]$

Let $\sigma^2$ denote the variance of $D$, i.e., $\sigma^2 = \mathrm{var}(D) = \mathbb{E}[D^2] - (\mathbb{E}[D]^2)$

Let the random variable $D^n$ denote the normal approximation of $D$

Let $F^n$ denote the cumulative distribution function of $D^n$

($F^n$ is cumulative dist. function of normal distribution with mean $\mu$, variance $\sigma^2$)

Let $F^s$ denote the distribution function of standard normal distribution.

$$\mu = \mathbb{E}[D] = \sum_{i=1}^{8} d_i \times \mathbb{P}\{D = d_i\}$$

where $d_i$'s are realized values of $D$. (e.g. $d_1 = 0$, $d_2 = 5$, $d_3 = 10$, ...)

Then,

$$\mu = \mathbb{E}[D] = 0 \times 0.05 + 5 \times 0.1 + \ldots + 35 \times 0.05$$ (using the values in table)

$$= 18$$

Moreover,

$$\mathbb{E}[D^2] = \sum_{i=1}^{8} d_i^2 \; \mathbb{P}\{D = d_i\}$$

$$= [(0^2 \times 0.05) + (5^2 \times 0.01) + \ldots + (35^2 \times 0.05)]$$

$$= 402.5$$

Therefore,

$$\sigma^2 = \mathbb{E}[D^2] - (\mathbb{E}[D])^2 = 402.5 - 18^2 = 78.5$$

and $\sigma = \sqrt{\sigma^2} = 8.86$

Result 1: $D^n \sim N(\mu, \sigma^2)$ with $\mu = 18$, $\sigma^2 = 78.5$

Now, we can find $Q^*$ when the demand distribution is approximated by a normal dist., similar to Example 5.1 on page 261 of our textbook (7th ed.)

The closest $z$ value which corresponds to

$$F^s(z) = 0.84375$$

is equal to $z^* = 1.01$ (without interpolation) by table A-4 on p. 807.

Moreover, we know from the properties of normal distribution that

$$F^s(z) = F^n(\sigma z + \mu)$$

Therefore, if $Q^* = \sigma z^* + \mu$, then $F^n(Q^*) = 0.84375$, which is what we want while finding $Q^*$. Consequently,

$$Q^* = \sigma z^* + \mu$$
$$= 8.86 \times 1.01 + 18$$
$$= 26.94$$ bagels should be baked each day.

## Question 2

This problem can be solved by using the Newsvendor Model on p. 258 of our textbook (7th ed.)

(a) $c_u = p - c = 260 - 68 = 192$

$c_o = c - s = 68 - 40 = 28$

critical ratio $= \dfrac{c_u}{c_u + c_o} = \dfrac{192}{220} = 0.8728$

(b) Introduce the following notation

Let the random variable $D$ denote the demand for sweatshirts

Let $\mu$ denote the mean of $D$, ie., $\mu = 1500$

Let $\sigma$ denote the standard deviation of $D$, ie, $\sigma = 300$

Let $F$ denote the cumulative distribution function of $D$

($F$ is distribution function of normal distribution with mean $\mu$, variance $\sigma^2$)

Let $f$ denote the probability distribution function of $D$.

Let $F^S$ denote the cumulative distribution function of standard normal distribution.

The closest $z$ value which corresponds to

$$F^{s}(z) = 0.8728$$

is equal to $z^* = 1.14$ (without interpolation) by table A-4 on p. 808

Moreover, we know from the properties of normal distribution that

$$F^{s}(z) = F^{n}(\sigma z + \mu)$$

Therefore, if $Q^* = \sigma z^* + \mu$, then $F^{n}(Q^*) = 0.8728$, which is what we want while finding $Q^*$. Consequently,

$$\begin{aligned} Q^* &= \sigma z^* + \mu \\ &= 300 \times 1.14 + 1500 \\ &= 1842 \text{ (in thousands) is the ordering quantity.} \end{aligned}$$

(c) $\mathbb{E}[\max\{0, Q^* - D\}]$

$$= \int_{-\infty}^{Q^*} (Q^* - x) f(x)\,dx$$

$$= \int_{-\infty}^{+\infty} (Q^* - x) f(x)\,dx - \int_{Q^*}^{+\infty} (Q^* - x) f(x)\,dx$$

$$= Q^* \underbrace{\int_{-\infty}^{+\infty} f(x)\,dx}_{1} - \underbrace{\int_{-\infty}^{+\infty} x f(x)\,dx}_{\mathbb{E}[D]} + \underbrace{\int_{Q^*}^{+\infty} (x - Q^*) f(x)\,dx}_{n(Q^*)}$$

$$= Q^* - \mathbb{E}[D] + n(Q^*)$$

where $n(Q^*) = \mathbb{E}[\max\{D - Q^*, 0\}]$ defined in p. 270 of textbook (7th ed)

We also know from p. 271 that if the p.d.f. of $D$ is the pdf. of normal distribution, then we can use the formula

$$n(Q^*) = \sigma L\left(\frac{Q^* - \mu}{\sigma}\right).$$ Hence,

$\mathbb{E}[\max\{0, Q^* - D\}]$

$= \sigma L\left(\frac{Q^* - \mu}{\sigma}\right) + Q^* - \mu$

$= 300 \times L\left(\frac{1842 - 1500}{300}\right) + 1842 - 1500$

$= 300 \times L(1.14) + 1842 - 1500$

$= 300 \times 0.0634 + 1842 - 1500$

$= 361.02$ is expected number of shirts not sold.

(d) $\mathbb{E}[\max\{0, D - Q^*\}]$

$= \int_{Q^*}^{+\infty} (x - Q^*) f(x)\,dx$

$= n(Q^*)$

$= \sigma L\left(\frac{Q^* - \mu}{\sigma}\right)$

$= 300 \times 0.0634$ (from part c)

$= 19.02$ is the expected excess demand