IE 375
Exam 1 November 9, 2020
Duration: 10 minutes

Academic integrity is always expected of all students of Bilkent University, whether in the presence or absence of members of the faculty.

Understanding this, I declare that I shall not give, use, or receive unauthorized aid in the examination.

Name:
Section: 01 KEY
ID Number:
Signature:

Question	Grade	Your Grade
1	35	
2	20	
3	25	
4	20	
Total	100	

- READ ALL QUESTIONS CAREFULLY.
- SHOW ALL YOUR WORK.

Useful Formula:

$F_{t+1} = F_{t,t+1} = S_t = \alpha D_t + (1-\alpha) S_{t-1}$

$G_t = \beta (S_t - S_{t-1}) + (1-\beta) G_{t-1}$

$c_t = \gamma (D_t / S_t) + (1-\gamma) c_{t-N}$

$F_{t,t+\tau} = (S_t + \tau G_t) c_{t+\tau-N}$

$\sigma_e = 1.25\ MAD$

$MAD = 1/n \sum |e_t|$

$S_t = \alpha D_t + (1-\alpha)(S_{t-1} + G_{t-1})$

$S_t = \alpha (D_t / c_{t-N}) + (1-\alpha)(S_{t-1} + G_{t-1})$

$d_t = (\mu + G_t) c_t + \varepsilon_t$

$F_{t,t+\tau} = (S_t + \tau G_t)$

$e_t = F_{t-1,t} - D_t$

$\sigma^2(\text{for } T \text{ periods}) = T\, \sigma^2(\text{for one period})$

Economic Order Quantity Model

$$G(Q) = \frac{K\lambda}{Q} + \frac{hQ}{2} \quad Q^* = \sqrt{\frac{2K\lambda}{h}} \quad G(Q^*) = \sqrt{2K\lambda h}$$

Economic Production Quantity Model

$$G(Q) = \frac{K\lambda}{Q} + \frac{h(1-\lambda/P)Q}{2} \quad Q^* = \sqrt{\frac{2K\lambda}{h(1-\lambda/P)}} \quad G(Q^*) = \sqrt{2K\lambda h(1-\lambda/P)}$$

Question 1 (35 points)

Double exponential smoothing (Holt's method) is used to obtain forecasts for a product. The following table summarizes some of the information.

t	D_t	S_t	G_t	$F_t = F_{t-1,t}$	e_t
1	500	500	0	500	0
2	400	475	-5	500	100
3	300	427.5	-13.5	470	170
4	400	410.5	-14.2	414	14
5	450	409.7	-11.5	396.3	-53.7
6	350	386.2	-13.9	398.2	48.2
7	200	329.2	-22.5	372.2	172.2

The MAD calculated as the average of seven absolute value of error terms is 79.73.

a) **(20 points)** Fill in the missing values in the table (i.e., D_4, F_4, e_4, S_5 and G_5). Note that you need to find the values for the smoothing constants used in the computations as well.

→ Excel Solution – Correct Model: Holt's double exp. smoothing

→ Use earliest info to minimize comp. errors

→ $S_2 = \alpha D_2 + (1-\alpha)(S_1+G_1)$

$S_2 = \alpha D_2 + S_1 + G_1 - \alpha(S_1+G_1)$

$S_2 - S_1 - G_1 = \alpha(D_2 - S_1 - G_1)$

$475 - 500 - 0 = \alpha(400 - 500 - 0)$

$\alpha = \frac{-25}{-100} = 0.25$

→ $G_2 = \beta(S_2 - S_1) + (1-\beta)G_1$

$G_2 = \beta(S_2 - S_2) + G_1 - \beta G_1$

$G_2 - G_1 = \beta(S_2 - S_1 - G_1)$

$-5 - 0 = \beta(475 - 500 - 0) \Rightarrow \frac{-5}{-25} = 0.20 = \beta$

The remaining becomes routine application of formulas.

D_4 $S_4 = \alpha D_4 + (1-\alpha)(S_3+G_3) \Rightarrow \frac{S_4 - (1-\alpha)(S_3+G_3)}{\alpha} = D_4$

$\Rightarrow \frac{410.5 - (0.75)(427.5 - 13.5)}{0.25}$

$D_4 = \frac{410.5 - 310.5}{0.25} = 400$

$F_{3,4} = 414$ $e_4 = 14$ $S_5 = 409.7$ $G_5 = -11.5$

IE375 - Exam I - Solution of the common forecasting problem for both sections.

Part a)

t	D_t	S_t	G_t	$F_t=F_{t-1,t}$	e_t			t
1	500.0	500.0	0.0	500.0	0.0			1
2	400.0	475.0	-5.0	500.0	100.0			2
3	300.0	427.5	-13.5	470.0	170.0			3
4	400.0000	410.5	-14.2	414.0	14.0			4
5	450.0	409.7	-11.5	396.3	-53.7			5
6	350.0	386.2	-13.9	398.2	48.2			6
7	200.0	329.2	-22.5	372.2	172.2			7
								8
	0.250							9
α=	0.250					α=	0.250	10
	0.200							11
β=	0.200					β=	0.200	12
								13
ABS(e_4)=	14.01							14

Part b) 282.1

Part c) 808.3

Grading – No partial if another model is considered.

α, β +4 for correct formulas
+2 for correct comp.

Each unknown +2 for correct formula

All unknows (if formulas are correct) +4 for correct computation
[–1 for each incorrect comp.]

b) **(5 points)** Assume that $D_8 = 300$. Find a forecast for period 9, taking into account the latest demand information.

$$F_{8,9} = S_8 + 1 \cdot G_8 = 305.0 - 22.9 = 282.1$$

+3 for the formula
+2 computation

(as long as consistent with part a), no points are lost, if formula is correct, & correction computation according to part a).

c) **(10 points)** Assume that $D_8 = 300$. Find a forecast for the sum of demands in periods 11 through 14, taking into account the latest demand information. This sum is needed to plan for the total demand for those four periods in the future.

$$F_{8,11} = S_8 + 3G_8 = 305 - 3(22.9) = 236.3$$
$$F_{8,12} = S_8 + 4G_8 = 305 - 4(22.9) = 213.4$$
$$F_{8,13} = S_8 + 5G_8 = 305 - 5(22.9) = 190.5$$
$$F_{8,14} = S_8 + 6G_8 = 305 - 6(22.9) = 167.6$$

807.8 (only two digits)
(Excel computation – 808.3)

Formulas +6
Computations +3
Sum +1

(Same exception as part b)

Question 2 (20 points)

A textile firm is developing its aggregate production plan for the next T months.

The firm owns two plants that are closely located: a yarn manufacturing plant that converts cotton and other synthetic fibers into yarn, and a fabric manufacturing plant that converts yarn into fabric. We consider a single type yarn, and an aggregate item is defined for the fabric.

The demand for fabric (in meter squares) in month t is given by D_t. One ton of yarn can be used to produce e^{YF} meter squares of fabric. The firm can keep inventory either as yarn or as fabric. Monthly inventory holding costs for one ton of yarn is h^Y and each meter square of fabric is h^F. Production costs in month t for one ton of yarn and each meter square of fabric are c_t^Y is c_t^F, respectively.

Monthly physical (technological) capacity of the yarn plant is B^Y tons and the fabric plant is B^F meter squares. The two plants can share the workers as they are located closely. Producing a ton of yarn requires a^Y man months and a meter square of fabric requires a^F man months. The firm has a total permanent workforce of W^p which is paid c^p per employee per month. Additional seasonal workers can be employed in any given month for a wage of c^S per employee per month. There are no hiring and firing costs for the seasonal employees.

The firm came up with the following mathematical program to plan its production in two plants for the next T months:

$$Min \sum_{t=1}^{T} \left(c_t^F P_t^F + c_t^Y P_t^Y + h_t^F I_t^F + h_t^Y I_t^Y + c^P W^P + c^S W_t^S \right)$$

$$I_t^F = I_{t-1}^F + P_t^F - D_t \qquad t = 1..T \qquad (1)$$

$$P_t^Y \le B^Y \qquad t = 1..T \qquad (2)$$

$$P_t^F \le B^F \qquad t = 1..T \qquad (3)$$

$$I_T^F = I_T^Y = I_0^F = I_0^Y = 0 \qquad (4)$$

a) **(10 points)** What are the decision variables of the program? Define them properly, as is done in modeling. Be precise and do not include parameters.

cotton etc. → [ ] → yarn → [ ] → fabric $\quad D_t$

$1 \longrightarrow e^{YF}$

Summary

$I_t^Y \quad I_t^F$

$h^Y \quad h^F$

$c_t^Y \quad c_t^F$

B^Y (ton) $\quad B^F$ (m^2)

a^Y/ton $\quad a^F/m^2$

W^P

c^P, c^S

$\uparrow$

N_t^S

W_t^S – seasonal workers hired for period t (assume hired for the month)

I_t^Y – Yarn inventory at the end of period t

I_t^F – Fabric inventory at the end of period t.

P_t^F production of fabric in period t

P_t^Y production of yarn

q_t^Y – yarn amount used in production of fabric in period $t \equiv \dfrac{P_t^F}{e^{YF}}$

$t = 1, 2, \ldots, T$

+2 each if $t=1,\dots,T$ is missing -1
if t is missing -5

any additional $-(-1,-2,-3)$ penalties.

b) (4 points) The amount of yarn used in the production of fabric is one of the decision variables, and let q_t be defined as the quantity in period t. Using this notation, write the constraint regarding material balance for the yarn production.

$$I_{t-1}^{Y} + P_t^{Y} - q_t = I_t^{Y} \qquad t=1,\dots,T$$

$$q_t \equiv q_t^{Y} \quad +3 \qquad +1$$

can be misunderstood — $q_t = \frac{P_t^F}{e^{YF}}$ -2

c) (6 points) What are the missing set or sets of constraints in this formulation, other than the non-negativity of the decision variables and the one asked in part b)? You only need to specify what they are; you do not need to explicitly write them. Make sure that you write them clearly.

manpower constraint

$$a^F \cdot P_t^F + a^Y P_t^Y \leq W^P + W_t^S \qquad t=1,\dots,T \quad +3$$

$$q_t^Y = \frac{P_t^F}{e^{YF}} \qquad t=1,\dots,T$$ conversion of yarn to fabric $+3$

inconsistent with a) — you lose points $(-1,-2,-3)$

consistent with a) — you do not lose any points in both, if correctly written (described)

Question 3 (25 points)

A company wants a high level, aggregate production plan for the next 4 months. Projected orders for the company's product are listed in the table. Over the 4-month period, units may be produced in one month and stored in inventory to meet some later month's demand. Because of seasonal factors, the cost of production is not constant, as shown in the table. The cost of holding an item in inventory for 1 month is $\$h$/unit, charged for those items in the inventory at the end of the period. The maximum number of units that can be held in inventory is I_{max}. The initial inventory level at the beginning of the planning horizon is I_0 units; the desired final inventory level at the end of the planning horizon is I_f.

The problem is to determine the optimal amount to produce in each month so that demand is met while minimizing the total cost of production and inventory.

Aggregate planning data

Period	1	2	3	4
Demand (units)	D_1	D_2	D_3	D_4
Cost of production ($/unit)	CP_1	CP_2	CP_3	CP_4

a) **(7 points)** Formulate the problem as an LP model. Assume that no shortages are allowed, and hence you plan to meet all the demand on time. Note that you need to define decision variables you use.

decision variables

P_t . production quantity at time t

$t = 1, 2, 3, 4$

+2

I_t – inventory at the end of period t, $t=1,..,4$

+1 $\min \sum_{t=1}^{4} [CP_t \cdot P_t + h I_t]$

s. to $I_t = I_{t-1} + P_t - D_t$ $t=1,..,4$

+2 $I_t \le I_{max}$ $t=1,..,4$ (no problem even if $I_0 \le I_{max}$ – all given)

+2 $P_t, I_t \ge 0$ $t=1,..,4$

each redundant equation –1

b) **(7 points)** Now consider the situation where you are allowed to have backorders in periods 2 and 3, meaning that you can delay the satisfaction of demand in periods 2 and, but all backordered demand should be satisfied latest at the end of period 4. The following data summarizes the cost of backorder per unit per period for each period.

Period	1	2	3	4
Cost of backorder ($/unit/period)	Not allowed	CB_2	CB_3	Not allowed

How is your formulation going to change? The problem is to determine the optimal amount to produce in each month so that the total cost of production, inventory and backorders is minimized. You need to define new decision variables, as well as use some of the previous.

+1 Define $I_t = I_t^+ - I_t^-$ $\quad I_t^+, I_t^- \geq 0$

I_t^- stands for backorders

$I_1^- = I_4^- = 0$

+3 O.F. $\sum_{t=1}^{4} [CP_t P_t + h I_t^+ + CB_t I_t^-]$ $\quad CB_1 = CB_4 =$ arbitrary # (or no need to write)

+3 $I_t^+ - I_t^- = I_{t-1}^+ - I_{t-1}^- + P_t - D_t$

$I_t^+, I_t^- \geq 0$

c) **This question has three independent parts:**

i. **(3 points)** When you solve the above problems, part a) and part b), let the optimal objective function value you have obtained is OF_a and OF_b, respectively. Write an inequality describing their relation.

$OF_a \geq OF_b$

ii. **(3 points)** What if you noticed that the inventory limitation described in the problem is $2I_{max}$ value rather than I_{max}, i.e. two times greater than what you thought it was. When you solve the problem again according to this change (let us say part **a**), are you going to obtain a lower, equal, or higher objective function value?

$OF(I_{max}) \geq OF(2I_{max})$

iii. **(5 points)** At optimality, you observe that the dual variable corresponding to the constraint regarding the inventory limitation I_{max} attains a value, V. How would you interpret the meaning if $V = 0$? If $V > 0$?

+2 $V = 0 \Rightarrow$ constraint is not binding

+3 $V > 0 \Rightarrow$ constraint is binding, for that period — would improve the OF (i.e. a smaller OF value) if constraint RHS is increased.

increased -1

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Question 4 (20 points)
A company faces a fixed rate of demand for a particular item with $\lambda = 600$ units per year. The company can either produce or outsource the item. If the company decides to produce the item, a fixed cost of \$8000 must be paid to setup the current machine and each item costs \$5. If the company decides to outsource the item, a fixed ordering cost of \$$A$ must be paid per order. However, the unit cost is \$6. Backorders and/or lost sales are not allowed. Holding cost computations are based on I=20%annual interest rate, i.e., $h = Ic$; where c is the unit cost. Hence, h will change if the item is manufactured or outsourced.

a) **(10 points)** Step by step describe how you would answer the following question: Should the company produce or outsource? At each step, if you are using a formula, write the formula and explain explicitly what the value of each term in the formula is. DO NOT COMPUTE.

b) **(10 points)** Give a formula for A (fixed ordering cost, if outsourced) so that company will be indifferent to produce or outsource.

a) $\lambda = 600$ units/year

→ produce → $K_p = \$8000$/setup, $C_p = \$5$/unit, $h_p = 5 \times I$

→ outsource → A/order, $C_o = \$6$/unit, $h_o = 6 \times I$

backorders not allowed.

Produce

1. Compute EOQ

$EOQ_p = \sqrt{\dfrac{2 K_p \lambda}{C_p \cdot I}}$

2. Compute Averaged Total Cost

$ACT_p(EOQ_p) = \sqrt{2 K_p \lambda C_p I} + \lambda C_p$

Outsource

1. Compute EOQ

$EOQ_o = \sqrt{\dfrac{2 A \lambda}{C_o \cdot I}}$

2. Compute Aver. T. Cost

$ACT_o(EOQ_o) = \sqrt{2 A \lambda C_o I} + \lambda C_o$

3) Select the min one, i.e. $\min\{ACT_p, ACT_o\}$

b)

$$ACT_p = ACT_o$$

$$\sqrt{2K_p \lambda C_p I} + \lambda \underset{5}{c_p} = \sqrt{2A\lambda C_o I} + \lambda \underset{6}{C_o}$$

$$\sqrt{2 \times 8000 \times 600 \times 5 \times 0.20} + 600(5-6)$$
$$= \sqrt{2 \times 600 \times 6 \times 0.20}\sqrt{A}$$

$\times 10$

$= \dots$

-3 for not having first less x's.

<u>or formula</u>

$$\frac{\sqrt{2K_p \lambda C_p I} - \lambda}{\sqrt{2\lambda C_o I}} = \sqrt{A_t} \qquad (*)$$

If $\sqrt{2K_p \lambda C_p I} \le \lambda$ always produce.

o.w. you can always find such an A.

<u>our case</u> $\sqrt{1200 \times 8000} \ge 600$

hence $(*)$ gives the desired A.

for any $A > A_t$ — produce
$A \le A_t$ — outsource.