Chapter 4 Part IV

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Remaining Problems of Interest

- We are done with the following:
  - Basic EOQ
  - Lead-time
  - Production Rate
  - Multi-item
- Quantity Discounts
- EOQ with backorders
- Practicality of the EOQ Solution and powers-of-two restriction
- Periodic Review – Lot Sizing Problem

Quantity Discount Models

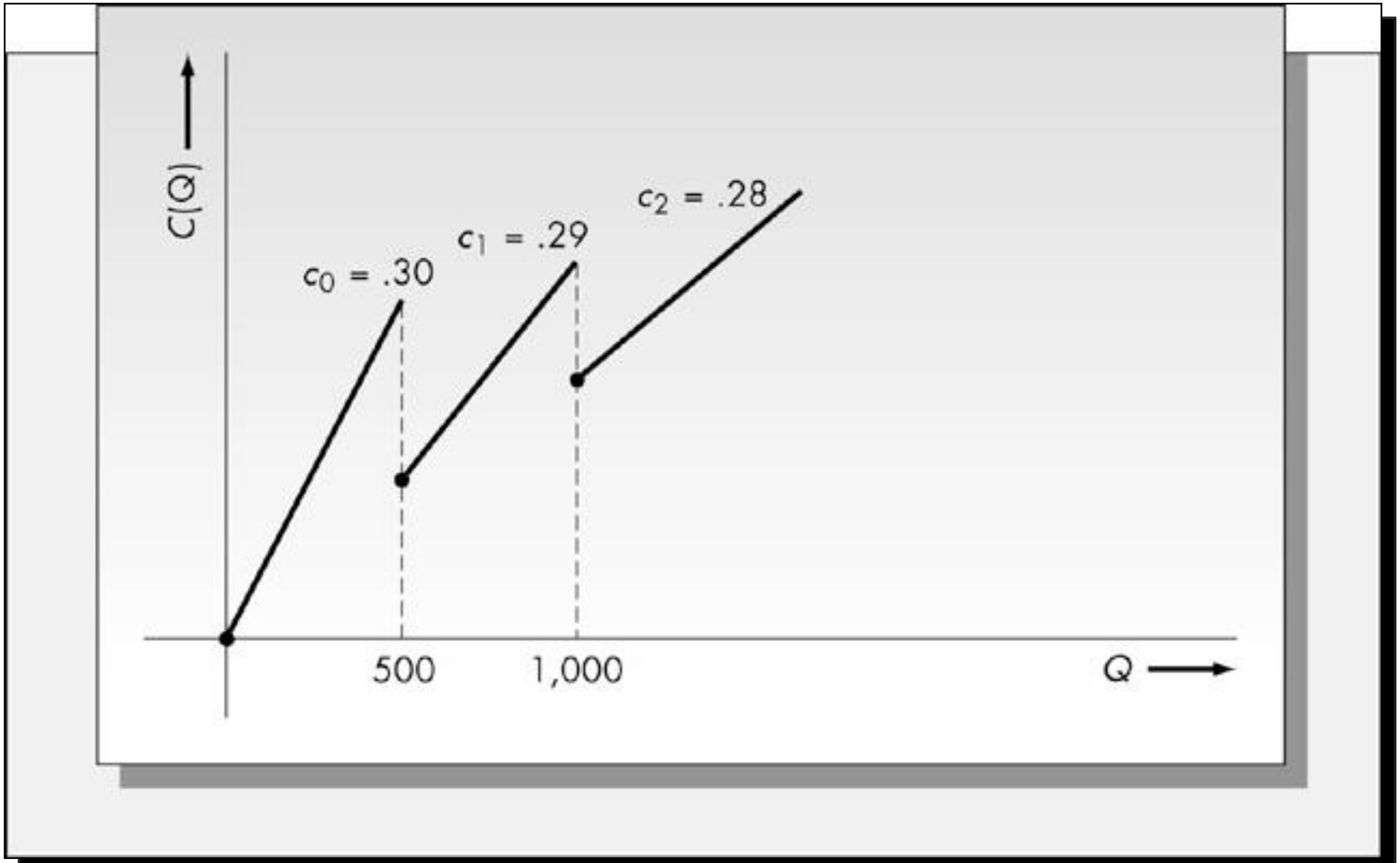
- In EOQ models, the unit cost c is independent of the size of the order
- However, usually suppliers are willing to charge less per unit for larger orders
- Two type of discount schedules
- All-units discount
  - The discount is applied to all units
- Incremental quantity discounts
  - The discount is applied to additional units beyond the breakpoint

All units quantity discounts

- Example
  - orders that are less than 500 units are charged 30 cents per unit
  - orders of 500 or more, but less than 1000 units are charged 29 cents per unit
  - order of 1000 or more are charged 28 cents per unit
- The purchase cost function C(Q) can be written as

$$C(Q) = \begin{cases} 0.30Q & \text{for} \quad 0 \le Q < 500 \\ 0.29Q & \text{for} \quad 500 \le Q < 1000 \\ 0.28Q & \text{for} \quad 1000 \le Q \end{cases}$$

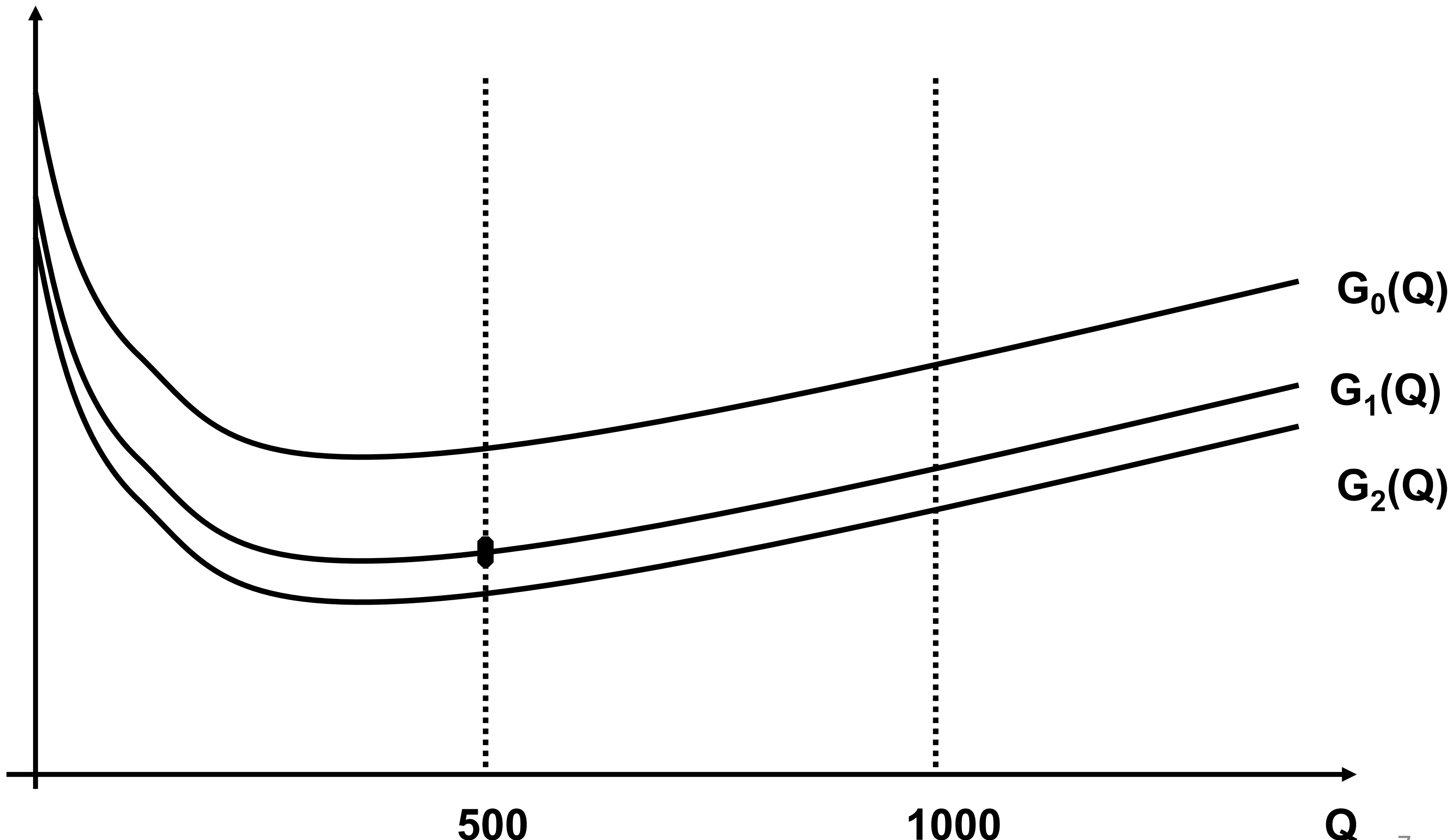
All-Units Discount Order Cost Function



All units quantity discounts – finding optimal Q

- Determine the largest realizable EOQ value. Start from the lowest price first, continue with the next higher price. Stop when the first EOQ value is realizable.
- Compare the value of the average annual cost at the largest realizable EOQ and all of the price breakpoints that are greater than the largest realizable EOQ. The optimal Q is the point at which the average annual cost is a minimum
- Example continued
  - **Annual demand of 600 units. If K=$8 per order and a 20% inventory holding cost is applied on cost, find the optimal order quantity for all units quantity discounts scheme**

Average annual cost for all-units discount

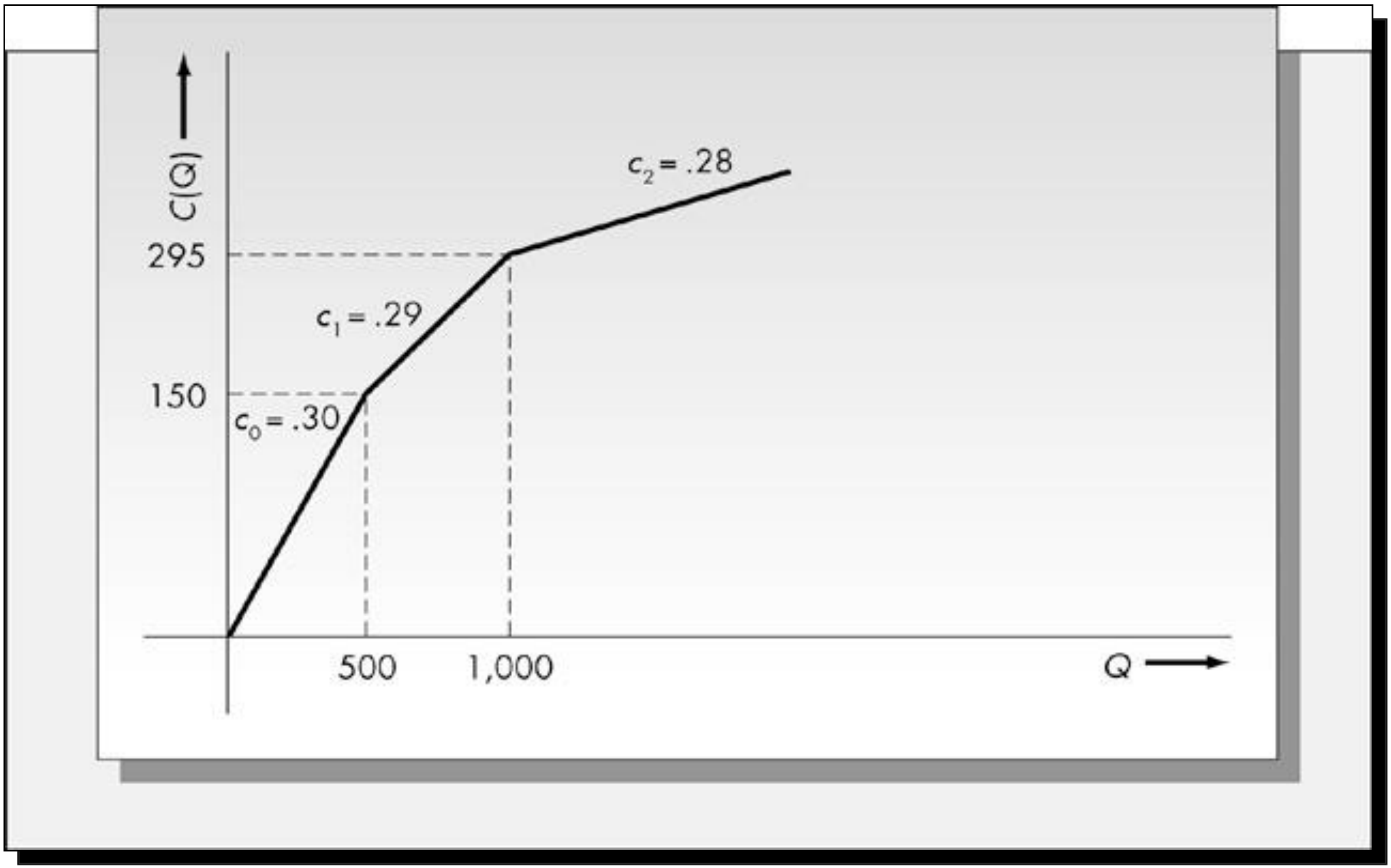


Incremental quantity discounts

- Example
  - For orders that are less than or equal to 500 units are charged 30 cents per unit
  - For orders more than 500, but less than or equal to 1000 units, 500 units are charged 30 cents per unit, remaining units are charged 29 units
  - For order of more than 1000 units, 500 units are charged 30 cents per unit, 500 units are charged 29 cents per unit, remaining units are charged 28 cents per unit
- The purchase cost function C(Q) can be written as

$$C(Q) = \begin{cases} 0.30Q & \text{for} \quad 0 \le Q \le 500 \\ 150 + 0.29(Q - 500) & \text{for} \quad 500 < Q \le 1000 \\ 295 + 0.28(Q - 1000) & \text{for} \quad 1000 < Q \end{cases}$$

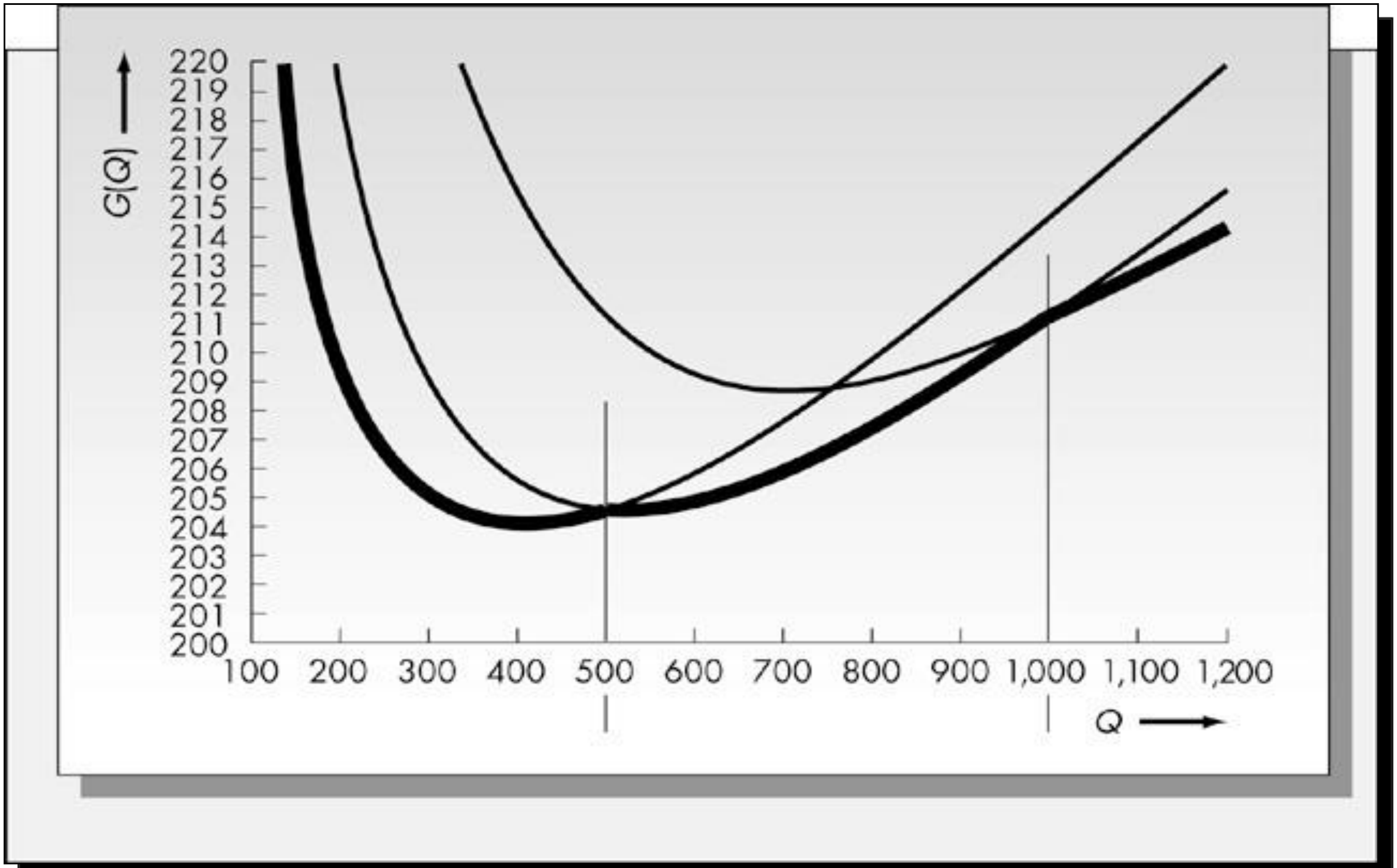
Incremental Discount Order Cost Function



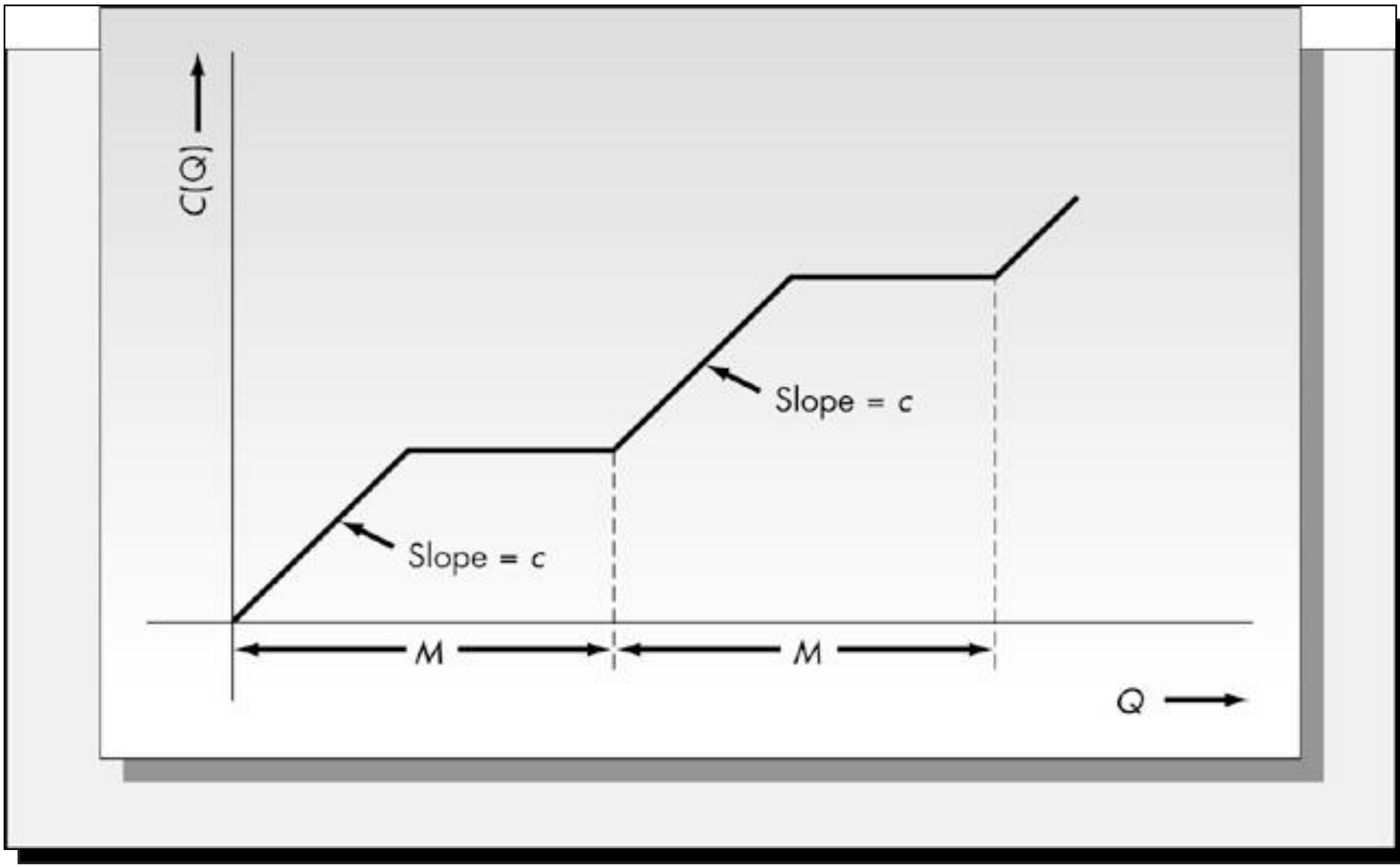
Incremental quantity discounts – finding optimal Q

- Determine the algebraic expression for C(Q) corresponding to each price interval. Use this to find C(Q)/Q
- Use C(Q)/Q to find G(Q) for each price interval. Find the optimal value of Q for each price interval.
- Determine which of the minima computed are realizable. Compare the values of the average annual cost at the realizable EOQ values and pick the lowest.
- Example continued
  - **Annual demand of 600 units. If K=$8 per order and a 20% inventory holding cost is applied on cost, find the optimal order quantity for all units quantity discounts scheme**

Average Annual Cost Function
for Incremental Discount Schedule



Other discount schedules: carload discount schedule

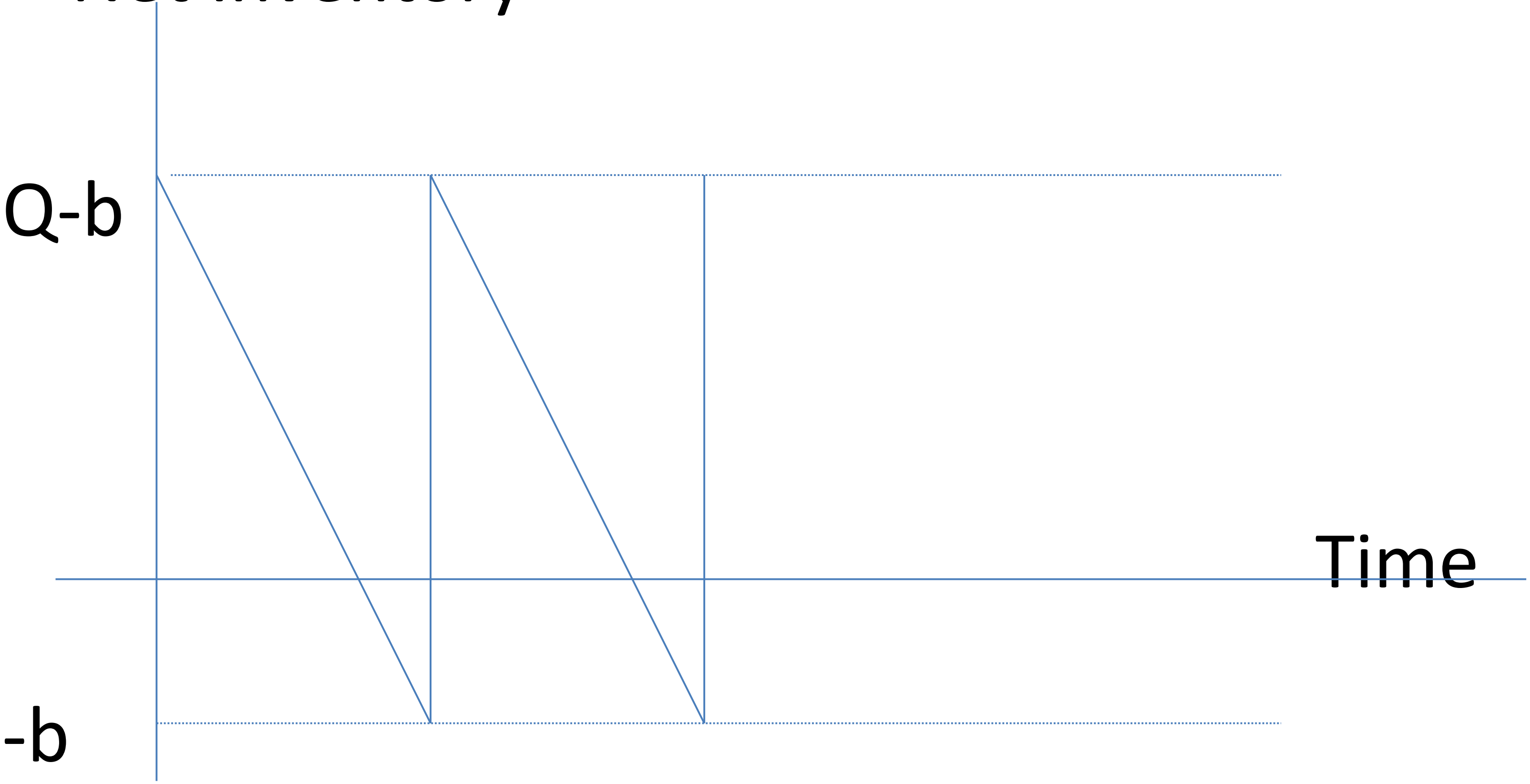


EOQ with Backorders

- Consider the extension of EOQ which allows for backorders.
- Cyclic structure holds: We order, Satisfy previous backorders, satisfy demand from inventory and backorder at the end – true for every cycle.
- As additional parameters, we solve for b, where b is defined as the maximum number of backorders allowed. We also define p as the cost of backorders/unit/unit time.

EOQ with Backorders

- Net Inventory



EOQ with Backorders

- Formulation – similar to EOQ - Cycle Time = $T = Q/\lambda$
- Write average fixed ordering cost , average inventory carrying cost (area/T) and average backorder cost (area/T)
- One can obtain the optimal solution to this problem as follows:
- $b^* = \{h/(p+h)\}Q^*$

  and $Q^* = \{(2K\lambda/h)^*[(p+h)/p]\}^{1/2}$
- Using these results, write the average total cost function and show that it is always less than the average total cost obtained by EOQ with no backorders.

Powers-of-two Policies

- Practicality of the EOQ Solution.
- Multiple items
- Correction for realistic times
- T_L: base-time (1 hour, 1 week)
- Limit your cycle times to $2^k T_L$ for k=0, 1, 2, …
- It is shown that with this restriction at most an additional 6% of the optimal EOQ average cost will be paid.

Statement of the Lot Sizing Problem

- Known set of requirements *(d_1, d_2, . . . d_T)* over an *T* period planning horizon.
- Costs: set up cost, *K,* and the holding cost, *h,* per unit in the ending inventory per period
- The objective is to determine production quantities *(X_1, X_2, . . ., X_T)* to meet the requirements at minimum cost.
- The feasibility condition to assure there are no stockouts in any period is:

$$I_{t-1} + X_t - d_t = I_t \text{ for all } t$$

Lot Sizing Problem

- Periodic Review where periods are given
- Formulation (we already did in class)
  - minimize total cost (excluding sum of unit purchasing costs which is constant)

Min $\sum_t \{\delta(X_t)K + hI_t\}$

$I_{t-1} + X_t - d_t = I_t$ for all t

$\delta(X_t)=1$ if $X_t>0$; 0 o.w.

$X_t \geq 0$, $I_t \geq 0$

Dynamic Programming

Lot Sizing Problem

- Special Case – Observations
- (1) Do we order when there is inventory? NO
- (2) So to ensure (1) how much do we order, if we order in a period *t*?

  d_t, or d_t+d_{t+1}, or $d_t+d_{t+1}+d_{t+2}$, …
- Solution: DP + shortest path problem:

  *Node 0 – corresponds to the beginning of time 1(end of time 0); and node T corresponds to end of time T (end of horizon)

  *Arc(i,j) – corresponds to the cost of ordering at then end of i (or beginning of i+1) to satisfy all the demand requirements from i+1 to j (including demand of j), j>i