Recitation 2 Example: Linear Programming Problem

An oil company makes two blends of fuel by mixing three oils. Figures on the costs and daily availability of the oils are given
in Table 1 below.

Oil	Cost ($/litre)	Amount available (litres)
A	0.30	6,000
B	0.40	10,000
C	0.48	12,000

Table 1. Costs and daily availability of the oils
Also, the requirements of the blends of fuel are given in Table 2 below.

Blend 1	at least 30% of *A* at most 50% of *B* at least 30% of *C*
Blend 2	at most 40% of *A* at least 35% of *B* at most 40% of *C*

Table 2. Requirements of the blends of fuel
Each litre of blend 1 can be sold for $1.10 and each litre of blend 2 can be sold for $1.20. Long-term contracts require at least
10,000 litres of each blend to be produced. Formulate this blending problem as a linear programming problem:
(a) Find the optimal solution
(b) Determine the shadow prices for each of the constraints and interpret your results.

1 . Linear Programming Problem

Complete the blending problem from the in-class part [included below]

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Table 2. *Requirements of the blends of fuel*

Each litre of blend 1 can be sold for $1.10 and each litre of blend 2 can be sold for $1.20. Long-term contracts require at least 10,000 litres of each blend to be produced. Formulate this blending problem as a linear programming problem:
(a) Find the optimal solution (use both the Large Scale Method and the Medium Scale Method in MATLAB)
(b) Determine the shadow prices for each of the constraints and interpret your results.

Solution

The problem wants to maximize the profit obtained from making and selling the two types of blends of fuel.
Define the (decision) variables:

$$\begin{aligned}
x_1 &= \text{amount of A oil used in blend 1}\\
x_2 &= \text{amount of B oil used in blend 1}\\
x_3 &= \text{amount of C oil used in blend 1}\\
x_4 &= \text{amount of A oil used in blend 2}\\
x_5 &= \text{amount of B oil used in blend 2}\\
x_6 &= \text{amount of C oil used in blend 2}
\end{aligned}$$

The total revenue and cost is given by

$$Revenue = 1.1(x_1 + x_2 + x_3) + 1.2(x_4 + x_5 + x_6)$$

$$Cost = 0.3(x_1 + x_4) + 0.4(x_2 + x_5) + 0.48(x_3 + x_6)$$

and the objective function to be maximized is the linear function

$$Profit = Revenue - Cost$$

Next we identify the constrains:

Amount of oil available:

$$\begin{aligned}
x_1 + x_4 &\leq 6,000\\
x_2 + x_5 &\leq 10,000\\
x_3 + x_6 &\leq 12,000
\end{aligned}$$

Long term contract requirements:

$$x_1 + x_2 + x_3 \geq 10,000$$
$$x_4 + x_5 + x_6 \geq 10,000$$

Requirement on the composition of the fuels:

$$x_1/(x_1 + x_2 + x_3) \geq 0.3$$
$$x_2/(x_1 + x_2 + x_3) \leq 0.5$$
$$x_3/(x_1 + x_2 + x_3) \geq 0.3$$
$$x_4/(x_4 + x_5 + x_6) \leq 0.4$$
$$x_5/(x_4 + x_5 + x_6) \geq 0.35$$
$$x_6/(x_4 + x_5 + x_6) \leq 0.4$$

Each of the inequalities above need to be rewritten in linear form $a_{k1}x_1 + \ldots a_{k6}x_6 \leq b_k$, for example the first one becomes:

$$x_1 \geq 0.3(x_1 + x_2 + x_3), \text{ or, equivanently, } -0.7x_1 + 0.3x_2 + 0.3x_3 \leq 0.$$

We then setup this linear programming problem in MATLAB, recalling that the standard form of the constraints in the `linprog` command is

$$\min f^T x, \text{ subject to } Ax \leq b$$

```
R=[1.1; 1.1; 1.1; 1.2; 1.2; 1.2];
C=[0.3; 0.4; 0.48; 0.3; 0.4; 0.48];

f=R-C;

A=[1 0 0 1 0 0;
   0 1 0 0 1 0;
   0 0 1 0 0 1;
   -1 -1 -1 0 0 0;
   0 0 0 -1 -1 -1;
   -0.7 0.3 0.3 0 0 0;
   -0.5 0.5 -0.5 0 0 0;
   0.3 0.3 -0.7 0 0 0;
   0 0 0 0.6 -0.4 -0.4;
   0 0 0 0.35 -0.65 0.35;
   0 0 0 -0.4 -0.4 0.6];

b=[6000; 10000; 12000; -10000; -10000; 0; 0; 0; 0; 0; 0];

lb=zeros(6,1);
```

We employ first the Large Scale Method (default in MATLAB):

```
[x,fval,exitflag,output,lambda] =  linprog(-f,A,b,[],[],lb);
x, fmax=-fval

slack=b-A*x

shadowprice=lambda.ineqlin


Optimization terminated.

x =

       3414.41
```