Chapter 4 Part IV

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Remaining Problems of Interest

- We are done with the following:
  - Basic EOQ
  - Lead-time
  - Production Rate
  - Multi-item
- Quantity Discounts
- EOQ with backorders
- Practicality of the EOQ Solution and powers-of-two restriction
- Periodic Review – Lot Sizing Problem

Quantity Discount Models

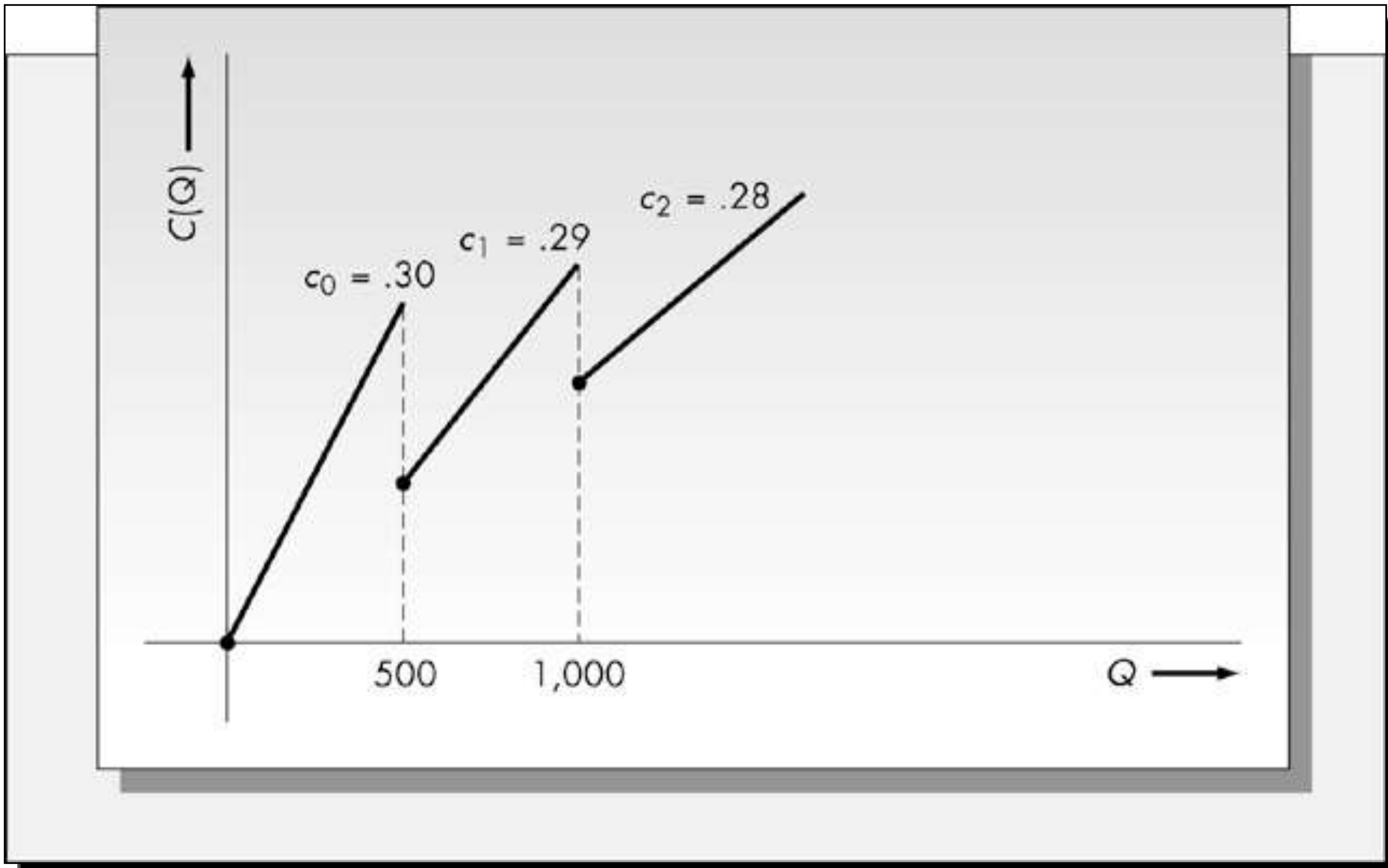
- In EOQ models, the unit cost c is independent of the size of the order
- However, usually suppliers are willing to charge less per unit for larger orders
- Two type of discount schedules
- All-units discount
  - The discount is applied to all units
- Incremental quantity discounts
  - The discount is applied to additional units beyond the breakpoint

All units quantity discounts

- Example
  - orders that are less than 500 units are charged 30 cents per unit
  - orders of 500 or more, but less than 1000 units are charged 29 cents per unit
  - order of 1000 or more are charged 28 cents per unit
- The purchase cost function C(Q) can be written as

$$C(Q) = \begin{cases} 0.30Q & \text{for} \quad 0 \le Q < 500 \\ 0.29Q & \text{for} \quad 500 \le Q < 1000 \\ 0.28Q & \text{for} \quad 1000 \le Q \end{cases}$$

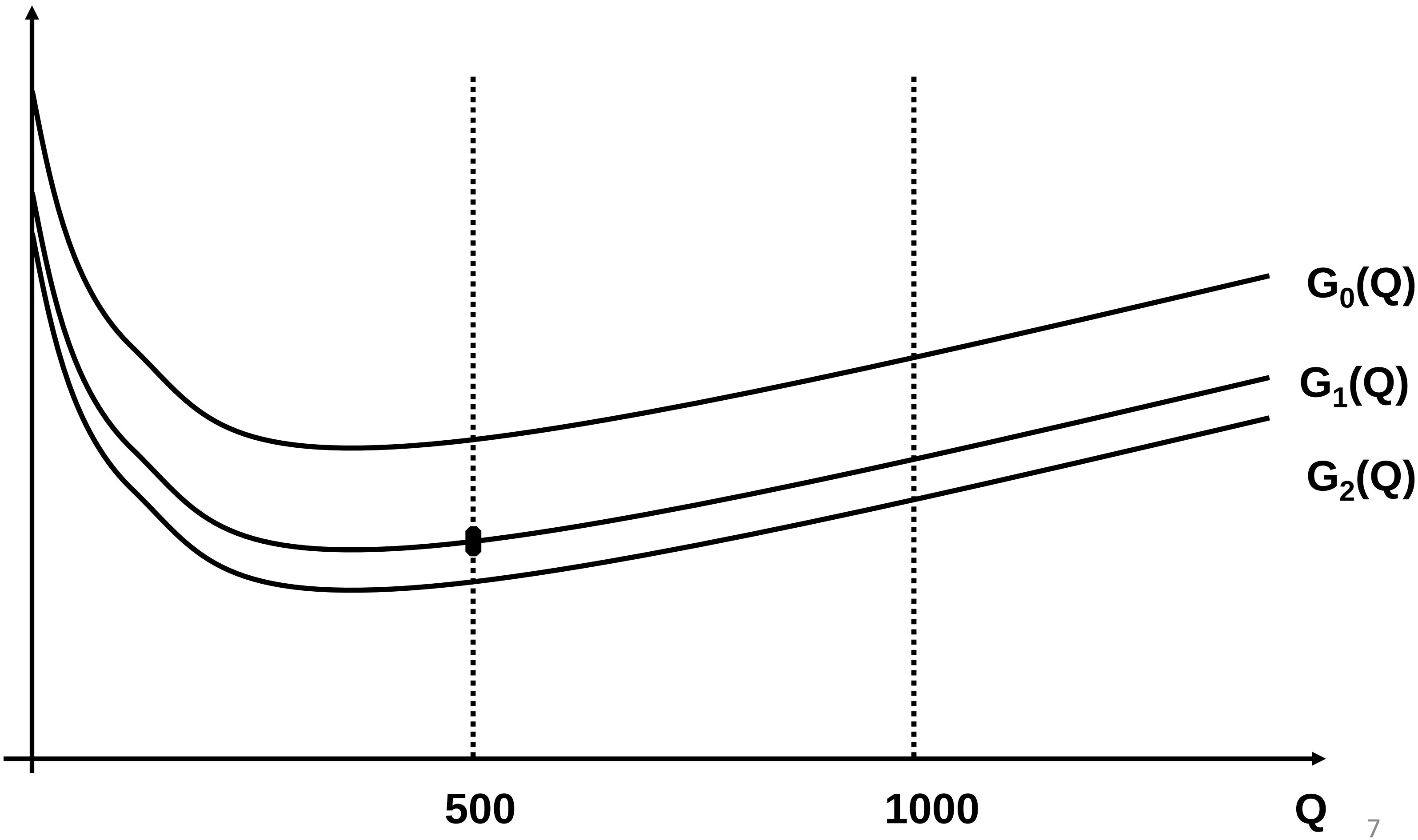
All-Units Discount Order Cost Function



All units quantity discounts – finding optimal Q

- Determine the largest realizable EOQ value. Start from the lowest price first, continue with the next higher price. Stop when the first EOQ value is realizable.
- Compare the value of the average annual cost at the largest realizable EOQ and all of the price breakpoints that are greater than the largest realizable EOQ. The optimal Q is the point at which the average annual cost is a minimum
- Example continued
  - **Annual demand of 600 units. If K=$8 per order and a 20% inventory holding cost is applied on cost, find the optimal order quantity for all units quantity discounts scheme**

Average annual cost for all-units discount

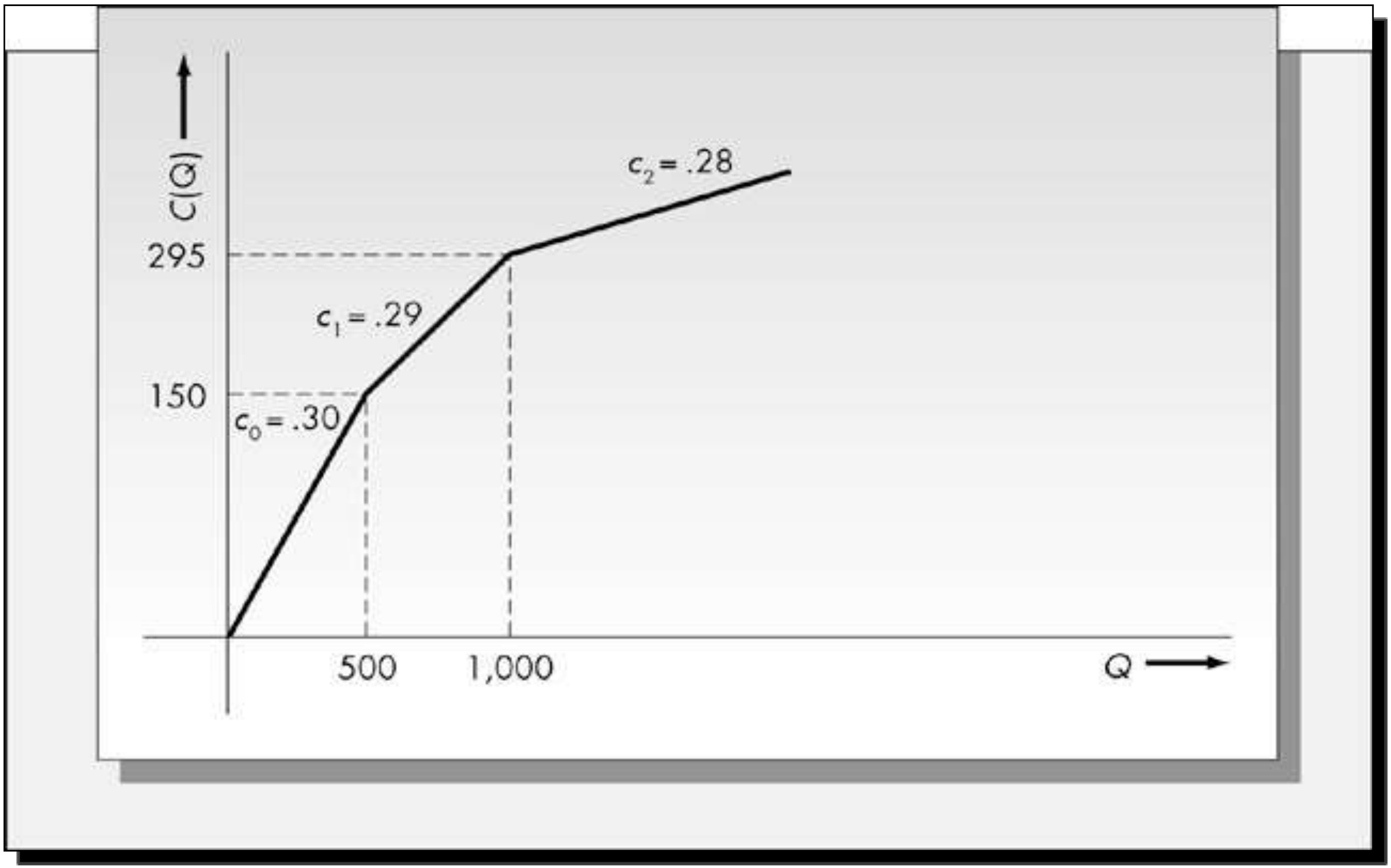


Incremental quantity discounts

- Example
  - For orders that are less than 500 units are charged 30 cents per unit
  - For orders of 500 or more, but less than 1000 units, 499 units are charged 30 cents per unit, remaining units are charged 29 units
  - For order of 1000 or more, 500 units are charged 30 cents per unit, 500 units are charged 29 cents per unit, remaining units are charged 28 cents per unit
- The purchase cost function C(Q) can be written as

$$C(Q) = \begin{cases} 0.30Q & \text{for} \quad 0 \le Q < 500 \\ 150 + 0.29(Q - 500) & \text{for} \quad 500 \le Q < 1000 \\ 295 + 0.28(Q - 1000) & \text{for} \quad 1000 \le Q \end{cases}$$

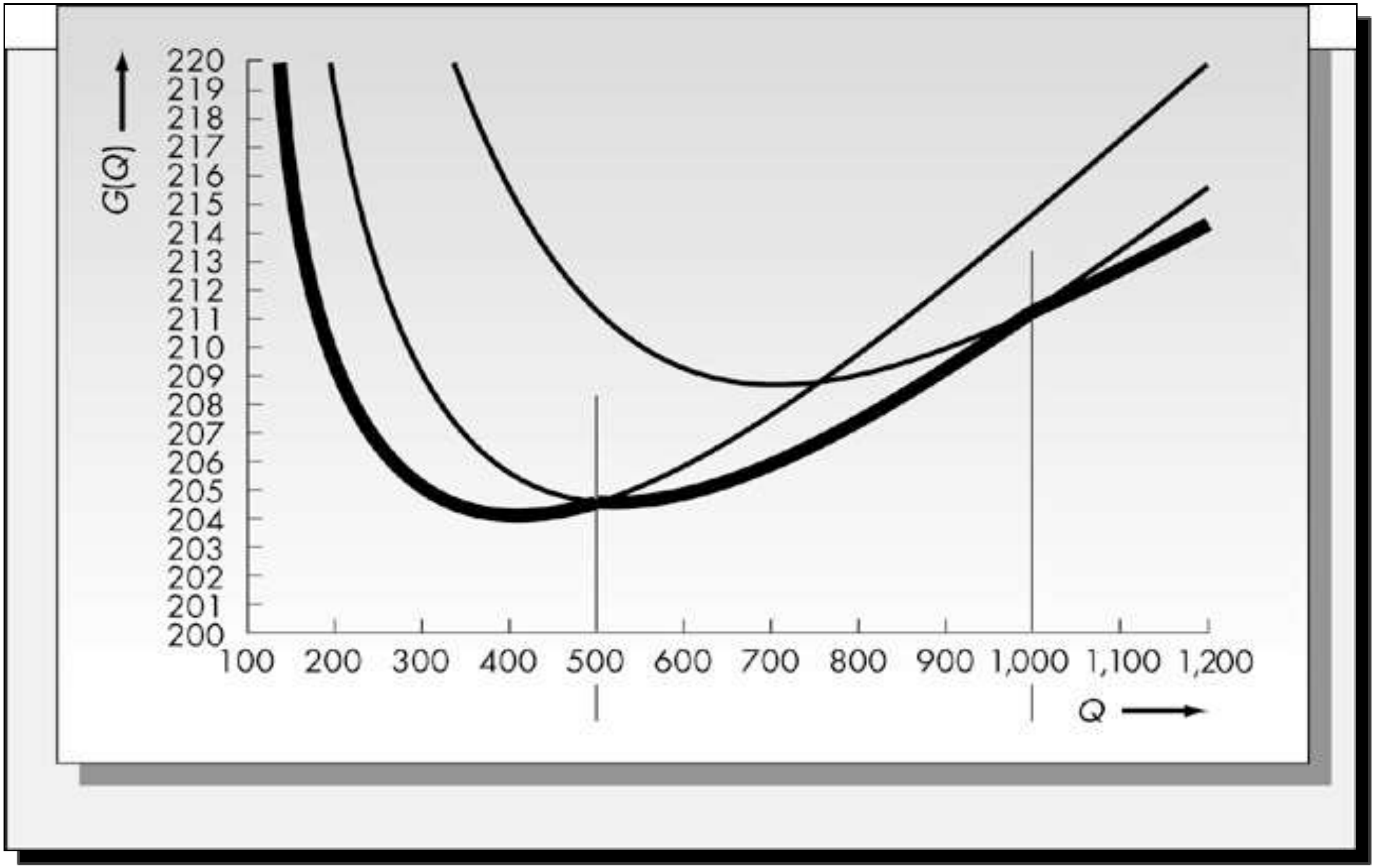
Incremental Discount Order Cost Function



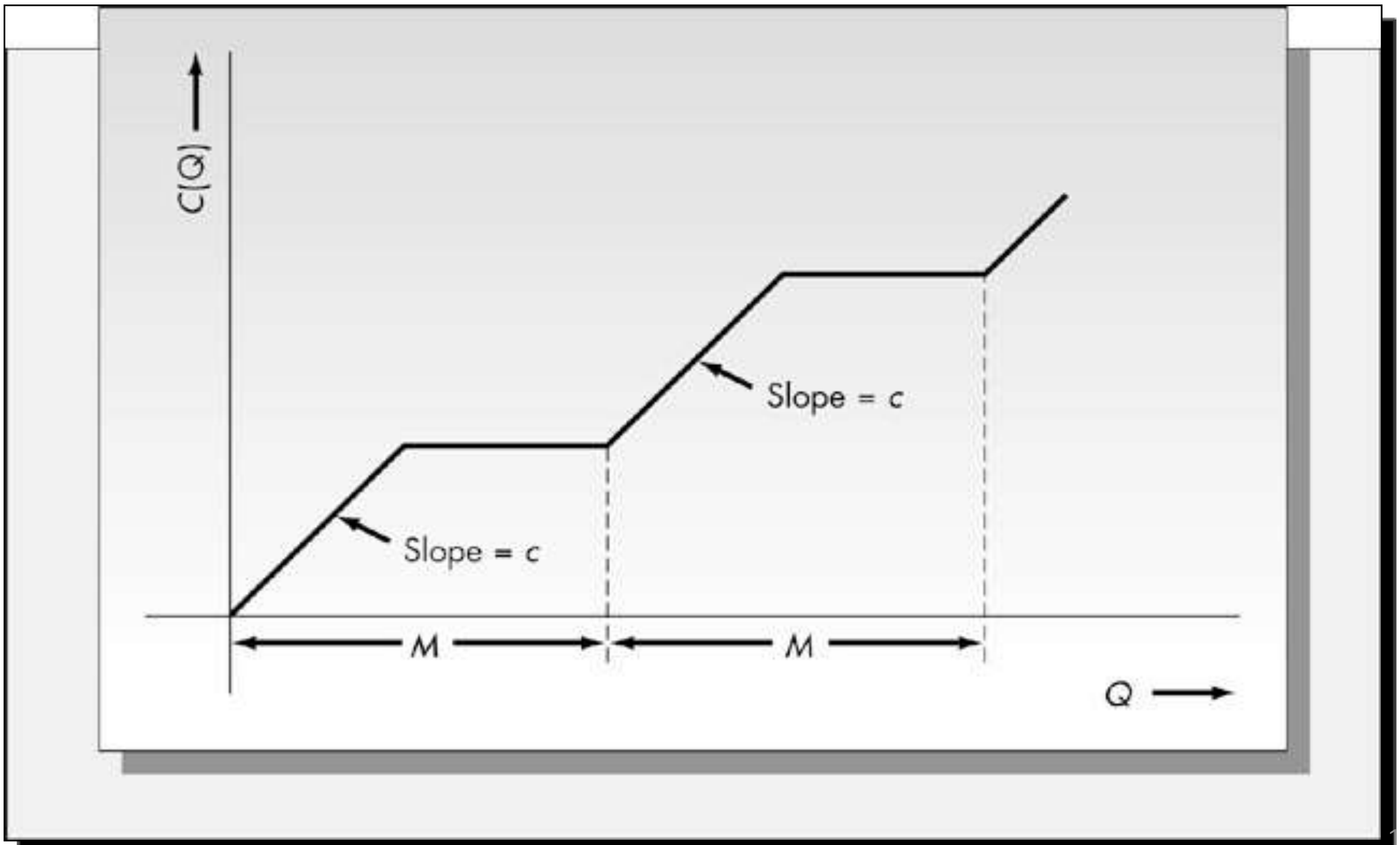
Incremental quantity discounts – finding optimal Q

- Determine the algebraic expression for C(Q) corresponding to each price interval. Use this to find C(Q)/Q
- Use C(Q)/Q to find G(Q) for each price interval. Find the optimal value of Q for each price interval.
- Determine which of the minima computed are realizable. Compare the values of the average annual cost at the realizable EOQ values and pick the lowest.
- Example continued
  - **Annual demand of 600 units. If K=$8 per order and a 20% inventory holding cost is applied on cost, find the optimal order quantity for all units quantity discounts scheme**

Average Annual Cost Function for Incremental Discount Schedule



Other discount schedules: carload discount schedule

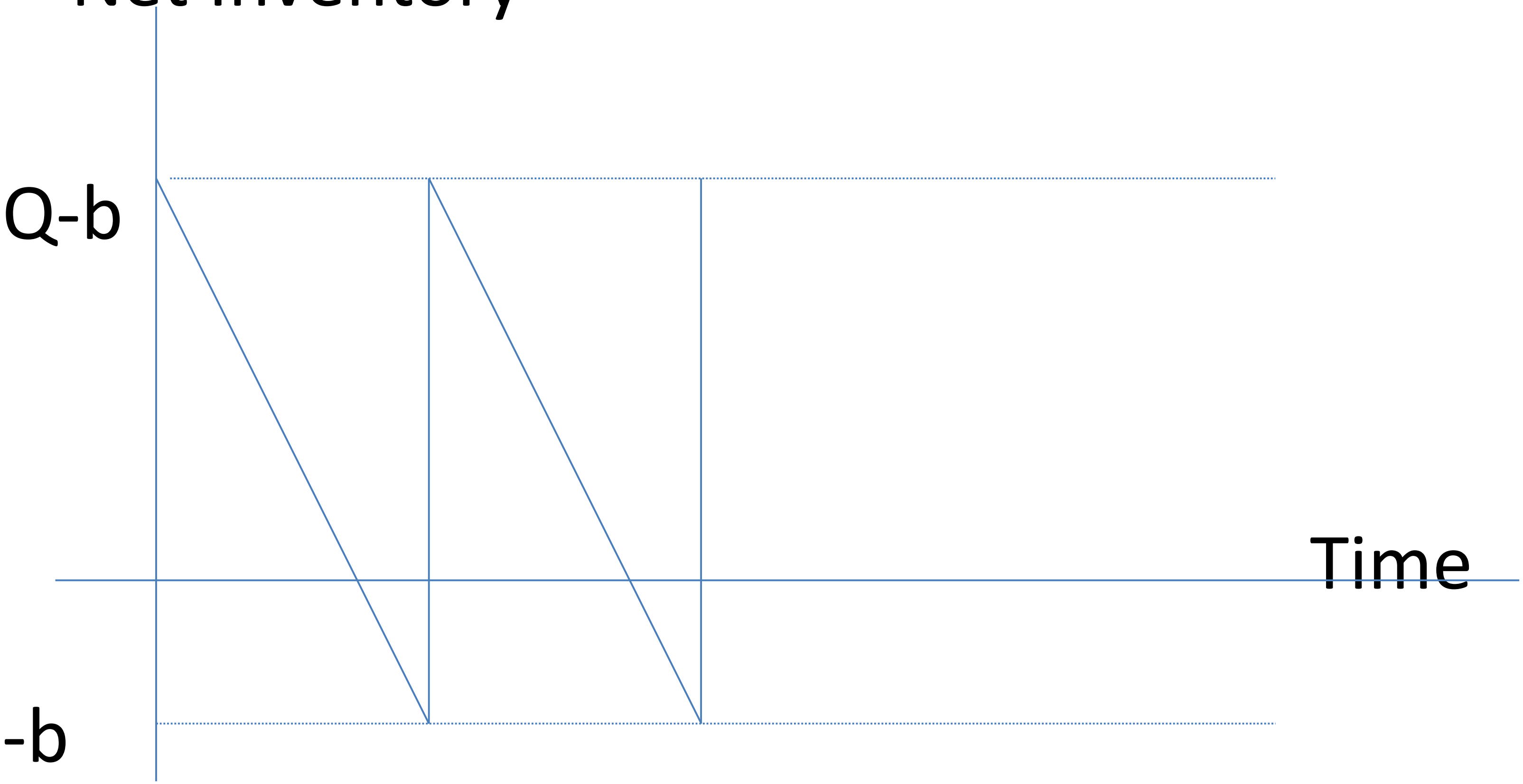


EOQ with Backorders

- Consider the extension of EOQ which allows for backorders.
- Cyclic structure holds: We order, Satisfy previous backorders, satisfy demand from inventory and backorder at the end – true for every cycle.
- As additional parameters, we solve for b, where b is defined as the maximum number of backorders allowed. We also define p as the cost of backorders/unit/unit time.

EOQ with Backorders

- Net Inventory



EOQ with Backorders

- Formulation – similar to EOQ - Cycle Time = $T = \lambda^* Q$
- Write average fixed ordering cost , average inventory carrying cost (area/T) and average backorder cost (area/T)
- One can obtain the optimal solution to this problem as follows:
- $b^*=\{h/(p+h)\}Q^*$

  and $Q^*=\{(2K\lambda/h)^*[(p+h)/p]\}^{1/2}$
- Using these results, write the average total cost function and show that it is always less than the average total cost obtained by EOQ with no backorders.

Powers-of-two Policies

- Practicality of the EOQ Solution.
- Multiple items
- Correction for realistic times
- T_L: base-time (1 hour, 1 week)
- Limit your cycle times to $2^k T_L$ for k=0, 1, 2, …
- It is shown that with this restriction at most an additional 6% of the optimal EOQ average cost will be paid.

Statement of the Lot Sizing Problem

- Known set of requirements *(d_1, d_2, . . . d_T)* over an *T* period planning horizon.
- Costs: set up cost, *K,* and the holding cost, *h,* per unit in the ending inventory per period
- The objective is to determine production quantities *(X_1, X_2, . . ., X_T)* to meet the requirements at minimum cost.
- The feasibility condition to assure there are no stockouts in any period is:

$$I_{t-1} + X_t - d_1 = I_t \text{ for all } t$$

Lot Sizing Problem

- Periodic Review where periods are given
- Formulation (we already did in class)
  - minimize total cost (excluding sum of unit purchasing costs which is constant)

Min $\sum_t \{\delta(X_t)K + hI_t\}$

$I_{t-1} + X_t - d_t = I_t$ for all t

$\delta(X_t)=1$ if $X_t>0$; 0 o.w.

$X_t \geq 0$

- Solution: Dynamic Programming in general

Lot Sizing Problem

- Special Case – Observations
- (1) Do we order when there is inventory? NO
- (2) So to ensure (1) how much do we order, if we order in a period t?

  d_t, or d_t+d_{t+1}, or $d_t+d_{t+1}+d_{t+2}$, …
- Solution: DP + shortest path problem:

  *Node 1 – corresponds to time 1; *node T+1 corresponds to time T+1 (end of horizon)

  *Arc(i,j) – corresponds to the cost of ordering at i to satisfy all the demand requirements from i to j-1, j>i