Mini-Homework 4 Key

Question 1

Note the following parameters

1 year = 50 weeks

Lead time = 2 weeks

Fixed cost: $K = \$300$ per order

Holding cost: $h = \$0.16$ / item-1week $\Rightarrow 0.16 \times 50 = \$8$ / item-1 year

Back order cost: $p = \$12$ / unit

Demand during two-weeks of lead time: $D \sim \text{uniform}([200, 800])$, $E[D] = 500$

Expected yearly demand: $\lambda = 25 \times E[D] = 25 \times 500 = 12500$

(a) Please see the comment file for this part.

For a fixed Q value, the minimizer R of $G(Q, R)$ satisfies the equation

$$1 - F(R) = Qh / p\lambda$$

by Equation (3), Appendix 5-C, p. 308 of our textbook (7th edition), where $F(.)$ is the cumulative dist. fnc. of demand during lead time (see Appendix 5-C, equation $n'(R) = -(1 - F(R))$)

Hence,

$$1 - F(R) = \frac{2500 \times 8}{12 \times 12500} = 0.13 \Rightarrow F(R) = 0.87$$

Since $F(.)$ is cdf of $D \sim \text{uniform}([200, 800])$, we obtain

$$F(R) = \int_{200}^{R} \frac{1}{800-200}\, dx = \frac{R-200}{600}$$

$F(R) = 0.87$ implies that

$$\frac{R-200}{600} = 0.87 \Rightarrow R = 722$$

(b) By p. 274 of our textbook (7th ed.), fill rate β is found by the formula

$$n(R)/Q = 1-\beta$$

where $n(R)$ is defined in p.270 by

$$n(R) = \int_{R}^{+\infty} (x-R) f(x)\, dx$$

with $f(.)$ being the pdf of D. Since $D \sim \text{uniform}([200,800])$, we have

$$n(R) = \int_{R}^{800} (x-R) \cdot \frac{1}{800-200}\, dx$$

$$= \frac{1}{600} \cdot \left[\int_{R}^{800} x\, dx - R \cdot \int_{R}^{800} .\, dx \right]$$

$$= \frac{1}{600} \left[\frac{800^2 - R^2}{2} - R \cdot (800-R) \right]$$

$$= \frac{1}{600} \left[\frac{800^2}{2} - R \cdot 800 + \frac{R^2}{2} \right]$$

$\Rightarrow n(722) = 5.07$

$\Rightarrow \beta = 1 - 5.07/2500 = \%\,99.8$

(c) $$n(R) = \int_R^{800} (x-R) \cdot \frac{1}{800-200}\, dx$$

$$= \frac{1}{600} \cdot \left[\int_R^{800} x\, dx - R \cdot \int_R^{800} .\, dx \right]$$

$$= \frac{1}{600} \left[\frac{800^2 - R^2}{2} - R \cdot (800 - R) \right]$$

$$= \frac{1}{600} \left[\frac{800^2}{2} - R \cdot 800 + \frac{R^2}{2} \right]$$

$$= \frac{(800-R)^2}{600 \times 2}$$

$$F(R) = \int_{200}^{R} \frac{1}{800-200}\, dx = \frac{R-200}{800-200}$$

(d) Let $Q_0 = EOQ = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2 \times 300 \times 12500}{8}} = 968.25$

Apply the formulae

$$1 - F(R) = Qh / p\lambda$$

$$Q = \sqrt{\frac{2\lambda [K + p\, n(R)]}{h}}$$

iteratively. Using part (c), we shorten the calculation as

$$1-\frac{R-200}{600}=\frac{Q\times 8}{12\times 12500}\Rightarrow R=\left[1-\frac{8Q}{12\times 12500}\right]\times 600+200 \quad (1)$$

$$Q=\sqrt{\frac{2\times 12500\times\left[300+12\times\frac{(800-R)^2}{600\times 2}\right]}{8}} \quad (2)$$

Applying the formulae (1) and (2) iteratively, we obtain

$EOQ = 968.25$ $\quad R_0 = 769.016$

$Q_1 = 983.61$ $\quad R_1 = 768.52$

$Q_2 = 984.1$ $\quad R_2 = 768.50$

$Q_3 = 984.11$ $\quad R_3 = 768.50$

(e) By using the model on p. 213 of our textbook (7th ed) we find the order quantity and reorder level as

$$EOQ=\sqrt{\frac{2K\lambda}{h}}=\sqrt{\frac{2\times 300\times 12500}{8}}=968.25$$

R = demand during lead time = 500 (lead time < T)

Therefore, the fill rate when $D\sim$ uniform($[200,800]$) is

$$\beta=1-\frac{n(R)}{Q}\underset{\text{part c}}{=}1-\frac{\frac{(800-R)^2}{600\times 2}}{Q}=1-\frac{\frac{(800-500)^2}{600\times 2}}{968.25}=\%92.2$$

Question 2

Item	Annual Sales Volume (units)	Average Cost per unit ($)	Average Contribution to Profit /unit ($)	Supplier
Greeting Cards	3870	1.5	0.4	S1
T-shirts	1550	10.5	1.25	S2
Men's jewelry	875	15	4.5	S3
Novelty gifts	2050	22.5	12.25	S1
Children's clothes	575	20	6.85	S2
Chocolate cookies	7000	0.25	0.1	S4
Earrings	1285	10	3.5	S3
Other costume jewelry	1900	25	15	S3

Part a.

Annual contribution to profit = Annual Sales Volume * Avg. contribution to profit / unit

Item	Annual Sales Volume (units)	Average Cost per unit ($)	Average Contribution to Profit /unit ($)	Supplier	Annual contribution to profit
Other costume jewelry	1900	25	15	S3	28500
Novelty gifts	2050	22.5	12.25	S1	25112.5
Earrings	1285	10	3.5	S3	4497.5
Children's clothes	575	20	6.85	S2	3938.75
Men's jewelry	875	15	4.5	S3	3937.5
T-shirts	1550	10.5	1.25	S2	1937.5
Greeting Cards	3870	1.5	0.4	S1	1548
Chocolate cookies	7000	0.25	0.1	S4	700

Category A: Other costume jewelry, Novelty gifts

Category B: Earrings, Children's clothes, Men's jewelry, T-shirts, Greeting Cards, and Chocolate cookies

Part b.

Annual variable cost = Annual sales volume * Avg. cost per unit

Item	Annual Sales Volume (units)	Average Cost per unit ($)	Average Contribution to Profit /unit ($)	Supplier	Annual Variable Cost
Other costume jewelry	1900	25	15	S3	47500
Novelty gifts	2050	22.5	12.25	S1	46125
T-shirts	1550	10.5	1.25	S2	16275
Men's jewelry	875	15	4.5	S3	13125
Earrings	1285	10	3.5	S3	12850
Children's clothes	575	20	6.85	S2	11500
Greeting Cards	3870	1.5	0.4	S1	5805
Chocolate cookies	7000	0.25	0.1	S4	1750

Category A: Other costume jewelry, Novelty gifts
Category B: Earrings, Children's clothes, Men's jewelry, T-shirts, Greeting Cards, and Chocolate cookies

Part c.

First, let us sort the suppliers with respect to their names and write annual variable cost for each item found in part b.

Item	Annual Sales Volume (units)	Average Cost per unit ($)	Average Contribution to Profit /unit ($)	Supplier	Annual Variable Cost
Novelty gifts	2050	22.5	12.25	S1	46125
Greeting Cards	3870	1.5	0.4	S1	5805
T-shirts	1550	10.5	1.25	S2	16275
Children's clothes	575	20	6.85	S2	11500
Other costume jewelry	1900	25	15	S3	47500
Men's jewelry	875	15	4.5	S3	13125
Earrings	1285	10	3.5	S3	12850
Chocolate cookies	7000	0.25	0.1	S4	1750

Summing their annual variable cost yields

Supplier	Annual Variable Cost
S3	73475
S1	51930
S2	27775
S4	1750

Category A: S1 and S3
Category B: S2 and S4

Part d.
Uses for ranking the item categories in decreasing order of the annual contribution to profit:
Better forecasting methods may be implemented for more profitable items.
Periodic review inventory policies may be used for less profitable items.
Less profitable items may be removed from the product assortment.

Uses for ranking the item categories in decreasing order of annual variable cost:
We may want to build more complex supply chain models for the items having higher variable cost.
Some examples which we have seen are
- Asking for quantity discounts from the suppliers.
- Delivering multiple items in one vehicle.
- Using safety stock to prevent backorders.

More advanced methods can be found in Chapter 6 of our textbook (7th ed.)

Uses for ranking the suppliers in decreasing order of their associated annual costs:
Other suppliers may give better offers. In this case, we may compare
the average annual total cost of the potential suppliers.
We may analyze the option of producing the item in-house.
We may analyze the feasibility of better contracts with the suppliers.

Part e
Collective use of groups may lead to more detailed analyses and results. For instance, we can create a group of items having low profit margin and high unit variable cost. We may consider analyzing those items earlier than the others.

Part f
On the one hand, the retailer may want to attract the customers and make them happy while they are visiting the store.
On the other hand, we can observe that the sales volume of chocolate cookies is quite large. Although the chocolate cookies have low profit margin, they may still bring a considerable amount of profit.
It is a nice coincidence that I answered this question while eating chocolate cookies :)