Luxe Glasses Part II Solution key

List of Equations used

A. EOQ = $\sqrt{\frac{2K\lambda}{h}}$

B. T = Q/λ

C. R = λ ∗ τ (where τ is the lead time in years)

D. Fixed ordering cost = $\frac{K\lambda}{Q}$

E. Purchase cost = λ ∗ c

F. Holding cost = $\frac{h*Q}{2}$

G. Total cost = $\frac{K\lambda}{Q} + \lambda * c + \frac{h*Q}{2}$

H. Take four weeks a month and 12 weeks. Hence, lead-time demand x ~ N(τμ, τσ^2) where τ, μ and σ^2 are all specified in terms of weeks.

I. 1-F(R) = $\frac{Qh}{p\lambda}$

J. Q = $\sqrt{\frac{2\lambda[K+pn(R)]}{h}}$

K. Expected annual holding cost = $h(\frac{Q}{2} + R - \lambda\ \tau)$

L. Expected annual setup cost = $\frac{K\lambda}{Q}$

M. Expected annual shortage cost = $\frac{p\lambda n(R)}{Q}$

N. Expected total cost = G (Q, R) = $h(\frac{Q}{2} + R - \lambda\ t) + \frac{K\lambda}{Q} + \frac{p\lambda n(R)}{Q}$

O. Probability of Stock-out in a cycle = 1-F(R) = α

P. *For Shortest Path:* D_{ij} = cost of satisfying the demand of all periods through *j*, including *j* with a setup in *i*. Hence, include the setup cost and associated holding costs; *i*=0, 1, 11, *j*>*i* for *j*=1, 2, ,12

Q. c_u = p – c where p is sales price and c is unit cost

R. c_o = c – s where c is unit cost and s is salvage value (salvage value is a negative cost)

S. c_o = c +12h – s where c is unit cost, h is unit cost of carrying per month and s is salvage value (salvage value is a negative cost)

T. c_u = p – c – 12h where p is sales price, h is unit cost of carrying per month and c is unit cost

U. c_o = c +12h – s where c is unit cost, h is unit cost of carrying per month and s is salvage value (salvage value is a negative cost)

V. F(z*) = $\frac{c_u}{c_u + c_o}$

W. Q = z* σ + μ (where z is the standardized variate of F(y*), σ is the standard deviation of total annual demand and μ is the expected value of total annual demand)

X. Expected number of excess demands = L(z*) * σ (where σ is the standard deviation of total annual demand)

Y. Expected number of units sold = Expected number of yearly demand- expected number of excess demand

Z. Expected number of units not sold = Ordering Quantity – Expected number of units sold

Regular Glasses

A. Optimal ordering quantity, cycle time and reorder point are calculated using Equations A,B and C respectively.

B. Annual ordering, purchase, holding costs and total cost are calculated using Equations D, E, F and G respectively.

C. Optimal ordering quantity is (supposed to be – check the numbers) larger than the capacity constraint and hence it is not implementable in the presence of ordering capacity constraint. Since the cost function is Equation G is a convex function, the optimal ordering quantity with ordering capacity constraint is 1500 units. Cycle length is calculated using Equation B and the annual ordering, purchase, holding costs and total cost are calculated using Equations D, E, F and G respectively.

Comments: In parts A), B) and C) no consideration is given for possible backorders. Hence the cost computed is possibly a lower bound.

D. Use H to find lead time demand distribution. Optimal R and Q values are calculated by solving Equations I and J together. You need to use the iterative algorithm. Stop the computations when two consecutive Q values are not different if expressed without any digits after the decimal point (with rounding). Expected annual holding cost, ordering cost, penalty cost and total costs are calculated using Equations K, L, M and N, respectively.

Comments: Better representation that parts A), B) and C) as consideration is given for possible backorders. Note that the validity of the solution (approximation as described in the textbook) depends on the penalty cost being considerably larger than the holding cost. Check how different is the Q found compared to EOQ as an exercise.

E. Use H to find the new lead time demand distribution. Optimal R and Q values are calculated by solving Equations I and J together, iteratively. Stop the computations when two consecutive Q values are not different if expressed without any digits after the decimal point (with rounding). Expected annual holding cost, ordering cost, penalty cost and total costs are calculated using Equations K, L, M and N, respectively.

Compare expected total annual costs. Select the one which is better.

Comments/Extra: It is not sufficient as it depends when the actual payment take place. Depending on who is the owner during the lead time, an additional cost of inventory carrying should be considered for items in transit.

Sunglasses

A. First Part – Safety Stock computation
First Step: Use Equation H to compute the mean and variance of demand during response time. Then use Equation O to compute R. Then set $SS = R- \lambda(\tau + one\ month)$.
Second Step: Notice that acquiring the SS quantity in the beginning is sufficient. So, you can add that to the demand of the first period or exclude it from the computation as it will not change the decisions. If you are using standard lot-sizing formulation and solve it with a solver, you can say that ending inventories each period should be at least SS.

Second Part: Solution of the Dynamic Demand Part:

Solution 1: Use shortest path model

Sets:

N = Set of nodes
A = Set of arcs

Parameters:

D_{ij} = cost of using arc (i,j)

Decision Variables:

$x_{ij} : \begin{cases} 1, if\ arc\ (i,j)\ is\ used \\ 0, otherwise \end{cases}$ for all $(i,j) \in A$

z: total cost

Model for Shortest Path

Minimize z

Subject to

$z = \sum_{(i,j) \in A} D_{ij} * x_{ij}$

$$\sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(j,i) \in A} x_{ij} = \begin{cases} 1, if\ i = 0 \\ -1, if\ i = 12 \\ 0, otherwise \end{cases}$$

Summarize results: (Can be with SS or not)

Months ordered:

Total Setup plus Holding Costs:

Comments: A heuristic approach to handle multi-item, multi-period stochastic dynamic demand problem. This problem is usually implemented with the rolling horizon concept and of course one can allow for SS adjustment depending on the information obtained by the forecasting method.

B. How to implement this case as a newsvendor?

Calculate Demand Distribution: Sum of normally distribute demands – Sum is also normal with mean as sum of the means, and variance as the sum of variances.

Calculation of critical Ratio: There are possibilities for this case:

i) Disregard the inventory carrying cost within the months. Critical ratio is calculated using Equations Q and R. and P.
ii) Consider the inventory carrying cost for those items which are left at the end of the season only – Use Equations Q and S.
iii) Consider the inventory carrying cost assuming that on the average demand comes uniformly within the period. Note that in this case you directly add the 12 month holding cost to the unit cost (and there are some deductions which are not a function of quantity, and hence are not considered). Use Equations T and U.

Calculation of the Optimal ordering: Use Equation V, and Equation W.

Comment: Any solution is reasonable. More will be discussed in comments.

C. Expected number of excess demands, expected number of units sold and expected number of units not sold are calculated using Equations X, Y and Z, respectively.