IE375 Spring 2020 Final Exam – Solution

Question 1) (40 points) Formulate the above Aggregate Production Planning Problem for a month as an LP to minimize total cost. Make sure that you define all decision variables you use.

Decision Variables:

3 R_{jk}: Number of tool j to be produced for Stage k during regular time, j=1, 2, 3, 4; k=1, 2

O_{jk}: Number of tool j to be produced for Stage k during overtime time, j=1, 2, 3, 4; k=1, 2

S_j: Number of tool j subcontracted for Stage 1 operations, j=1, 2, 3, 4

Parameters to be used (relate them with the information supplied)

cr_{jk}: Cost of regular time for product j in stage k, k=1, 2; j=1, 2, 3, 4. These costs are given in Table 4, Column 1 and Column 3.

co_{jk}: Cost of overtime for product j in stage k, k=1, 2; j=1, 2, 3, 4. These costs are given in Table 4, Column 2 and Column 4.

p_{ij}: Unit processing time needed in operation i for product j, i=1, 2, 3, 4; j=1, 2, 3, 4. These processing times are supplied in Table 1 for i=1 and i=2 (stamping and drilling), in Table 2 for i=3 and i=4 (finishing and packaging). Note that Assembly operations are not considered as there is no capacity limit which is considered.

m_j: Sheet metal required per unit of tool j, j=1, 2, 3, 4. These quantities are supplied in the first column of Table 3.

No parameters are defined for demand or capacity limitations to ease the notation.

Objective function:

5 TC = Total Cost, where $TC = \sum_{j=1}^{4} \{\sum_{k=1}^{2}(cr_{jk}R_{jk} + co_{jk}O_{jk}) + cr_{j1}(1.4) * S_j\}$

Min TC

Constraints:

In-house Regular Capacity:

9 Stamping: $\sum_{j=1}^{4}(p_{1j}\, R_{j1}) \leq 500$

Drilling: $\sum_{j=1}^{4}(p_{2j}\, R_{j1}) \leq 400$

Finishing and Packaging: $\sum_{j=1}^{4}\{(p_{3j} + p_{4j})\, R_{j2})\} \leq 1200$

4 **In-house Overtime Capacity:**

All operations: $\sum_{j=1}^{4}\{ (p_{ij}O_{j1}\} \leq 100$ i=1,2

All operations: $\sum_{j=1}^{4}\{ (p_{ij}O_{j2}\} \leq 100$ i=3,4

8 **Relate Stage 1 and Stage 2 operations with Subcontracting;**

$R_{j1} + O_{j1} + S_j \geq R_{j2} + O_{j2}$ for j=1, 2, 3, 4

Sheet Metal Constraint:

$\sum_{j=1}^{4} \{m_j (R_{j1} + O_{j1})\} \leq 10000$ 3

Demand Satisfaction:

Trowel: $R_{12} + O_{12}: \geq 1800$

Hoe: $R_{22} + O_{22}: \geq 1800$

8 Rake: $R_{32} + O_{32}: \geq 1800$

Shovel: $R_{42} + O_{42}: \geq 1800$

Non-negativity:

$R_{jk} \geq 0$, $O_{jk} \geq 0$, $S_j \geq 0$, j=1, 2, 3, 4; k=1, 2

Question 2) (10 points) Compute monthly capacities needed for each operation to satisfy the demand if subcontracting is not an option. Also compute the sheet metal needed to satisfy the demand.

Define d_j as the demand for tool j.

9 Stamping: $\sum_{j=1}^{4}(p_{1j}\, d_j)$ = 0.04*1800+0.17*1400+0.06*1600+0.12*1800 = 622 hours

Drilling: $\sum_{j=1}^{4}(p_{2j}\, d_j)$ = 0.05*1800+0.14*1400+0.14*1800 = 538 hours

Finishing and Packaging: $\sum_{j=1}^{4}\{(p_{3j} + p_{4j})\, d_j)\}$ = (0.05+0.03)*1800 + = 1 194 hours

1 Sheet Metal: $\sum_{j=1}^{4} \{m_j\, d_1\}$ = 1.2*1800+ 1.6*1400+ …..= 12 080 sqaure feet

Question 3) (25 points) Consider a simplified version of the scheduling problem described. Assume there are two workers one specialized in finishing operations (Worker 1) and the other in packaging operations (Worker 2). There are 8 jobs to be completed in 150 hours which is working hours in a given month. The following table summarizes the data needed as explained in the text above:

For **Mode 1)** Total Job Processing time by Worker

Job	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Worker 1	56	65.2	54.8	108.8	11.3	7.52	6.56	9.76
Worker 2	60.8	70.8	66.8	132.8	9.8	6.56	7.28	10.88

For **Mode 2)** Job Processing times in hours for each operation

Job	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Finishing	20	23.2	32.3	63.8	5.3	3.68	4.4	6.4
Packaging	24.8	28.8	18.8	36.8	5.3	3.68	2.24	3.04

Question 3-i) (7 points) Compute the makespan for **Mode 1)** if Worker 1 performs Job 3 and Job 4 whereas Worker 2 performs all the other jobs. Compute the overtime needed.

Completion times:

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Worker 1: 54.8 + 108.8 = 163.6 hours

Worker 2: 60.8 + 70.8 + 9.8 + 6.56 + 7.28 +10.88 = 166.12 hours 32

Makespan = max {166.12, 163.6} = 166.12 hours

Overtime = Worker 1 Overtime + Worker 2 overtime = 13.6 + 16.12 = 29.72 hours 2

Question 3-ii) (18 points) Determine an optimal sequence of jobs to minimize the makespan for the two-machine flow shop problem, as dictated by **Mode 2)**. Compute the associated makespan and any overtime if Johnson's Algorithm implemented to the data available in **Mode 2)** Table.

Johnson's Algorithm:

SET I={1,2}; SET II={3,4,5,6,7,8} **(Alternatives exist if you break the ties in a different way.)** 3 11

5

Optimal Sequence: 1-2-4-3-5-6-8-7 **(Alternatives exist.)**

Makespan: 176.86 hours

Overtime: Worker 1 overtime + Worker 2 overtime = 9.48 + 26.86 = 36.34 hours 2

(**Note:** You need to check the completion time of Worker 1, simply sum of the finishing times. Also please observe that Worker 2 is idle in the first 20 hours as there are no jobs which s/he can process.)

Question 4) (10 points) Compute the implied value for the fixed ordering cost if the order quantity was found following EOQ.

Use the EOQ formula to find fixed ordering cost, K.

Q = 10,000; = 120,000; h = 0.25 * 1.75 = 0.4375

Hence: K = 10,000*10.000*0.4375/(2*120,000) = 182.3$

Question 5) (15 points) We were not presented any information regarding the forecasting procedure in the Case. One would expect that even if demand is forecasted to be following a stationary pattern, there is likely to be a forecast error. Presume that forecasts are made monthly and standard deviation of forecast error term is known, and is used to as the standard deviation for monthly demand. (Recall that forecast error is normally distributed.) Now assume that we implement a continuous review policy and lead time to Japan is 2 months. Finally, Q=10, 0000 square feet is used as the ordering quantity by Spring Garden Company.

Under stochastic demand, you decided to use a service level criterion to find the reorder level. Let the desired fill rate be 0.95. Explain how you would find the reorder point. Note that you cannot calculate any quantity as tables are not given, as well as mean and standard deviation of demand is presumed to be given with no specified values. Hence, you need to explain step by step how you will proceed with the computations. Specifically you first need to explain how to compute the value you will look up from the table; then you need to explain how to use the table, and finally explain what to do with the value that you find from the table.

Use fill rate formula to find *R*, reorder level.

5 Q = 10,000; = 0.95; n(R) is the expected number of shortages when R is the order point. Hence, using the formula:

n(R) = Q*(1-) = 10,000*0.05 = 500

Interpretation: Find R using tables and appropriate lead-time demand distribution such that the expected number of shortages is 500 units.

Lead-time demand distribution:

4 Expected monthly demand = ; Standard deviation of monthly demand =

Expected lead-time demand = 2; Standard deviation of lead-time demand = √2*

To use the normal tables:

Define z, standardized normal value corresponding to R: (R-2)/√2*

3 Use the formula for normal distributed loss functions: n(R) = L(z)* √2*

Then L(z) = n(R) /√2* can be computed.

Table look-up:

From the standard normal tables find z corresponding to L(z), call it z_R.

3 **Compute *R*:**

$R = z_R$* √2*+2