IE 375 HW5 SOLUTIONS

(Q1) By page 506 our textbook (7th ed.), Lawler's algorithm can minimize the maximum tardiness.

Precedence relationships are:

$2 \rightarrow 6 \rightarrow 3$

$1 \rightarrow 4 \rightarrow 7 \rightarrow 8$

Jobs without precedence relation: 5

The relevant data is

Job	Processing Time	Due Date
1	2	5
2	3	4
3	2	13
4	1	6
5	4	12
6	3	10
7	2	15
8	2	19

Let τ denote the total processing time for the remaining jobs that are not scheduled yet.

Step 1: Candidate jobs to be scheduled last are 3-5-8

$Z = 2+3+2+1+4+3+2+2 = 19$

$\min\{19-13, 19-12, 19-19\} = \min\{6, 7, 0\} = 0$

Therefore, Job 8 is scheduled last.

Step 2: Candidate jobs to be scheduled at 7^{th} position: 3-5-7

$Z = 17$

$\min\{17-13, 17-12, 17-15\} = \{4, 5, 2\} = 2$

Therefore, job 7 is scheduled in the 7^{th} position.

Step 3: Candidate jobs to be scheduled at 6^{th} position: 3-5-4

$Z = 15$

$\min\{15-13, 15-12, 15-6\} = \min\{2, 3, 9\} = 2$

Therefore, job 3 is scheduled in the 6^{th} position.

Step 4: Candidate jobs to be scheduled at 5th position: 4-5-6

$\tau = 13$

$\min\{13-6, 13-12, 13-10\} = \min\{7, 1, 3\} = 1$

Therefore, job 5 is scheduled in the 5th position.

Step 5: Candidate jobs to be scheduled at 4^{th} position: 4-6

$\tau = 9$

$\min\{9-6, 9-10\} = \min\{3, -1\} = -1$

Therefore, job 6 is scheduled in the 4^{th} position.

Step 6: Candidate jobs to be scheduled at 3^{rd} position: 2-4

$\tau = 6$

$\min\{6-4, 6-6\} = \min\{2, 0\} = 0$

Therefore, job 4 is scheduled in the 3^{rd} position.

Step 7: Candidate jobs to be scheduled at 2^{nd} position: 1-2

$\tau = 5$

$\min\{5-5, 5-4\} = \min\{0, 1\} = 0$

Therefore, job 1 is scheduled in the 2^{nd} position. and job 2 is scheduled at 1^{st} position.

The sequence is: 2-1-4-6-5-3-7-8

Job	Processing Time	Completion Time	Due Date	Tardiness
3	3	3	4	0
2	2	5	5	0
1	1	6	6	0
4	3	9	10	0
6	4	13	12	1
5	2	15	13	2
3	2	17	15	2
7	2	19	19	0
8	2			

Maximum tardiness = 2

(2a) We have a single worker and five cars (jobs). The worker is responsible for both refitting and repainting. In addition, there is no precedence relation between the cars. We can use Earliest Due Date (EDD) scheduling. EDD minimizes the maximum tardiness in an order of jobs with respect to their due dates.

Job (Car)	Processing Time	Completion Time	Due Date	Tardiness
E	3+9=12	12	16	0
B	16+14=30	30+12=42	40	2
A	4+5=9	9+42=51	42	9
D	12+11=23	23+51=74	43	31
C	8+7=15	15+74=89	70	19

Maximum tardiness is 31 (minimized with EDD)

(2b) We can use Johnson's algorithm.
Note that there is a work flow from worker 1 to worker 2

Car	Worker 1	Worker 2	
A	4	5	(2)
B	16	14	(3)
C	8	7	(5)
D	12	11	(4)
E	3	9	(1)

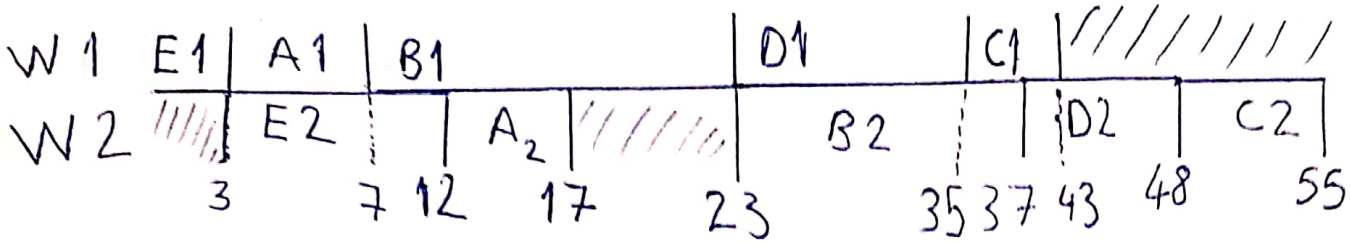
The first step is to identify the minimum job time. It is 3, for car E on worker 1. Because it appears in Worker 1, car E is scheduled first, and its row is crossed out.

The next smallest processing time is 4, for car A on worker 1. It appears in worker 1, so car A is scheduled next.

The next smallest processing time is 7, for car C on worker 2. Because it appears in worker 2, car C is scheduled last.

The next smallest processing time is 11, for car D on worker 2. It appears in Worker 2, so car D is scheduled 4th.
Thus, car B is scheduled 2nd

E - A - B - D - C Makespan = 55



(2c) In (2a), the schedule is in an order: E-B-A-D-C, and the completion time of the last job C, which is 89, gives us the makespan value.

It is 34 hours more than the makespan value in (2b) due to the number of workers.

In (2b), the schedule is an order: E-A-B-D-C. From the previous Gantt chart, we can observe the completion time of each job, and find the tardiness values.

Job	Completion Time	Due Date	Tardiness
E	12	16	0
A	17	42	0
B	37	40	0
D	48	43	5
C	55	70	0

Maximum tardiness is 5, which is less than maximum tardiness value found in (2a) although we minimized the maximum tardiness in (2a) via EDD algorithm. However, note that there is a single worker in (2a) and there are two workers being responsible for different jobs in (2b).