IE375 Fall 2020 Case 2

Coco's Toys Solutions

PART A Production Planning

I) (25 points)

Standard/General approach: If you do not consider the given statements in detail, you can simply formulate the problem in a rather standard way.

Sets:

I = {1,2,3} (Set of toys, 1: Car, 2: Doll, 3, Lego)

K= {1, 2, 3, … 12} (Set of months, 1: January 2: February…)

Parameters (as given in the question):

r_i : Cost of producing one car of type i in regular time
o_i : Cost of producing one car of type i in over time
h_i : Inventory holding cost per unit car per month
a_i : Labor time required for shaping of toy type i
b_i : Labor time required for assembly of toy type i
c_i : Labor time required for painting of toy type i
d_{ik} : Demand for toy type i in period k

Decision Variables:

x_{ik} : Number of toys of type i produced in in month k in regular time
y_{ik} : Number of toys of type i produced in month k in overtime
I_{ik} : Amount of inventory of cars of type i in month k

Model:

Maximize $\sum_{i\epsilon I}\sum_{k\epsilon K}$ $(r_i*x_{ik} + o_i*y_{ik} + h_i*I_{ik})$

Subject to

I:	$x_{ik} + y_{ik} + I_{i(k-1)} = I_{ik} + d_{ik}$	for all i ϵ I, for all k ϵ K
II:	$I_{i0} = I_{i12} = 0$	for all i ϵ I
III:	$a_1*x_{1k} + a_2*x_{2k} + a_3*x_{3k} \leq 6400$	for all k ϵ K
IV:	$b_1*x_{1k} + b_2*x_{2k} + b_3*x_{3k} \leq 4800$	for all k ϵ K
V:	$c_1*x_{1k} + c_2*x_{2k} + c_3*x_{3k} \leq 3200$	for all k ϵ K
VI:	$1.2*y_{1k} + 1*y_{2k} + 0.4*y_{3k} \leq 450$	for all k ϵ K
VII:	$x_{ik} \geq 0,\ y_{ik} \geq 0,\ I_{ik} \geq 0$	for all i ϵ I, for all k ϵ K

Equations (I) and (II) are for material balance; Equations (III), (IV) and (V) are regular time capacity constraints; Equation (VI) is for overtime capacity (considering all the process of the item will be carried out in overtime); Equation (VII) represents non-negativity constraints.

Problem specific approach: Consider properties of the specific problem: **(1)** Observe that you are not told to process one whole product in overtime. To take the advantage of this possibility reformulation is needed. Here we assume that any of the processes for any toy type can be left for overtime with overtime costs incurring for the whole toy. **(2)** Observe that demands are identical in every period and you are not allowed to have any shortages in any period. Also starting and ending inventories are stated as zero. Hence the solution is obviously identical in every period (as we assumed equal working days in every period). Hence, your solution will not require any inventory carrying, and we can now solve formulate the single-period problem.

Sets:

$I = \{1,2,3\}$ (Set of toys, 1: Car, 2: Doll, 3, Lego)

$J = \{1,2,3\}$ (Set of processes, 1: Shaping, 2: Assembly, 3: Paint)

Parameters (as given in the question):

r_i : Cost of producing one car of type i in regular time
o_i : Cost of producing one car of type i in over time
h_i : Inventory holding cost per unit car per month
a_i : Labor time required for shaping of toy type i
b_i : Labor time required for assembly of toy type i
c_i : Labor time required for painting of toy type i
d_i : Demand for toy type i

Decision Variables:

x_{ij} : Number of toys of type i completed process j during regular time
y_{ij} : Number of toys of type i completed process j in overtime
z_i : Number of complete toys of type i produced in regular time
w_i : Number of toys of type i completed in overtime

Model:

Maximize $\sum_{i \epsilon I}$ $(r_i*z_i + o_i*w_i)$

Subject to

I:	$a_1*x_{11}+ a_2*x_{21} + a_3*x_{31} \leq 6400$	
II:	$b_1*x_{12}+ b_2*x_{22} + b_3*x_{33} \leq 4800$	
III:	$c_1*x_{13}+ c_2*x_{23} + c_3*x_{33} \leq 3200$	
IV:	$z_1 \leq x_{1j}$	for all $j \epsilon J$
V:	$a_1*y_{11}+a_2*y_{21}+a_3*y_{31}+b_1*y_{12}+b_2*y_{22}+b_3*y_{33}+c_1*y_{13}+c_2*y_{23}+c_3*y_{33} \leq 450$	
VI:	$z_i + w_i \leq x_{ij} + y_{ij}$	for all $i \epsilon I$, for all $j \epsilon J$
VII	$d_i \leq z_i + w_i$	for all $i \epsilon I$
VIII:	$x_{ij} \geq 0,\ y_{ij} \geq 0,\ z_i \geq 0,\ w_i \geq 0$	for all $i \epsilon I$, for all $j \epsilon J$

Equations (I), (II) and (II) are regular time capacity constraints; Equation (IV) determines the number of completed items during regular time; Equation (V) represents overtime capacity constraint; Equations (VI) determines the number of completed items during regular time and overtime; Equation (VII) represents demand satisfaction constraint; Equation (VIII) represents non-negativity constraints.

II) (5 points)

Here it is assumed that all individual demand values would increase proportional, hence we can represent demand by $(1+\delta)$ * original demand for all demands where δ is any non-zero positive real number. Hence question asked can be rephrased by the following statement:

Question: Find the largest possible δ_{max} such that if $\delta > \delta_{max}$, the problem formulated in **Part I)** will be infeasible.

Answer: Generic way to find: Parametric Analysis feature of LP.

Answer using **Problem specific approach** described above can be found manually, without resorting any solver, as problem structure is simple (Of course, it is even easier to find it under the structure assumed by the **Standard/General approach.**

Answer with another model: Formulate the problem to maximize δ_{max} with all the original constraints. This model will push the boundary of feasibility to maximize satisfied demand. Of course, cost minimization issue is no-longer considered, as you may get an arbitrary one among the alternative solutions (if any).

PART B Material Ordering Policy

Supplier	Fixed Ordering Cost	Unit Cost	Holding Cost	Yearly Demand Rate
Plastic	10000	20	2.3	125400
Metal	12500	30	3.4	38100
Paint	13000	15	1.6	37260

Annual demand rates of raw materials are calculated by multiplying the demand rates of the toys with the amount of raw material they require – this step should not be missed!

Formulas and Equations:

A. Cycle time in Powers-of-Policy $T = T_L * 2^k$ (for $k \geq 0$) where T_L is a base time.

B. $Q = \lambda * T$

C. $G(Q) = \frac{K*\lambda}{Q} + \lambda * c + \frac{h*Q}{2}$

D. $G(Q) = \frac{K}{T} + \sum_i [\lambda_i * c_i + \frac{h_i Q_i}{2}]$

E. $\sum_i \lambda_i / P_i \leq 1$

F. $h'_i = (1 - \frac{\lambda_i}{P_i}) * h_i$

G. $K_i = 120 * s_i$

H. $s'_i = s_i / 2400$

I. $T* = sqrt\,(\frac{2\sum_{j=1}^{n} K_j}{\sum_{j=1}^{n} h'_j \lambda_j})$

J. $T_{min} = \frac{\sum_{j=1}^{n} s'_j}{1 - \sum_{j=1}^{n} \frac{\lambda_j}{P_j}})$

K. $T = \max\{T*, T_{min}\}$

L. $G(T) = \sum_{j=1}^{n} (\frac{K_j}{T} + \frac{h'_j \lambda_j T}{2})$

M. Setup Cost = $\sum_i \frac{K_i}{T}$

N. Holding cost = $\sum_i (\frac{h'_i \lambda_i T}{2})$

O. Total cost = Annual Holding Cost + Annual Setup Cost

P. Uptime = $\sum_i \frac{Q_i}{P_i}$

Q. Proportion of idle time = $\frac{T - Uptime}{T}$

R. $R = \tau * \lambda$ where τ is the lead time in terms of years.

I) Company's ordering policy requires implementation of Powers-of-Two Policy. Cycle times are calculated using Formula A. Since the base time is given as one month in the question, the possible cycles times are 1, 2, 4, 8 or 16. Separately for each toy, lot sizes are calculated using Formula B for each possible cycle time and total costs are calculated using Formula C. The optimal cycle time considering company's policy (separately for each toy) is determined as the cycle time that gives the least total cost. Reorder levels are calculated using Equation

R. Nevertheless, you can implement the optimal solution in the text by first finding EOQ and only checking two neighboring cycle times (as shown with * in the below tables).

Plastic	Quantity	Fixed Ordering Cost	Proportional Order Cost	Holding Cost	Total Cost
T=1 Month	10450	120000	2508000	12017.5	2 640 017.5
*T=2 Months	20900	60000	2508000	24035	2 592 035
*T= 4 Months	41800	30000	2508000	48070	2 586 070
T= 8 Months	83600	15000	2508000	96140	2 619 140

Metal	Quantity	Fixed Ordering Cost	Proportional Order Cost	Holding Cost	Total Cost
T=1 Month	3175	150000	1143000	5397.5	1 298 397.5
*T=2 Months	6350	75000	1143000	10795	1 228 795
*T= 4 Months	12700	37500	1143000	21590	1 202 090
T= 8 Months	25400	18750	1143000	43180	1 204 930

Paint	Quantity	Fixed Ordering Cost	Proportional Order Cost	Holding Cost	Total Cost
T=1 Month	3105	156000	558900	2484	717 384
T=2 Months	6210	78000	558900	4968	641 868
*T= 4 Months	12420	39000	558900	9936	607 836
*T= 8 Months	24840	19500	558900	19872	598 272
T= 16 Months	49680	9750	558900	39744	608 394

Optimal cycle times are 4 months for plastic and metal and 8 months for paint.

	Reorder Levels
Plastic	15675
Metal	4762.5
Paint	4657.5

Total average cost = 2 586 070 + 1 202 090 + 598 272 =$4 386 432

II) Rotational Cycle Policy: EOQ solution:

Implement formula I to find T where summation of setup costs is simply $30 000 and h' is simply h for each item.

T* = 0.3544 years or approximately 17 weeks.

Total cost is computed using equations M and N and adding the unit costs.

Cycle Length.	Fixed Ordering Cost	Proportional Order Cost	Holding Cost	Total Cost
T=0.3544	84 650	4 209 900	84 626	4 379 176

This cost is less than what we found in Part I. Of course, there are two other factors which are not stated in the case:

a) When do you pay for the goods you buy? If you pay at the time of arrival to your facility, then the above computations will hold. However, if you pay at the suppliers' facility, then you need to charge for the in-transit inventory. Hence, this will make the second case more profitable as lead time is smaller.
b) Risks involved: By relying on a single supplier, you are probably decreasing your risk as now you become a more important customer. However, we are not given any information on the variability of lead time with the single supplier, which is a factor that will probably work against the single supplier.

As an alternative, one can apply a similar powers-of-two policy in this case, as well.

When it is possible to given only one order for all the raw materials, fixed ordering c using Formula 2 and the total cost is calculated using Formula 4. The cycle time that provides the least total cost is chosen as the optimal solution.

Cycle Length.	Fixed Ordering Cost	Proportional Order Cost	Holding Cost	Total Cost
T=1 Month	360000	4209900	19899	4 589 799
T=2 Months	180000	4209900	39798	4 429 698
T= 4 Months	90000	4209900	79596	4 379 496
T= 8 Months	45000	4209900	159192	4 414 092

Optimal cycle length is 4 months. Note that the cost obtained is remarkably close to the first solution.

Part C Automation of Production

Toy	Annual Production Rate	Setup Time	Setup Cost	Unit Cost
Car	100000	2.4	288	11
Doll	100000	3.2	384	8
Lego	200000	1.5	180	5
Total		0.002958333	852	

I) For a rotational cycle policy problem to be feasible Equation E should be satisfied. If the equation is binding and the summation is equal to 1, then the greatest proportion of demand that can be satisfied with the machine will be satisfied. The following model is solved in MS Excel Solver to find the maximum percentage.

Decision Variables:

x: The maximum percentage of demand that the machine can satisfy

Model:

Maximize x

Subject to

$\sum_{i=1}^{n} \frac{\lambda_i * x}{P_i} \leq 1$ where demands are annual demands (note that monthly demands are given in the beginning of the case description). Solution is x = 0.59

II-III) T* is calculated using Equations F,G and I (the demand annual demand rates should be divided by two since only half of the demand will be satisfied from the machine). T_{min} is calculated using Equation H and J and optimal cycle length is calculated using Equation K. Optimal lot sizes are calculated using Equation 2.

Toy	Demand/Production Rate	Modified Holding Cost	Holding Cost*Demand
Car	0.36	1.0752	38 707.2
Doll	0.33	0.804	26 532
Lego	0.108	0.64224	13 872.4
Total			79 111.6

T=	0.146762383
Tmin =	0.014645215

Setup and holding costs are calculated using Equations M and N, respectively.

Toy	Lot Size	Setup Cost	Holding Cost	Total Cost
Car	5283.45	1 962.36	2840.387	4 802.74
Doll	4843.16	2 616.47	1946.95	4 563.42
Lego	3170.07	12 290.70	1017.97	13 308.68
	TOTAL	16 869.53	5805.30	22 674.84

IV) Uptime is calculated using Equation P (in terms of years). Proportion of idle time is calculated using formula Q.

Toy	Lot Size/Production Rate
Car	0.052834458
Doll	0.048431586
Lego	0.015850337
Uptime	0.117116382

Proportion of Idle Time= 0.202

V) The answer is straightforward if you can keep track of costs charged for the correct time. So, consider a year and let us compute the cost within the year. Assume the machine works 50 hours a week – of course this number is a bit arbitrary for the solution of the problem.

Workers will be working on all the time that the machine is up. Uptime is converted to hours by multiplying Uptime in years with 2400 hours (2400 hours a year = 48 weeks * 50 hours/week) - in a year). Cost of manpower is calculated by multiplying uptime in hours with hourly wage. The total yearly cost of production with the machine is calculated by summing worker cost, holding cost, investment cost and setup cost.

Uptime as hours	(1-0.202)*2400 = 1680 hours
6 Workers, 10$ per hour	$100 800 per year
Operating costs	$22 674.84
Maintenance Cost	50 000
Unit costs	11*36000+8*33000+5*19600=$758 000
Total Annual Cost	$931 474.84