HW 4 Answers

(1a) $c_u = p - c = 130 - 32 = 98$

$c_o = c - s = 32 - 20 = 12$

critical ratio $= \frac{c_u}{c_u + c_o} = \frac{98}{98+12} = 0.8909$

(1b) D: demand for sweatshirts

μ: the mean of D, i.e, $\mu = 150$

σ: the standard deviation of D, i.e, $\sigma = 30$

F: the cumulative distribution function of D
(Normal distribution with mean μ, variance σ^2)

f: The probability distribution function of D

F^S: The cumulative distribution function of standard normal distribution

The closest z value which corresponds to
$F^S(z) = 0.8909$
is equal to $z^* = 1.23$ (without interpolation) by table A-4 on pg. 808.

We know from the properties of normal distribution that $F^S(z) = F(\sigma z + \mu)$.

Therefore, if $Q^* = \sigma z^* + \mu$, then $F(Q^*) = 0.8909$, which is what we want while finding Q^*.

Consequently,

$$Q^* = \sigma z^* + \mu$$

$$= 30 . 1.23 + 150$$

$$= 186.9$$ (in thousands) is the ordering quantity.

1c)

$E[\max\{0, y^* - D\}]$

$$= \int_{-\infty}^{y^*} (y^* - x)\, f(x)\, dx$$

$$= \int_{-\infty}^{+\infty} (y^* - x)\, f(x)\, dx - \int_{y^*}^{+\infty} (y^* - x)\, f(x)\, dx$$

$$= y^* \underbrace{\int_{-\infty}^{+\infty} f(x)\, dx}_{1} - \underbrace{\int_{-\infty}^{+\infty} x f(x)\, dx}_{E[D]} + \underbrace{\int_{y^*}^{+\infty} (x - y^*)\, f(x)\, dx}_{n(y^*)}$$

$$= y^* - E[D] + n(y^*)$$

where $n(y^*) = E[\max\{D - y^*, 0\}]$ defined in pg. 270 of textbook (7th ed.). We also know from p.271 that if the pdf of D is the pdf of normal distribution, then we can use the formula $n(y^*) = \sigma L\left(\frac{y^* - \mu}{\sigma}\right)$. Hence,

$E[\max\{0, y^* - D\}]$

$$= \sigma L\left(\frac{y^* - \mu}{\sigma}\right) + y^* - \mu$$

$$= 30 L\left(\frac{186.9 - 150}{30}\right) + 186.9 - 150$$

$$= 30 \times L(1.23) + 186.9 - 150$$

$$= 30 \times 0.0527 + 186.9 - 150$$

$= 38.481$ (in thousands) is expected number of shirts not sold.

(1c) $E[\max\{0, y^o - D\}]$

$$= \int_{-\infty}^{y^o} (y^o - x)\, f(x)\,dx$$

$$= \int_{-\infty}^{+\infty} (y^o - x) f(x)\,dx - \int_{y^o}^{+\infty} (y^o - x) f(x)\,dx$$

$$= y^o \underbrace{\int_{-\infty}^{+\infty} f(x)\,dx}_{1} - \underbrace{\int_{-\infty}^{+\infty} x f(x)\,dx}_{E[D]} + \underbrace{\int_{y^o}^{+\infty} (x - y^o) f(x)\,dx}_{n(y^o)}$$

$= y^o - E[D] + n(y^o)$

where $n(y^o) = E[\max\{D - y^o, 0\}]$ defined in pg. 270 of textbook (7th ed.). We also know from p.271 that if the pdf of D is the pdf of normal distribution, then we can use the formula $n(y^o) = \sigma L\left(\frac{y^o - \mu}{\sigma}\right)$. Hence,

$E[\max\{0, y^o - D\}]$

$$= \sigma L\left(\frac{y^o - \mu}{\sigma}\right) + y^o - \mu$$

$$= 30 L\left(\frac{186.9 - 150}{30}\right) + 186.9 - 150$$

$$= 30 \times L(1.23) + 186.9 - 150$$

$$= 30 \times 0.0527 + 186.9 - 150$$

$= 38.481$ (in thousands) is expected number of shirts not sold.

1d $E[\max\{0, D-y^{*}\}]$

$$= \int_{y^{*}}^{+\infty} (x-y^{*})\, f(x)\, dx$$

$$= n(y^{*})$$

$$= \sigma L\left(\frac{y^{*}-r}{\sigma}\right)$$

$$= 30 \times 0.0527 \text{ (from 1c)}$$

$= 1.581$ (in thousands) is the expected excess demand.

Recall, Expected Demand = Quantity Ordered + Expected Shortage − Expected Remaining

Question 2

Note the following parameters

1 year = 50 weeks

Lead time = 4 weeks (τ)

Fixed cost: $K = \$300$

Holding cost: $h = \$0.18$ / item − 1 week $\Rightarrow$ \$9 / item − 1 year

Backorder cost: $p = \$6$/unit

Demand during 4 weeks of lead time: $D \sim$ uniform$([200, 800])$

$$E[D] = 500$$

Expected yearly demand: $\lambda = \frac{50}{4} \times E[D] = 6250$

(2a) $$G(Q,R) = h\left(\frac{Q}{2} + R - \lambda\tau\right) + \frac{K\lambda}{Q} + \frac{p\lambda n(R)}{Q}$$

(2b) $$Q = \sqrt{\frac{2\lambda[K + p\,n(R)]}{h}}$$

$$1 - F(R) = \frac{Qh}{p\lambda}$$

(2c) $$n(R) = \int_R^{800} (x - R)\,\frac{1}{800-200}\,dx$$

$$= \frac{1}{600}\left[\int_R^{800} x\,dx - R\int_R^{800} dx\right]$$

$$= \frac{1}{600}\left[\frac{800^2 - R^2}{2} - R.(800 - R)\right]$$

$$= \frac{1}{600}\left[\frac{800^2}{2} - R.800 + \frac{R^2}{2}\right]$$

$$= \frac{(800-R)^2}{600 \times 2}$$

$$F(R) = \int_{200}^{R} \frac{1}{800-200}\,dx = \frac{R-200}{800-200}$$

Let $Q_0 = EOQ = \sqrt{\frac{2K\lambda}{h}} = \sqrt{\frac{2.300.6250}{9}} = 645.50$

Apply the conditions in (2b) iteratively.

$$1 - \frac{R-200}{600} = \frac{Q \cdot 9}{6 \times 6250} \Rightarrow R = \left[1 - \frac{9Q}{6 \times 6250}\right] \times 600 + 200 \quad (1)$$

$$Q = \sqrt{\frac{2 \times 6250 \times \left[300 + 6 \times \frac{(800-R)^2}{1200}\right]}{9}} \quad (2)$$

Applying the formulae (1) and (2) iteratively, we obtain

$EOQ = 645.50 \quad R_0 = 707.05$

$Q_1 = 690.41 \quad R_1 = 700.58$

$Q_2 = 696.64 \quad R_2 = 699.71$

$Q_3 = 697.50 \quad R_3 = 699.56$

$Q_4 = 697.66 \quad R_4 = 699.53$

$Q_3 - Q_4$ and $R_3 - R_4$ are very close to each other, so we may terminate computations.

We conclude that the optimal values of Q and R are $(Q^*, R^*) = (697.66, 699.53)$

In addition to this algorithm, we can directly solve the explicit equations.

$$F(R) = 1 - \frac{Qh}{p\lambda}$$

$$\frac{R-200}{600} = 1 - \frac{\sqrt{\frac{2\lambda[K+pn(R)]}{h}}\, h}{p\lambda}$$

$$= 1 - \sqrt{\frac{2[K+pn(R)]h}{p^2\lambda}}$$

$$= 1 - \sqrt{\frac{2Kh + 2hp\frac{(800-R)^2}{1200}}{p^2\lambda}}$$

$$= 1 - \sqrt{\frac{2 \cdot 300 \cdot 9 + 2 \cdot 9 \cdot 6\frac{(800-R)^2}{1200}}{36 \cdot 6250}}$$

$$\frac{R-200}{600} = 1 - \sqrt{\frac{150}{6250} + \frac{(800-R)^2}{2500000}}$$

$$\Downarrow$$

$R^* = 699.53$ and $Q^* = 697.66$

2d $n(R^*) = \dfrac{(800-699.53)^2}{1200} = 8.41$

By 2a;

$$G(Q^*, R^*) = h\left(\frac{Q^*}{2} + R^* - \lambda\tau\right) + \frac{K\lambda}{Q^*} + \frac{p\lambda n(R^*)}{Q^*}$$

$$= 9\left(\frac{697.66}{2} + 699.53 - 6250 \times \frac{4}{50}\right)$$

$$+ \frac{300 \times 6250}{697.66} + \frac{6 \times 6250 \times 8.41}{697.66}$$

$$= \$8074.84$$

2e

Type 1 Service

$$\alpha = F(R^*) = F(699.53) = \frac{699.53 - 200}{800 - 200} = 0.83$$

We can interpret α as the proportion of cycles in which no stock-out occurs.

Type 2 Service

$$\underbrace{n(R^*)}_{} = Q^*(1-\beta)$$

(By 2d) $8.41 = 697.66(1-\beta)$

$$1-\beta = 0.01$$

$$\beta = 0.99$$

We can interpret β as the proportion of demands that are met from stock.

For these type-1 and type-2 service levels, we can find the imputed shortage cost with Q^* and R^* values in 2d.

$$p = \frac{Qh}{(1-F(R))\lambda} = \frac{697.66}{\left(1-\frac{699.53-200}{600}\right)6250} = \$0.67$$