# IE375 Spring 2020 On-line

Further Issues in Scheduling

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# Characteristics of the Job Shop Scheduling Problem

- Machine environment :
  - Number and Variety of Machines,
  - Flow Patterns
  - Processing Characteristics
- Sometimes decomposable
- Bottleneck machine
- Objective
- Complicated: Evaluation of Alternative Rules
  - Job Arrival Pattern
  - Number and skill level of workers

# Static Stochastic Scheduling

- All jobs are available at time zero
- Processing times are uncertain
- Single machine scheduling
  - Objective is to minimize expected weighted flow time
    - Use Shortest Weighted Expected Processing Time
    - $E(t_{[1]})/u_{[1]}<E(t_{[2]})/u_{[2]}<\ldots<E(t_{[n]})/u_{[n]}$
  - Objective is to minimize maximum over all jobs of the probability that a job is late
    - Use EDD if the due dates are deterministic
    - Use expected earliest due date if the due dates are random

# Multiple machines stochastic case

- Processing times are exponential
- Parallel machine scheduling
  - Processing times $t_1$, $t_2$,…,$t_n$ are exponential with rates $\mu_1$, $\mu_2$,…,$\mu_n$. Expected processing times are $1/\mu_1$,$1/\mu_1$,…, $1/\mu_n$.
  - List schedule with longest expected processing time (LEPT) minimizes makespan

    (Remember LPT list schedule is not always optimal in the deterministic case)

# Multiple machines stochastic case

- Two machine flow shop
  - Deterministic case job j precedes job j+1 if

    $\min(A_j, B_{j+1}) < \min(A_{j+1}, B_j)$ (Check why)
- Stochastic case: Processing times on machine 1: $A_1, A_2, \ldots, A_n$ and processing times on machine 2: $B_1, B_2, \ldots, B_n$ are exponential with rates $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$
- $E[\mathrm{Min}(A_j, B_{j+1})] = 1/(a_j + b_{j+1})$ $E[\min(A_{j+1}, B_j)] = 1/(a_{j+1} + b_j)$
- Job j precedes job j+1 if

  $1/(a_j + b_{j+1}) < 1/(a_{j+1} + b_j)$, or $a_j - b_j > a_{j+1} - b_{j+1}$

# Stochastic flow shop example

- What is the optimal sequence of jobs on both machines if the expected processing times are given as below?

| Job | Expected Times | |
|---|---|---|
| | A | B |
| 1 | 5 | 2 |
| 2 | 1 | 6 |
| 3 | 9 | 7 |
| 4 | 3 | 8 |
| 5 | 10 | 4 |

# Dynamic Stochastic Scheduling

- Jobs are arriving randomly over time
- Queuing theory is used to analyze the system
- Wait time in the queuing system is equivalent to flow time in scheduling terminology
- If there is a single machine, processing times and inter-arrival times are exponential, it is an M/M/1/FCFS queue.
  - Processing rate is $\mu$ and arrival rate is $\lambda$, $\rho=\lambda/\mu$
  - Pr {number of jobs in system=i}=$\rho^i(1-\rho)$
  - Expected waiting time = E[W] = $1/(\mu-\lambda)$
  - Variance of the waiting time = Var[W]= $1/(\mu-\lambda)^2$

# Stochastic Scheduling: Dynamic Analysis

- Means of analyzing the results
  - M/M/1, ..., G/GI/1 queues: random arrivals to a single machine with random processing times.
  - **Little's Law :** $L=\lambda W$
- If the selection discipline does not depend on the flow times, the mean flow times are the same, but the variance of the flow times will differ.
- If job times are realized when the job joins the queue rather than when the job enters service, SPT generally results in lowest expected flow time.

# Objectives in Job Shop Scheduling

- Meet due dates
- Minimize work-in-process (WIP) inventory
- Minimize average flow time
- Maximize machine/worker utilization
- Reduce set-up times for changeovers
- Minimize direct production and labor costs

(note: that these objectives can be conflicting)

# Scheduling Problems in Operations

- Job Shop Scheduling
- Assembly Line Balancing
    - (6th Edition - 8.10  or 7th Edition –9.10)
- Project Scheduling
    - (6th Edition - Chapter 9 or 7th Edition – Chapter 10)
- Dynamic versus Static Scheduling
- Personnel Scheduling
- Vehicle Scheduling and Routing
- Scheduling Courses, Exams

# Use of simulation

- A reasonable way of evaluating a few policies
- A reasonable way of showing your policy versus current
- Not straightforward!
  - Detail level
  - Comparison metrics
  - You need to have an approach to simulate
  - Analysis of output

# How to approach a Real Machine Scheduling Problem?

- Analyze and deal only with the “significant” part
  - Bottleneck Scheduling
- Set measures that are important
- Check whether there is a reported “optimal” approach for any of the measures
- If none – heuristic – use of known results to come up with an intuitive approach (approaches)
  - Check special cases – should work
  - Approaches to improvement of the solution
  - Test your approach – artificial data – simulation - ...