IE 375-HW2 ANSWERS

1A)

Model	Period 1	Period 2	Period 3	Weight(kg/unit)
HS	100	200	100	150
SS	250	150	200	200
GS	150	100	300	250
MS	200	250	150	300
OS	100	100	200	200
Aggregate Demands	182500	180000	215000	

We multiplied each demand with weight of related model, then sum up them for each period.

1B)

Set

J={1,2,3} : periods

Parameters:

D_j: Aggregate demand for period j (in kilograms)

c_r: production cost per kilogram for regular time

c_o: production cost per kilogram for overtime

c_s: production cost per kilogram for subcontracting

c_h: inventory carrying cost per kilogram per period

$rcap_j$: regular time production capacity for period j (in kilograms)

$ocap_j$: overtime production capacity for period j (in kilograms)

scapj: subcontracting production capacity for period j (in kilograms)

Inv_0: initial inventory = 4500 kg

Decision variables:

R_j: amount of engine blocks produced in regular time for period j (in kilograms)

O_j: amount of engine blocks produced in overtime for period j (in kilograms)

S_j: amount of engine blocks produced with subcontracting for period j (in kilograms)

Inv_j: inventory level at the end of period j (in kilograms)

Model:

$$\min \sum_{j=1}^{3} c_r R_J + c_o O_j + c_s S_j + c_h Inv_j$$

S.t.

$$Inv_j = Inv_{j-1} + R_j + O_j + S_j - D_j, \forall j \in J \ (1)$$

$$R_j \leq rcap_j, \forall j \in J \ (2)$$

$$O_j \leq ocap_j, \forall j \in J \ (3)$$

$$S_j \leq scap_j, \forall j \in J \ (4)$$

$$Inv_j \geq 0, \forall j \in J \ (5)$$

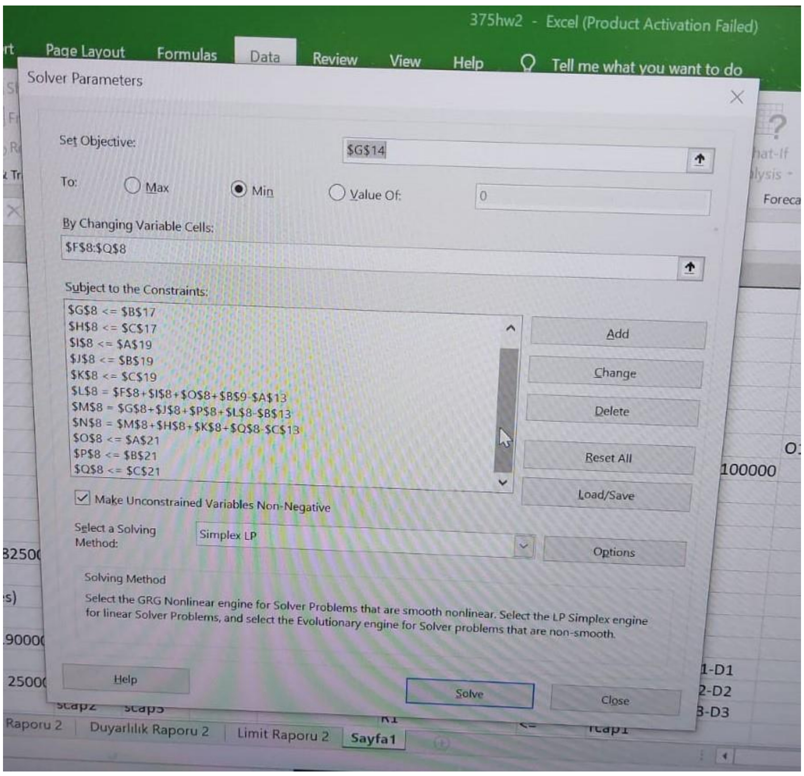
$$R_j \geq 0, \forall j \in J \ (6)$$

$$O_j \geq 0, \forall j \in J \ (7)$$

$$S_j \geq 0, \forall j \in J \ (8)$$

In the objective function, we aim to minimize the total cost. The constraint (1) is for inventory balance. Consraints (2), (3), (4) prevent overcapacity production. Constraints (5), (6), (7), (8) are nonnegativiti constraints. They prevent backorders.

1C)



Values of decision variables:

R_1	R_2	R_3
178000	200000	100000
O_1	O_2	O_3
0	20000	40000
S_1	S_2	S_3
0	10000	25000
Inv_1	Inv_2	Inv_3
0	50000	0

Dual variables of this model are also called shadow prices of releated constraints. The sensitivity analysis gives information about the value to the objective function of additional resources. This information is contained in the values of the shadow prices (dual variables) for the constraints.

The value of shadow price in the constraint "$Inv_1 = Inv_0 + R_1 + O_1 + S_1 - D_1$" equals to -2. It means that if we increase the right hand side of the constraint (i.e. $Inv_0 + R_1 + O_1 + S_1 - D_1 + 1$), the value of objective function (total cost) decreases by 2 TL.

The value of shadow price in the constraint "$Inv_2 = Inv_1 + R_2 + O_2 + S_2 - D_2$" equals to -4. It means that if we increase the right hand side of the constraint (i.e. $Inv_1 + R_2 + O_2 + S_2 - D_2 + 1$), the value of objective function (total cost) decreases by 4 TL.

The value of shadow price in the constraint "$Inv_3 = Inv_2 + R_3 + O_3 + S_3 - D_3$" equals to -6. It means that if we increase the right hand side of the constraint (i.e. $Inv_2 + R_3 + O_3 + S_3 - D_3 + 1$), the value of objective function (total cost) decreases by 6 TL.

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
L8	c_s Inv1	0	-2	0	178000	12000
M8	c_s Inv2	50000	-4	0	10000	26000
N8	c_s Inv3	0	-6	0	10000	26000

1d)

If we consider the production capacity constraints, the maximum of total regular and overtime production for each period = min {$rcap_1+ocap_1$, $rcap_2+ocap_2$, $rcap_3+ocap_3$} = $rcap_3+ocap_3$ = 140000. Hence, we can suggest a solution that will meet the demand as much as possible.

	Period 1	Period 2	Period 3
Initial Inventory	4500	0	0
R	140000	140000	100000
O	0	0	40000
S	32000	360000	25000
Demand	182500	180000	215000
Unmet Demand	6000	4000	50000
Ending Inventory	0	0	0

In this solution, although we use subcontracting production capacities at the top level, we cannot meet the demands of each period because of the capacity constraints. Backorders are not allowed, so our ending inventories are not equal to the negative of unmet demands of each period. According to this solution,

$$\textit{Total cost} = \sum_{j=1}^{3} c_r R_J + c_o O_j + c_s S_j = 1{,}252{,}000\ TL$$

In the optimal solution, we found total cost = 1,376,000 TL. New total cost provided from our alternative solution is 9% less than the total cost of optimal solution. However, we should be aware that we cannot meet demands, so we have compromised on feasibility in the alternative solution.

1e)

In 1c, we found the shadow prices of inventory constraints as -2, -4 and -6 for period 1, period 2, and period 3, respectively. Therefore, if we increase the right hand side of these constraints by one unit, the objective value will decrease by shadow prices. In the right hand side of these constraints, the parameters D_j's have minus sign so if the forecast values in Table 1 were increased, the right hand side of each constraint would be decreased. Thus, the optimal objective function value would increase by shadow price for each additional one unit in the forecasts.

1f)

Additionally, the reduced cost associated with the nonnegativity for each variable is the shadow price of that constraint. The reduced costs tell us how much the objective coefficients can be increased or decreased before the optimal solution changes. The reduced cost of a variable may be interpreted as the amount of penalty you would have to pay to introduce one unit of that variable into the solution.

Thus, reduced costs of R_j's must be different from 0 so that the optimal objective function value would change if we increase one of the regular time capacities in Table 2.

The reduced cost of R_1 is 0. Thus, if we increase $rcap_1$ by one unit, objective function value will be the same.

The reduced cost of R_2 is -2. Thus, if we increase $rcap_2$ by one unit, objective function value decreases by 2 TL.

The reduced cost of R_3 is -4. Thus, if we increase $rcap_3$ by one unit, objective function value decreases by 4 TL.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
F8	c_s R1	178000	0	2	1	0
G8	c_s R2	200000	-2	2	2	1E+30
H8	c_s R3	100000	-4	2	4	1E+30
I8	c_s O1	0	1	3	1E+30	1
J8	c_s O2	20000	-1	3	1	1E+30
K8	c_s O3	40000	-3	3	3	1E+30
L8	c_s Inv1	0	0	2	1E+30	0
M8	c_s Inv2	50000	0	2	1E+30	2
N8	c_s Inv3	0	8	2	1E+30	8
O8	c_s S1	0	2	4	1E+30	2
P8	c_s S2	10000	0	4	0	1
Q8	c_s S3	25000	-2	4	2	1E+30

2A)

Month	Workhours	# of initial workers	# of hired workers	# of total workers	Demand	Monthly total production	Initial Inv	Ending Inv
1	200	8	8	16	1400	1200	200	0
2	160	16	13	29	1800	1800	0	0
3	192	29	1	30	2400	2976	0	576>400

In the first month we need to produce 1400-200=1200 bicycles. 8 experienced workers produce (200/2)*8=800 bicycles in the first month. We need 400 more bicycles, so we need to hire 400/(200/4)= 8 more workers.

In the second month we need to produce 1800 bicycles. 16 experienced workers produce (160/2)*16=1280 bicycles in the second month. We need 520 more bicycles, so we need to hire 520/(160/4)= 13 more workers.

In the third month we need to produce 2400+400=2800 bicycles. 29 experienced workers produce (192/2)*29=2784 bicycles in the second month. We need 16 more bicycles, so we need to hire 16/(192/4)= 0.33, we round this value to 1, which is the smallest integer greater than 0.33. Monthly total production becomes 29*(192/2)+1*(192/4) = 2976 units in this period.

Total cost= Hiring cost+Holding cost+ Total monthly salaries

= 500(8+13+1)+5*576+2500(8+21+22)=141380 TL (without the salaries of constant 8 workers that exist since the beginnig of the first month)

2B)

Month	Workhours	# of initial workers	# of hired workers	# of total workers	Demand	Monthly total production	Initial Inv	Ending Inv
1	200	8	x	x+8	1400	50x+800	200	50x-400
2	160	x+8	0	x+8	1800	80x+640	50x-400	130x-1560
3	192	x+8	0	x+8	2400	96x+768	130x-1560	226x-3192>=400

In the first month, x new workers produce (200/4)x= 50x bicycles. 8 experienced workers also produce 8*(200/2)=800 bicycles. Thus, monthly total production becomes 50x+800. We find ending inventory = monthly total production+initial inventory-demand for each month.

If 226x-3192>=400, then x>=16. We update our table with x=16:

Month	Workhours	# of initial workers	# of hired workers	# of total workers	Demand	Monthly total production	Initial Inv	Ending Inv
1	200	8	16	24	1400	50x+800=1600	200	50x-400=400
2	160	24	0	24	1800	80x+640=1920	50x-400=400	130x-1560=520
3	192	24	0	24	2400	96x+768=2304	130x-1560=520	226x-3192=424

Total cost= Hiring cost+Holding cost+ Total monthly salaries

= 500*16+5*(400+520+424)+2500(16+16+16)= 134720 TL (without the salaries of constant 8 workers that exist since the beginnig of the first month)