IE375 Spring 2020

Case 2: Solution and Comments

(Explains the attached Excel Computation Sheets)

<u>General Comments:</u>

Two comments:

1) Computational work: You better use a spreadsheet or a program to compute similar things over and over again. Some of you looked very hesitant to use any supporting tool. Of course, when you use such a tool, you need to clearly describe the formulas you have used. This is what I did in this document which states the solution.

2) When you are working with numbers, make sure that you used a reasonable number of digits after the decimal point (and be consistent). The reason is obvious: The readers want to get an insight looking at the numbers and you need to facilitate their understanding by only presenting the more significant part. It is also important that you understand those numbers and realize any peculiarities (mostly computational errors).

3) If you are using a program, to submit a spread-sheet or a computer program output without any explanation is not acceptable.

4) Make sure that you have your names written on the report.

Below find relevant statistics regarding grades received.

	Exam
Average	86.82
Std. Dev.	18.64
MAX	100
MIN	0

Range	0-10	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
Freq.	2	0	0	0	0	0	0	6	12	31

Formulas Implemented in Excel:

Equation 1) $Q = \sqrt{\frac{2\lambda[K+p\,n(R)]}{h}}$

Equation 2) $1 - F(R) = \frac{Qh}{p\lambda}$

Equation 3) $G(Q,R) = h(Q/2 + R - \lambda\tau) + K\lambda/Q \ + p\lambda\, n(R)/\, Q$

Equation 4) $n(R) = \sigma L\left(\frac{R-\mu}{\sigma}\right) = \sigma L(z)$

Equation 5) $F(R) = \alpha$

Equation 6) $n(R)/Q = 1 - \beta$

Equation 7) $Q_0 = \sqrt{\frac{2K\lambda}{h}}$

Normal distribution computations:

Approach I - f(x), f(z) apply the formula involving pdf – simple;

F(x), F(z) use the built in functions; $F^{-1}(x)$, $F^{-1}(z)$ use built in functions.

L(z) = f(z) – z[1-F(z)] and L(x)=σL(z) property of normal distribution.

Approach II – use tables

Description of the Solution presented in Excel and Comments

I) Question I

Parts A-B-C-D

In *part A*, to calculate Q and R, we first start the algorithm by calculating ***(Equation 7)***. Next, we use ***(Equation 1)*** and ***(Equation 2)*** iteratively. While calculating $n(R)$, we use ***(Equation 4)***.

In *part B*, to calculate expected average total cost, we use ***(Equation 3)***.

In *part C*, to calculate type-1 service level, we use ***(Equation 5).*** You can use one of the calculation methods for normal distributions, outlined in the page where equations are presented.

In *part D,* to calculate type-2 service level, we use ***(Equation 6).***

The parameters used in the formulae are:

K	10000	TL
τ	2	weeks
	52	# of weeks/year
	3.14	Pi

Expected demand during lead time = 2*mean weekly demand

Standard deviation of demand during lead time = $\sqrt{2}$ * standard deviation of weekly demand.

I) Parts A-D					**~~Formula I~~**	**~~Formula 2~~**
Vaccine Type	**Mean Weekly demand (in boxes)**	**Stan. Dev. of weekly demand (in boxes)**	**Holding Cost (TL/box/year)**	**Shortage Cost (TL/box)**	**exp. demand during lead-time (in boxes)**	**stan. Dev. Of demand during lead-time (in boxes)**
A	2000	150	40	400	4000.00	212.13
B	3500	500	30	300	7000.00	707.11
C	2500	500	45	450	5000.00	707.11

The resulting values are:

Vaccine A										
Step	**z**	**F(x)=F(z)**	**phi(z)**	**loss(z)**	**n(R)**	**Q**	**R**	**EAC(Q,R)**	**Type 1**	**Type 2**
0						7211.10	4521.99			
1	2.46	0.99	0.02	0.00	0.48	7279.99	4521.26			
2	2.46	0.99	0.02	0.00	0.48	7280.71	4521.26	312,078.6	0.9930	0.9999

Vaccine B										
Step	z	F(x)=F(z)	phi(z)	loss(z)	n(R)	Q	R	EAC(Q,R)	Type 1	Type 2
0						11015.14	8774.19			
1	2.51	0.99	0.02	0.00	1.38	11240.48	8769.12			
2	2.50	0.99	0.02	0.00	1.41	11245.49	8769.01			
3	2.50	0.99	0.02	0.00	1.41	11245.60	8769.01	390,438.5	0.9938	0.9999

Vaccine C										
Step	z	F(x)=F(z)	phi(z)	loss(z)	n(R)	Q	R	EAC(Q,R)	Type 1	Type 2
0						7601.17	6782.79			
1	2.52	0.99	0.02	0.00	1.33	7824.77	6775.56			
2	2.51	0.99	0.02	0.00	1.37	7831.88	6775.34			
3	2.51	0.99	0.02	0.00	1.37	7832.11	6775.33	432,334.8	0.9940	0.9998

Total expected average cost from all vaccines is 1,134,851.85 TL.

<u>Comments regarding performance</u>

- In *Part A*, small numerical differences are accepted. Almost all of you did a great job in this part. Thanks!
- In *Part A*, while using the formula $R = \sigma z + \mu$, many multiplied the z value with standard deviation of weekly demand. However, we need to multiply the z value with standard deviation of lead-time demand.
- In *Part A*, few wrote standard deviation of lead-time demand = 2 * standard deviation of weekly demand.
- In *Part B*, almost all of you provided different numbers for the expected annual cost.
- In *Part B*, some forgot to add one of the main cost components (setup, holding, penalty).
- In *Parts C and D*, this part was doable and you did a great job. Thanks!

<u>Part E</u>

We use ***(Equation 5)*** with $\alpha = 0.97$ and find R from there. After that, we find Q from ***(Equation 1)***. The resulting values are:

I) Part E	R	z	F(x)=F(z)	phi(z)	loss(z)	n(R)	Q	EAC(Q,R)
A	3601.02	-1.88	0.03	0.07	1.89	401.44	29782.50	1,191,300.2
B	5670.08	-1.88	0.03	0.07	1.89	1338.14	70655.17	2,119,655.0
C	3670.08	-1.88	0.03	0.07	1.89	1338.14	59472.13	2,676,245.9
							total expected average cost=	5,987,201.15

Comment: First note that the cost term does not include unit costs as we backorder. Second, note that the way we implemented a better version of Type I service level compared to the case where we do not know backorder cost. By using the order given in the text, we found R and then the cost minimizing Q given that R. Third, note that the total expected average cost difference is huge. The reason is obvious: The relatively large backorder costs require a much higher service level than 0.97. Finally, note that the optimal lot sizes in Part A) correspond to demand which are in between 2-3 weeks T value. If you look at those in Part E) they correspond to 3 to 4 times of those cycle times. Hence, this may create an additional problem with respect to perishability and other storage problems with respect to warehouse capacity.

II) <u>Question 2</u>

<u>Part A</u>

There are alternative ways to formulate this problem, including the formulations which yield an exact solution or heuristics. One possible formulation aiming for an exact solution is the following.

Let T be the time between orders. This will be a periodic review system and hence for any item there will be an optimal order-up-to level, call it S_j. The cost components are as follows.

Fixed cost of ordering:

$$\frac{K}{T}$$

Inventory holding cost:

$$h_j\left[\frac{T\lambda_j}{2} + S_j - (T+\tau)\lambda_j\right]$$

Note that $[S_j - (T+\tau)\lambda_j]$ is the expected inventory level at the arrival of an order, and it is always non-negative. The expected order size is

$$T\lambda_j$$

Expected number of backorders in a cycle is

$$n(S_j) = \int_{S_j}^{\infty} (x - S_j) f_j(x)\, dx$$

Where $f_j(x)$ is the pdf of demand distribution over $(T+\tau)$ periods. (Please note that up to this point, all the components can be followed from the textbook as well as the slides.) The backorder cost component is

$$\frac{b_j}{T} n(S_j)$$

Finally, the objective function is

$$\min_{T} \frac{K}{T} + \sum_{i=1}^{3} \left\{ h_j \left[\frac{T\lambda_j}{2} + S_j - (T+\tau)\lambda_j \right] + \frac{b_j}{T} N(S_j) \right\}$$

As $S_j - (T+\tau)\lambda_j \geq 0$, we can say that $SS_j = S_j - (T+\tau)\lambda_j$. However, note that SS_j is a function of T as well as S_j. Rearranging terms gives another representation of the objective function

$$\frac{K}{T} + \frac{T}{2}\sum_{j=1}^{3} h_j\lambda_j + \frac{1}{2}\sum_{j=1}^{3} h_j SS_j + \frac{1}{T}\sum_{j=1}^{3} b_j n(S_j)$$

Which is also equivalent to

$$\frac{K + \sum_{j=1}^{3} b_j \, n(S_j)}{T} + \frac{T}{2}\sum_{j=1}^{3} h_j\lambda_j + \frac{1}{2}\sum_{j=1}^{3} h_j SS_j$$

With $SS_j \geq 0$. The main difficulties while solving this problem is that the decision variable T form the demand function. Moreover, SS_j is both a function of T and S_j. A possible heuristic to solve this problem is the following.

Heuristic 1

Consider a given T (In practice, you do not have much choice with respect to such intervals. Remember the discussion of power-of-two policies in Chapter 4). For a given T,

$$SS_j = S_j - (T+\tau)\lambda_j$$

$$S_j = SS_j + (T+\tau)\lambda_j$$

So that the problems are separable with respect to j. Now, $f_j(x)$ is not a decision variable. We know that the demand distribution over $(T+\tau)$ is normally distributed with

$$x \sim N(\,(T+\tau)\mu_j, (T+\tau)\sigma_j^2)$$

One can take derivative with respect to S_j and find their optimal values. After that, we can evaluate optimal expected average costs for different T values and select the optimal.

Heuristic 2

- For a given T, determine SS_j using a service level.
- Evaluate overall possible T values and the respective expected average costs to select the minimum.

Part B

Not clear, as the cost structure of those two problems are different. However, we can say that total expected average cost from Question II + $2K/T_{(I)}$ is greater than or equal to total expected average cost from Question I. Note that $2K/T_{(I)}$ is the difference between costs, where $T_{(I)}$ is an optimal T value in Question I.

Comments on the performance

- In *Part A*, any reasonable approach received more than 10 points.
- In *Part A*, many of you decided to formulate this problem as a linear programming problem. This is accepted. However, please note that formulating a problem does not necessarily mean that the formulation should be a linear programming formulation.
- In *Part A,* few did not consider penalty cost while formulating the model.
- In *Part B*, almost all of you stated that the model in Part B of I) yields a better solution. There might be a number of reasons that makes Part B of I) better than Part A of II). However, we should also think about the conditions under which Part A of II) can yield a better solution.