KONU/SUBJECT: TARİH/DATE:

Recitation 3: (Recitation 4 in Fall 2019)

1) X: Demand, r.v. $X \sim Uniform(0, 1000)$

c: purchase cost = \$12.05

s: selling price = \$32

p: shortage cost = \$14

v: salvage ~~cost~~ price = \$11

a) $Q^* = ?$

Remember that;

- pdf of uniform dist: $f(x) = \frac{1}{b-a}$ for $a \le x \le b$
  $X \sim U(a,b)$

$\Rightarrow f(x) = \frac{1}{1000}$

- cdf of uniform dist: $F(t) = \int_a^t \frac{1}{b-a} \cdot dx$ ($\Uparrow P(X \le t)$)

$$= \frac{t-a}{b-a}$$

$F(Q^*) = \frac{C_u}{C_u + C_o}$ where C_u: is cost of underage.

C_o: cost of overage.

$C_u = s - c + p = 32 - 12.05 + 14$
$= 33.95$

$C_o = h + c$? (holding, purchase)

$C_o = \overset{c-v}{\cancel{\quad}} = 12.05 - 11 = 1.05$

$F(Q^*) = \frac{33.95}{35} = 0.97$

$\frac{Q^* - 0}{1000} = 0.97 \Rightarrow Q^* = 970$ units.

what if $C_o > C_u$?

b) Minimum order quantity = 1000 units.

⇒ Order 1000 then salvage remaining flash drives (Expected ~~shortage~~ underage cost would be higher than expected overage cost)

c) Let $Q=600$ Expected gain from salvages?

If $Q \le x$ then there will be no salvage and the cost is 0.

$$v\,\mathbb{E}[\underbrace{(Q-x)^+}_{+\text{ sign means}}] = v\int_a^Q (Q-x)\cdot f(x)\,dx$$

$$(Q-x)^+ = \begin{cases} Q-x & \text{if } Q \ge x \\ 0 & \text{if } Q \le x \end{cases} \quad \max\{(Q-x), 0\}$$

$$= 11\int_0^{600} (600-x)\,\frac{1}{1000}\,dx = \$1980.$$

d) Let $Q=600$. Expected cost from shortages?

If $Q \ge x$, then there is no shortage cost will be 0.

$$\mathbb{E}[(X-Q)^+] = \int_Q^b p\,(x-Q)\cdot f(x)\,dx$$

$$= \int_{600}^{1000} 14\,(x-600)\,\frac{1}{1000}\,dx$$

$$= \$7280$$

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Q2) Exercise set III b), Question 3)

14)

(Q, R) inventory policy.

monthly demand $= D \sim N(28, 8)$

Replenishment lead time $= \tau = 14$ weeks $= \frac{14}{4}$ month

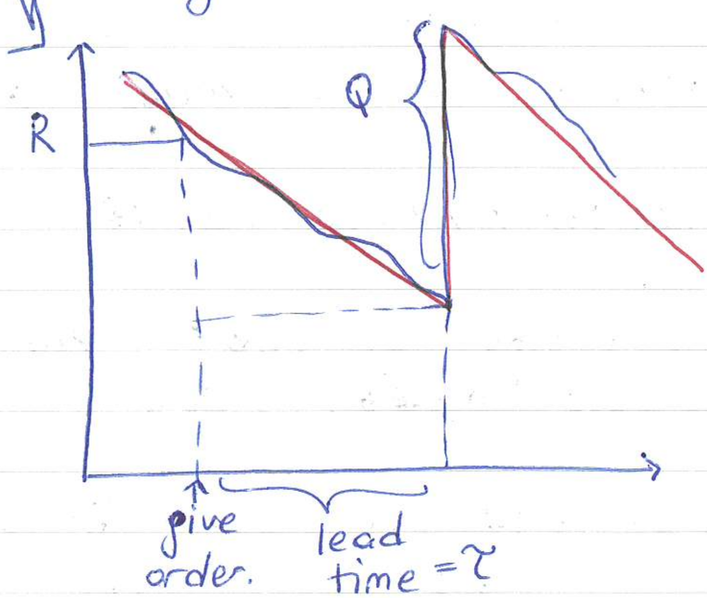
unit procurement cost $= c = \$6$.

loss of goodwill $u = p = \$10$.

Fixed cost $= K = \$15$ per order.

I = interest rate $= \%30 = 0.3$ (annual interest)

holding cost $= h = I.c = (0.3)6 = \$1.8$



λ (annual demand) $= 28 \times 12 = 336$

Demand during lead time: $\mu = \frac{14}{4} \cdot 28 = 98$ (mean)

$\sigma = 8 \cdot \sqrt{\frac{14}{4}} = 14.96$ (std dev)

$n(R)$: expected # of stock outs.

$n(R) = \sigma \cdot L\left(\frac{R-\mu}{\sigma}\right)$

Interpolation:

If you cannot find ~~exact~~ exact $F(z)$ value on table

$x_1 = F(z_1)$ $\quad$ $x_2 = F(z_2)$ $\quad$ $x_3 = F(z_3)$

$\Downarrow$ $\quad$ $\Downarrow$ $\quad$ $\Downarrow$

z_1 $\quad$ $z_2 = ?$ $\quad$ z_3

$$\frac{x_2 - x_1}{x_3 - x_1} = \frac{z_2 - z_1}{z_3 - z_1}$$

$n(R))$

$$1 - F(R) = \frac{Qh}{p \cdot \lambda}$$

$$EOQ = \sqrt{\frac{2k\lambda}{h}} = \sqrt{\frac{2 \times 15 \times 336}{1.8}} = 74.83$$

$$1 - F(R_0) = \frac{Q_0 \cdot h}{p\lambda} = \frac{74.83 \times 1.8}{10 \times 336} = 0.04$$

interpolation hatırlat $\Leftrightarrow$ By table, A4, $z = 1.75 \Rightarrow 1 - F(z) = 0.04$ for standart normal dist.

NOTE: Use normal distribution table ~~only~~ if the demand is normally distributed.

$$R_0 = \sigma z + \mu = (14.96)(1.75) + 98 = 124.18$$

$L(z) = L(1.75) = 0.0162$ by table A4

$n(R) = \sigma L(z) = (14.86)(0.0162) = 0.242$

Step 1: $Q_1 = \sqrt{\dfrac{2 \times 336\,(15 + 10 \times 0.242)}{1.8}} = 80.64$ (not close to Q_0)

$$1 - F(R_1) = \frac{Q_1 h}{p\lambda} = \frac{80.64 \times 1.8}{10 \times 336} = 0.0432$$

By table A4; $z = 1.71 \Rightarrow R = (14.86)(1.71) + 88$

$= 123.58$

$L(z) = 0.0178$

$n(R) = (14.86)(0.0178) = 0.266$

Step 2: $Q_2 = \sqrt{\dfrac{2 \times 336\,(15 + 10 \times 0.266)}{1.8}}$

$= 81.197$ (close to Q_1).

$1 - F(R_2) = \dfrac{Q_2 h}{p\lambda} = 0.0434$

By table A4 $z = \underbrace{1.71}_{\text{same w/ step 1.}} \Rightarrow R = 123.58$

$L(z) = 0.0178$ stop.

$Q^* = Q_2 = 81.197$

$R^* = R_2 = 123.58$

b) optimal safety stock $\Rightarrow R - \mu = 123 - 98 = 25.$