Mini Homework 5-6 Key

Question 1

Cars	Repair Time (Days)	Promised Date (in days from now)
1	3	14
2	2	12
3	4	9
4	1	11
5	5	8
6	2	15
7	1	18
8	4	21
9	1	18
10	3	27
11	2	32

Question 1 (a,b) (i): Shortest processing time and the cost for part b

Cars	Processing Time	Completion Time	Due Date	Tardiness
4	1	1	11	0
7	1	2	18	0
9	1	3	18	0
2	2	5	12	0
6	2	7	15	0
11	2	9	32	0
1	3	12	14	0
10	3	15	27	0
3	4	19	9	10
8	4	23	21	2
5	5	28	8	20

Avg Tardiness= 2.909
Max Tardiness = 20

Total Cost = 10*150 + 350 + 20*150
= 4850

Question 1 (a,b) (ii): Earliest Due Date and the cost for part b

Cars	Processing Time	Completion Time	Due Date	Tardiness
5	5	5	8	0
3	4	9	9	0
4	1	10	11	0
2	2	12	12	0
1	3	15	14	1
6	2	17	15	2
7	1	18	18	0
9	1	19	18	1
8	4	23	21	2
10	3	26	27	0
11	2	28	32	0

Avg Tardiness= 0.545
Max Tardiness = 2

Total Cost = 200 + 350 + 200 + 350
= 1100

Question 2

First, let us convert the numbers

Job	Printing Time	Bounding Time	Due Date
A	7	6	40
B	14	12	32
C	5	10	24
D	8	11	48
E	3	8	25

Part A

Question: Do you think that all the jobs can be completed before their due-dates?
Answer: No, since there is a single worker, the worker must complete all jobs one by one. The sum of all jobs is 84, which exceeds the due date of all jobs.

Question: If the worker was allowed to work overtime, what is a lower bound for hours of overtime needed to complete all the repairs before their due-dates?
Answer: There might be multiple lower bounds. One lower bound is 84 - 48 = 36

Part B

EDD rule minimizes the maximum tardiness (page 505 of our textbook, 7th ed.)
After eliminating job B from the sequence and sorting by their due dates, we obtain

Job	Printing Time	Bounding Time	Processing Time	Completion Time	Due Date	Tardiness
C	5	10	15	15	24	0
E	3	8	11	26	25	1
A	7	6	13	39	40	0
D	8	11	19	58	48	10

- The order is C-E-A-D.
- Since EDD rule yields the minimum of maximum tardiness, it is 10.
- We need at least 10+1=11 hours overtime.

Part C

By page 511 of our textbook (7th ed.), Johnson's algorithm can yield the minimum makespan.
Converting the data for our problem, we obtain

Job	Machine A (Printing)	Machine B (Bounding)
A	7	6
C	5	10
D	8	11
E	3	8

1- Minimum processing time is 3, for Job E on column A. Hence, Job E is scheduled first.
2- Next minimum processing time is 5, for Job C on column A. Hence, Job C is scheduled next.
3- Now, minimum processing time is 6, for Job A on column B. Hence, Job A is scheduled last.
Therefore, the sequence of jobs E-C-D-A minimizes the makespan.

We can see the minimum makespan from the Gannt chart below.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38

| E E E | C C C C C | D D D D D D D D | A A A A A A A |

| E E E E E E E E | C C C C C C C C C C | D D D D D D D D D D D | A A A A A A |

Therefore, the minimum makespan is 38. Now, let us see if this plan can process all the jobs before their due dates. The tardiness values are

Job	Completion Time	Due Date	Tardiness
E	11	25	0
C	21	24	0
D	32	48	0
A	38	40	0

Hence, while carrying this plan, all the jobs will be completed before their due-dates.

Question 3
Part A
By page 506 of our textbook (7th ed.), Lawler's algorithm can minimize the maximum tardiness.
Precedence relationships are

	Jobs w/o/ precedence relation
Job 2 ---> Job 1 ---> Job 6	Job 5
Job 3 ---> Job 4 ---> Job 6	Job 7

The relevant data is

Job	Processing Time	Due Date
1	2	5
2	3	17
3	2	17
4	1	6
5	4	12
6	3	10
7	2	15

Let `tau` denote the total processing time for the remaining jobs that are not scheduled yet.
Step 1: Candidate jobs to be scheduled last are 5-6-7
tau = 2+3+2+1+4+3+2 = 17
min{ 17-12, 17-10, 17-15 }= min{5,7,2} = 2
Therefore, Job 7 is scheduled last.

Step 2: Candidate jobs to be scheduled at 6th position: 5-6
tau = 15
min{15-12, 15-10} = min{3,5} = 3
Therefore, Job 5 is scheduled in the 6th position

Step 3: Candidate jobs to be scheduled at 5th position: 6
Therefore, Job 6 is scheduled in the 5th position

Step 4: Candidate jobs to be scheduled at 4th position: 1-4
tau= 2+3+2+1 = 8
min{8-5, 8-6}=min{3,2}=2
Therefore, Job 4 is scheduled in the 4th position.

Step 5: Candidate jobs to be scheduled at 3rd position: 1-3
tau = 2+3+2 = 7
min{7-5, 7-17, 7-17} = min{2, -10, -10} = -10
Therefore, Job 3 is scheduled in 3rd position.

Step 6: Candidate jobs to be scheduled at 2nd position: 2
Therefore, Job 2 is scheduled in 2nd position and Job 1 is scheduled at 1st position
The sequence is: 2 - 1 - 3 - 4 - 6 - 5 - 7

Tardiness values are

Job	Processing Time	Completion Time	Due Date	Tardiness
2	3	3	17	0
1	2	5	5	0
3	2	7	17	0
4	1	8	6	2
6	3	11	10	1
5	4	15	12	3
7	2	17	15	2

Max. tardiness = 3

Part B

Let Job A denote the jobs 1 and 2 where Job 1 comes immediately after Job 2.
Let Job B denote the jobs 3 and 4 where Job 4 comes immediate after Job 3.
New jobs are

Job	Processing Time	Due Date
A	5	17
B	3	17
5	4	12
6	3	10
7	2	15

and the precedence relations are

	Jobs w/o/ precedence relation
Job A ---> Job 6	Job 5
Job B ---> Job 6	Job 7

Again, Lawler's algorithm can minimize the maximum tardiness.

Step 1: Candidate jobs to be scheduled last are 5-6-7
tau= 5+3+4+3+2 = 17
min{17-12, 17-10, 17-15} = 2
Therefore, Job 7 is scheduled at the 5th position

Step 2: Candidate jobs to be scheduled at 4th position: 5-6
tau = 17-2 = 15
min{15-12, 15-10} = 3
Therefore, Job 5 is scheduled at 4th position

Step 3: Candidate jobs to be scheduled at 3rd position: 6
Therefore, Job 6 is scheduled at 3rd position

Step 4: Candidate jobs to be scheduled at 2nd position: A-B
tau = 5+3=8
min{8-17, 8-17} = -9
Job A is in the 2nd position
Job B is in the 1st position

The sequence is : B - A - 6 - 5 - 7 or A - B - 6 - 5 - 7